

# DETERMINING THE STEP-CHANGE CONDUCTIVITY PROFILES WITHIN LAYERED METAL STRUCTURES USING INDUCTANCE SPECTROSCOPY

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*Abstract – This paper presents an inverse method for determining the conductivity distribution of a flat, layered conductor using a multi-frequency electromagnetic sensor based on phase signature alone. Eddy current sensors are used in a wide range of non-destructive testing (NDT) applications. Single frequency sensors are very common, however, the potential of an eddy current sensor with spectroscopic techniques offer the ability to extract depth profiles and examine more fully the internal structure of the test piece. In this paper, we found a simplified model that can estimate the phase signature of a cylindrical coil above a conductor with an arbitrary conductivity profile. This simplified model improves the computational efficiency by many fold compared to the complete analytic solution. For inverse solution, a simplex search method was used to fit a set of multi-frequency phase values in a least-squared sense. Experimental eddy-current tests are performed by taking the difference in inductance of the coil when placed in free space and next to a layered conductor over the range 100Hz -1MHz. Good estimates for the conductivity profile from experimental and simulated data were obtained.*

**Index terms:** Multi-frequency electromagnetic sensor, simplified model, conductivity distribution, layered conductor

## I. INTRODUCTION

Determining the conductivity profile of a conductor is important in a range of technological applications such as coating, surface treatment and quality inspection. One approach is to measure the direct current (DC) impedance map and estimate the conductivity [1-2]. Another approach is eddy current inspection, which infers the conductivity profile from impedance (or



*A. The complete forward model*

The analytical solution for the inductance of a right-cylindrical air-cored coil placed above a finite number of layers with constant conductivity and permeability in each layer has been given by Cheng [13]. In this paper, only non-magnetic layered conductors will be dealt with, therefore, the relative permeability for all layers are assumed to be that of the free space, i.e. 1. Figure 1 shows the schematic diagram of the model. The base of the coil is at a height of  $h_1$  above the surface and the top of the coil is at  $h_2$ . The coil parameters of importance are number of turns  $N$ , inner and outer radii  $r_1$  and  $r_2$  and coil length  $L = h_2 - h_1$ . Note that the space below layer 1 is free space. In the forward model, Layer 0 is treated as a layer with infinite thickness. In practice, the inductance change induced by the layered conductor (1a) is compared with free space (1b). This arrangement reduces common measurement errors and facilitates subsequent reconstruction algorithm.

Cheng [13] considered a similar geometry without subtracting the coil impedance in free space. He first determined the impedance of a single turn delta-function filament by solving corresponding differential equations, and then derived the impedance for a right-cylindrical air-cored coil by superposition, assuming that the current density is uniform over the cross-section of the coil. In this paper, the results of Cheng are presented in a slightly different form to express the coil inductance difference for cases (1) the layered conductor and (2) free space.

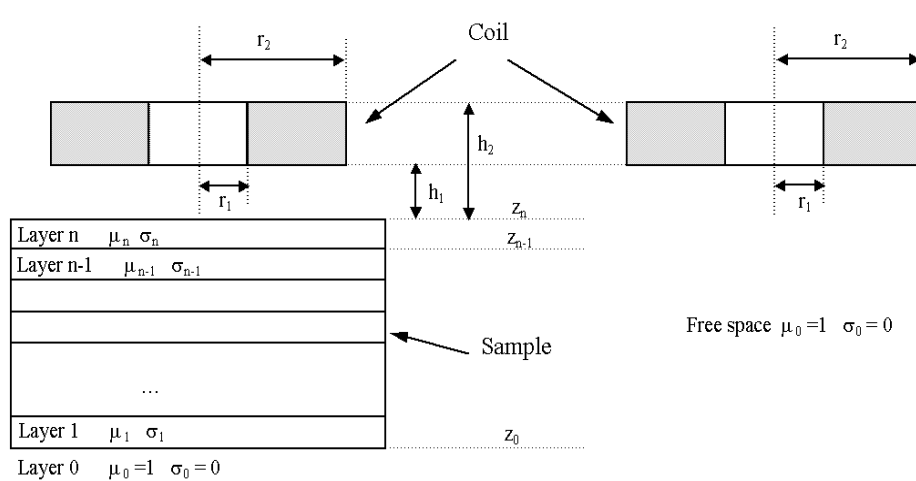


Figure 1. Schematic diagram of the model (a) the sensor placed next to the layered conductor (b) the sensor placed in free space



phase response, only the latter of which is believed to be strongly dependent on the conductivity distribution for a given coil.

To identify the simplified model, firstly, two limiting cases  $\omega=0$  and  $\infty$  are considered. Setting  $\omega=0$  gives the inductance change for zero-frequency. The real part and imaginary part of the inductance change are zero, which means the non-magnetic conductor causes no inductance change and the magnetic flux penetrates the plate as in free space. In the limit of arbitrarily large frequency, the inductance change is given by  $\Delta L = -\Delta L_0$ , where  $\Delta L_0 = K \int \frac{P^2(\alpha)}{\alpha^6} A(\alpha) d\alpha$ .  $\Delta L_0$  is dependent on lift-off for a given coil, but is independent of the conductivity distribution, which corresponds to the situation that the incident magnetic flux is totally excluded from the plate.

The simplification of the complete model is to evaluate the phase term  $\phi(\alpha)$  at  $\alpha_0$  and take it outside of the integral.

$$\Delta L(\omega) = \phi(\alpha_0) \Delta L_0 \quad (9)$$

This operation originates from the fact that  $\phi(\alpha)$  varies slowly with  $\alpha$  compared to the rest of the integrand, which reaches its maximum at a characteristic spatial frequency  $\alpha_0$ .  $\alpha_0$  is defined to be one over the smallest dimension of the coil.

Note that the phase term  $\phi(\alpha_0)$  solely depends on the conductivity profile of the conductor, and totally accounts for the frequency-dependent phase signature.  $\Delta L_0$  contributes to the strength of the signal, but is not related to the phase components. Therefore,  $\phi(\alpha_0)$  can be used to approximate the phase signature of a coil instead of equations (1-8).

### *C. The inverse problem*

The inverse problem in this case is to determine the conductive profile from the frequency-dependent phase measurements (phase signature). A simplex search method is used to find the conductivity profile to fit phase values (measured or simulated) in a least-squared sense.



Table 1. Coil geometry and model parameters

Figure 2 shows the differential inductance (subtracted from free space) of a layered conductor whose conductivity profile is shown in figure 3. The difference between the calculated data and the measured data is due to imperfect modelling of the coil, one factor being the undesirable capacitance of the coil.

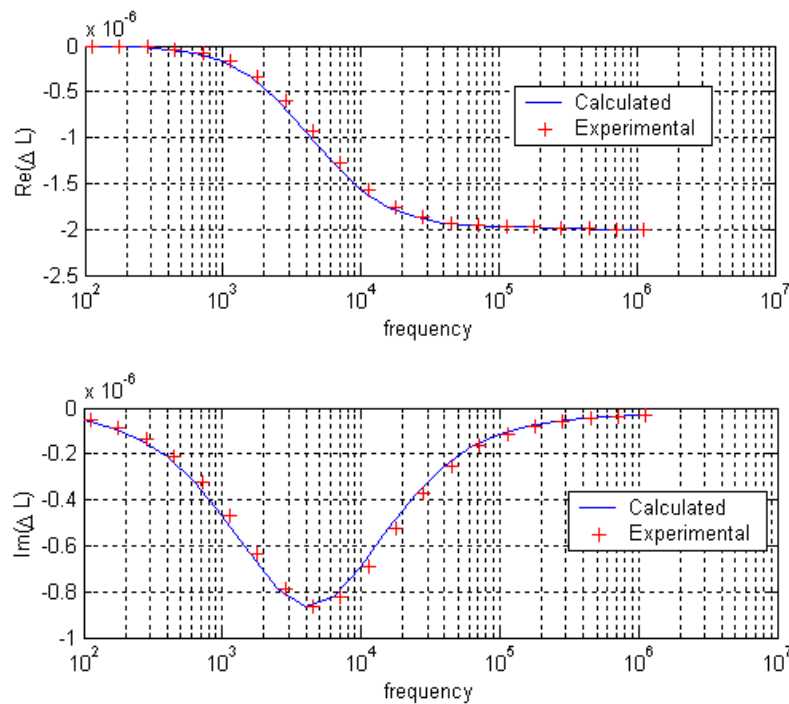


Figure 2. The real and imaginary parts of the inductance change for the sample whose conductivity profile is shown in Figure 4 (= layered – free space)





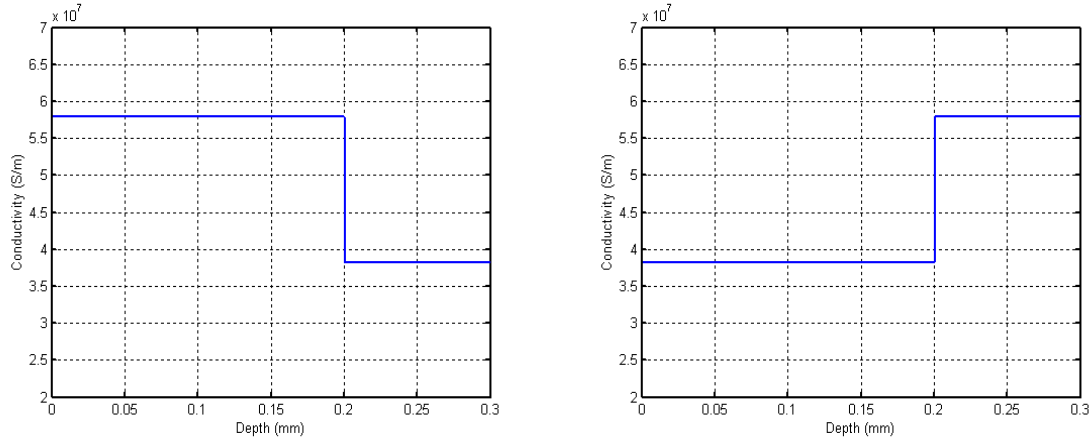


Fig. 4. Simulated conductivity profiles

First, the boundary coordinates are determined based on known conductivities. The results are shown in Table 2.

Table 2. Estimated and actual layer coordinates

	Profile 1		Profile 2		Profile 3		Profile 4	
	Actual	Estimated	Actual	Estimated	Actual	Estimated	Actual	Estimated
z1	0.1mm	0.095mm	0.1mm	0.097	-		-	
z2	0.2mm	0.208mm	0.2mm	0.209	0.2mm	0.214	0.2mm	0.206
z3	0.3mm	0.312mm	0.3mm	0.315	0.3mm	0.320	0.3mm	0.319

Second, the conductivities are determined based on known boundary coordinates. The results are shown in Table 3.

Table 3. Estimated and actual conductivities

	Profile 1		Profile 2		Profile 3		Profile 4	
	Actual (10 <sup>7</sup> S/m)	Estimated (10 <sup>7</sup> S/m)	Actual (10 <sup>7</sup> S/m)	Estimated (10 <sup>7</sup> S/m)	Actual (10 <sup>7</sup> S/m)	Estimated (10 <sup>7</sup> S/m)	Actual (10 <sup>7</sup> S/m)	Estimated (10 <sup>7</sup> S/m)
σ <sub>1</sub>	3.8	3.74	5.8	5.73	5.8	5.76	3.8	3.82
σ <sub>2</sub>	4.8	5.62	3.8	4.01	5.8	5.69	3.8	3.90
σ <sub>3</sub>	5.8	4.87	4.8	4.67	3.8	3.91	5.8	5.68

The estimated conductivities and boundary coordinates agree reasonably well with the actual ones for all cases. This confirms the potential of this method of being able to reconstruct more complex conductivity profiles than monotonic ones.



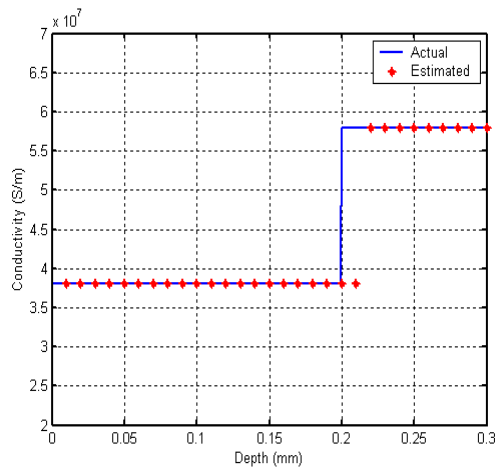


Figure 5. Actual and reconstructed profiles by measurements

#### IV. CONCLUSIONS

In this paper, a method is presented which has the potential to reconstruct complex step-change conductivity profile for a flat non-magnetic conductor from inductance spectroscopic measurements. A simplified model was found by approximating the complete forward analytic solution. In inverse solution, a simplex search method was used to find the conductivity profile to fit phase signatures (measured or simulated) in a least-squared sense. Conductivity profiles have been reconstructed from simulated and measured data, which verified this method.

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