SIMULTANEOUS PERIODIC OUTPUT FEEDBACK CONTROL FOR PIEZOELECTRIC ACTUATED STRUCTURES USING INTERVAL METHODS

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Abstract- In this paper, the problem of modeling, output feedback control design and the experimental implementation for the vibration control of smart cantilever beam with parameter uncertainties represented in interval form is addressed. The interval model of the system is obtained by introducing variation in the parameter of the identified model. However, Uncertainties are assumed in the model, identified through on line recursive least square parameter estimation. The control and identification process is done by using Simulink modeling software and dSPACE DS 1104 controller board. The output feedback controller design for the interval model is carried out through simultaneous output feedback controller design methodology by considering lower, nominal and upper bound models. The controllers designed are periodic output feedback is experimentally evaluated for their performance in suppressing the first vibration mode.

Index terms: Interval analysis, output feedback, piezoelectric, smart structure

I. INTRODUCTION

The objective of active vibration control is to reduce the vibration of a system by automatic modification of the systems structural response. An active vibration control system can take many forms but the important components of any such system are a sensor, a controller and an actuator. Active control of vibration in flexible structure through smart structure/intelligent structure concept is a new and developing area of research in et al. [1]. System identification is an established modeling tool in engineering and numerous successful applications in [2-3]. Many real systems are difficult to model and control due to incomplete knowledge of the system behavior and the availability of only short-term or unreliable data records. Hence, the uncertainty, which results from both the measurement and modeling process, should be taken into account when making predictions or designing systems. This kind of perturbations can introduce uncertainties in an identified model of piezoelectric bonded structures. The limitations of uncertainty analysis utilizing probabilistic and stochastic models can be seen in [4]. Interval analysis is a new and growing branch of applied mathematics in [5-6]. Interval arithmetic considers the uncertainty of all the parameters, treating them as interval numbers whose range contains the uncertainties in those parameters. The resulting computations calculated using interval arithmetic carry the uncertainties associated with the data throughout the analysis in [7]. Intelligent structural elements covered by piezoelectric materials for high-polymer is presented by [8].

This paper is organized as follows: Section II gives a brief review of interval arithmetic. The experimental set up used for identification and control is presented in section III. A brief review of a simultaneous periodic output sampling feedback control is given in section IV. Section V gives the controller design and experimental evaluation of a simultaneous periodic output sampling feedback control. The results and discussion are presented in section VI. Conclusions are drawn in section VII.

II.REVIEW OF INTERVAL ARITHMETIC

Interval arithmetic has been proposed and used as a potential tool for modeling uncertainty in [9-11]. This form of mathematics uses interval "number" which is actually an ordered pair of real numbers representing the lower and upper of the parameter range. Interval arithmetic is built upon a basic set of axioms. If we have two interval numbers $s = [a, b]$ and T = [c, d] with $a \leq b$ and $c \leq d$ then:

$$S+T = [a, b] + [c, d] = [a + c, b + d]$$

$S - T = [a, b] + (-[c, d]) = [a - d, b - c]$

$ST = [\min \{ac, ad, bc, bd\}, \max \{ac, ad, bc, bd\}]$

$1/T = [1/b, 1/a], 0 \notin [a, b]$

$S/T = [a, b]/ [c, d] = [a, b]*[1/d, 1/c], 0 \notin [c, d]$

Using the above representation, interval arithmetic comes with free error estimation. The width of the interval can be interpreted as the possible deviation of the numerical representation from a real number. This inclusion of the real or mathematical results is even inherited in proper arithmetic operations or functions i.e. as long as one applies well-defined, interval – specific operations, the computational results is guaranteed to enclosed the correct mathematical result. Due to this characteristic, interval arithmetic is tailor-made for applications where results are mission-critical. Furthermore, recent advances prove that interval techniques are more appropriate in specific applications like solving non-linear problems previously thought to be impossible to solve with floating-points techniques. Other fields where intervals proved that it's superiorly are global optimization and solving ordinary differential equation problems.

III. EXPERIMENTAL SET-UP AND ITS MODEL

A flexible aluminum beam with clamped end as shown in Figure 1 is considered in this paper. Two piezo ceramic patches are surface bonded at a distance of 10mm from the fixed end of the beam. The patch bonded on the bottom surface acts as a sensor and the one on the top surface acts as an actuator. To apply an excitation input to the structure another piezo ceramic patch is bonded on the top surface at a distance of 387.8mm from the fixed end. The dimensions and properties of the beam and piezo ceramic patches are given in table 1 and table 2.

The sensor output is given to the piezo sensing system which consists of charge to voltage converting amplifier. The conditioned piezo sensor signal is given as analog input to dSPACE1104 controller board. The control algorithm is developed using simulink software and implemented in real time on dSPACE 1104 using RTW and dSPACE real time interface tools. The simulink software is used to build control block diagrams and real time workshop is used to generate C code from the simulink model. The C code is then converted to target specific code by real time interface and target language compiler supported by dSPACE1104. This code is then deployed on to the rapid prototype hardware system to run hardware in-the-loop simulation. The control signal generated from simulink is interfaced to piezo actuation system through configurable analog input/output unit of dSPACE 1104. The piezo actuation system drives the actuator and the excitation signal is applied from simulink environment through a DAC port of dSPACE system.

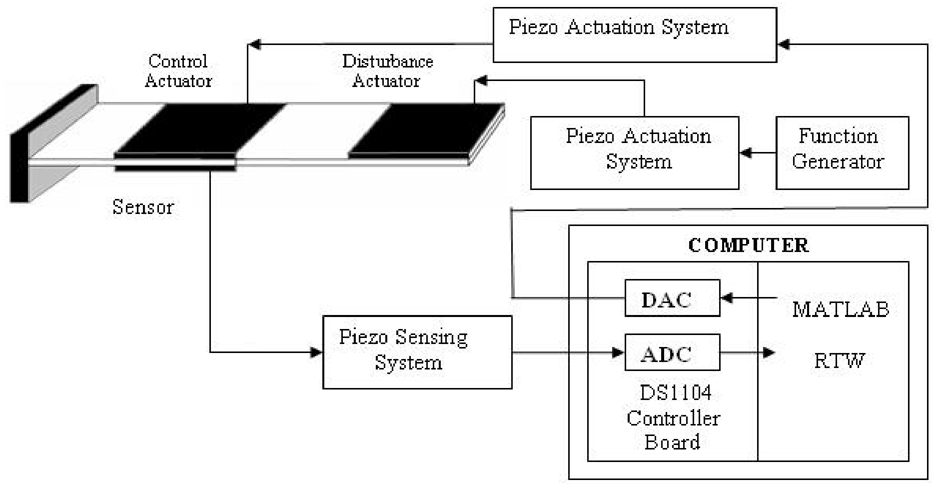


Figure.1. Schematic diagram of the experimental set up.

Table 1

Properties and dimensions of the Aluminium beam

Parameter	Numerical values
Length (m)	0.5205
Width (m)	0.013
Thickness (m)	0.002
Young's modulus (Gpa)	71
Density (kg/m^3)	2700
First natural frequency (Hz)	7.18

Table 2

Properties and dimensions of piezoceramic sensor/actuator

Parameter	Numerical Values
Length (m)	0.0762
Width (m)	0.0127
Thickness (m)	0.0005
Young's modulus (Gpa)	4762e9
Density (kg/m^3)	7500
Piezoelectric strain constant (m V^{-1})	-247x10^{-12}
Piezoelectric stress constant (V m N^{-1})	-9x10^{-3}

The model of a cantilever beam in figure 1 is obtained using recursive least square (RLS) method based on ARX model in [12]. The excitation signal, input signal and sensor output are given to MATLAB/simulink through ADC port of dSPACE 1104 system. The RLS algorithm is implemented by writing a C-file S-function used in MATLAB/simulink. The state space model derived from the identified second order ARX model parameters is

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}u + \mathbf{e}r\,; \quad y = \mathbf{c}^{T}\mathbf{x} \tag{1}$$

Where

$$\mathbf{A} = \begin{bmatrix} 92.81 & 103.79 \\ -103.15 & -93.42 \end{bmatrix}, \ \mathbf{b} = \begin{bmatrix} -0.27 \\ 0.25 \end{bmatrix}, \mathbf{e} = \begin{bmatrix} -0.0057 \\ 0.0053 \end{bmatrix}, \ \mathbf{c}^{T} = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

To include the variations in the system parameter $\pm 10\%$ variation is introduced in natural frequency. The state space models with these variations are identified as lower and upper bound system. The lower and upper bound system are

$$\dot{\mathbf{x}} = \mathbf{A}^{\mathbf{L}}\mathbf{x} + \mathbf{b}^{\mathbf{L}}u + \mathbf{e}^{\mathbf{L}}r\,; \quad y = (\mathbf{c}^{T})^{\mathbf{L}}\mathbf{x} \tag{2}$$

Where

$$\mathbf{A}^{\mathbf{L}} = \begin{bmatrix} 83.52 & 93.43 \\ -92.86 & -84.13 \end{bmatrix}, \ \mathbf{b}^{\mathbf{L}} = \begin{bmatrix} -0.27 \\ 0.25 \end{bmatrix}, \ \mathbf{e}^{\mathbf{L}} = \begin{bmatrix} -0.0057 \\ 0.0053 \end{bmatrix}, \ \mathbf{c}^{T} = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$\dot{\mathbf{x}} = \mathbf{A}^{\mathbf{U}}\mathbf{x} + \mathbf{b}^{\mathbf{U}}u + \mathbf{e}^{\mathbf{U}}r\,; \quad y = (\mathbf{c}^{T})^{\mathbf{U}}\mathbf{x} \tag{3}$$

Where

$$\mathbf{A}^{\mathbf{U}} = \begin{bmatrix} 103.14 & 115.29 \\ -114.59 & -103.75 \end{bmatrix}, \ \mathbf{b}^{\mathbf{U}} = \begin{bmatrix} -0.27 \\ 0.25 \end{bmatrix}, \ \mathbf{e}^{\mathbf{U}} = \begin{bmatrix} -0.0057 \\ 0.0053 \end{bmatrix}, \ \mathbf{c}^{T} = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

The lower and upper bound models in equation (2) and (3) are represented as an interval system using interval arithmetic in [6-7].The lower and upper bound systems are represented in interval form as follows:

$$\dot{\mathbf{x}} = \mathbf{A^I x} + \mathbf{b^I} u + \mathbf{e^I} r\,; \;\; y = (\mathbf{c}^T)^{\mathbf{I}} \mathbf{x} \tag{4}$$

Where

$$A^I = \begin{pmatrix} [83.52,103.14] & [93.43,115.29] \\ [-92.86,-114.59] & [-84.13,-103.75] \end{pmatrix} b^I = \begin{pmatrix} [-0.27,-0.27] \\ [0.25,0.25] \end{pmatrix} e^I = \begin{pmatrix} [-0.0057,-0.0057] \\ [0.0053,0.0053] \end{pmatrix} (c^T)^I = [1,0]$$

IV. REVIEW OF SIMULTANEOUS PERIODIC OUTPUT SAMPLING FEEDBACK CONTROL

Assume that output measurements are available from the system in equation (1) at time instants $t = k\tau$, where $k = 0,1,2.......$ Now design an output injection gain matrix $\mathbf{G}$, such that the eigen values of $\left(\Phi_\tau + \mathbf{G}\mathbf{c}^T\right)$ are inside the unit circle. For the system $(\mathbf{\Phi_\tau}, \mathbf{\Gamma}_\tau, \mathbf{c}^T)$ the control signal is generated according to

$$u(t) = K_l\, y(k\tau),\; k\tau + l\Delta \le t < k\tau + (l+1)\Delta,\; K_{l+N} = K_l \;, \tag{5}$$

for $l = 0,1,....N-1$, where a sampling interval τ is divided into N subintervals $\Delta = \dfrac{\tau}{N}$ and N is equal to or greater than the controllability index of $\left(\mathbf{\Phi_\tau}, \mathbf{\Gamma_\tau}\right)$. Note that the sequence of gain matrices $\{K_0, K_1, K_2, ...K_{N-1}\}$, when substituted into equation (5), it generates a time-varying, piecewise constant output feedback gain $K(t)$ for $0 \le t \le \tau$. To see the relationship between the gain sequence $\{K_l\}$ and closed loop behavior collect the gain matrices K_l into one matrix

$$\mathbf{K} = \begin{bmatrix} K_0 \\ K_1 \\ . \\ . \\ K_{N-1} \end{bmatrix}.$$

By applying a control input $u(t)$ as calculated in equation (5), a state space representation for the system sampled at rate $\frac{1}{\tau}$ is

$$\mathbf{x}(k+1) = \mathbf{\Phi}^{N}\mathbf{x}(k) + \mathbf{\Gamma}\ \mathbf{K}\ \mathbf{c}^{T}\mathbf{x}(k);\ y\ (k) = \mathbf{c}^{T}\mathbf{x}(k), \tag{6}$$

where

$$\mathbf{\Gamma} = \begin{bmatrix} \mathbf{\Phi}^{N-1}\Gamma & \mathbf{\Phi}^{N-2}\Gamma & \mathbf{\Phi}^{N-3}\Gamma & . & . & . & . & \Gamma \end{bmatrix}$$

If $(\mathbf{\Phi},\Gamma)$ system is controllable and $(\mathbf{\Phi}_\tau, \mathbf{c}^T)$ observable, one can first choose an output injection gain $\mathbf{G}$ to place the eigen values of $(\mathbf{\Phi}_\tau + \mathbf{G}\mathbf{c}^T)$ in the desired locations and then compute the gain sequence $\{K_l\}$ such that $\mathbf{\Gamma}\mathbf{K} = \mathbf{G}$ is satisfied. The problem with controllers obtained in this way is that, although they are stabilizing, they may cause an excessive oscillation between sampling instants. In [13-15], proposed the performance index so that $\mathbf{\Gamma}\mathbf{K} = \mathbf{G}$ need not be forced exactly. This constraint is replaced by a penalty function which makes it possible to enhance closed loop performance by allowing slight deviations from original design. The performance index is

$$J(k) = \sum_{l=0}^{\infty}\begin{bmatrix} x_l^T & u_l^T \end{bmatrix}\begin{bmatrix} \overline{\mathbf{Q}} & \mathbf{0} \\ \mathbf{0} & \mathbf{R} \end{bmatrix}\begin{bmatrix} x_l \\ u_l \end{bmatrix} + \sum_{k=1}^{\infty}(x_{kN} - x_{kN}^*)^T\,\overline{\mathbf{P}}\,\left(x_{kN} - x_{kN}^*\right) \tag{7}$$

where $\mathbf{R} \in \Re^{m\times m}, \overline{\mathbf{Q}}$ and $\overline{\mathbf{P}} \in \Re^{n\times n}$ positive definite and symmetric weight matrices. The first term represents averaged state and control energy and the second term penalizes deviation of the **G.**

V. PERIODIC OUTPUT FEEDBACK CONTROLLER DESIGN AND EXPERIMENTAL EVALUATION

A. Controller Design

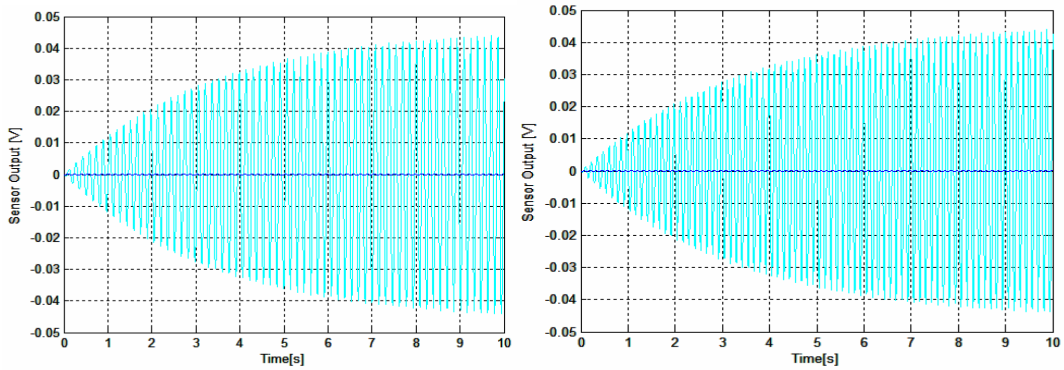
The periodic output feedback controller is designed to reduce the amplitude of vibration of a cantilever beam at resonance. A stabilizing output injection gain is designed for the system

$(\mathbf{\Phi}^{L}{}_{\tau}, \mathbf{\Gamma}^{L}{}_{\tau,}(\mathbf{c}^{T})^{L})$ and $(\mathbf{\Phi}^{U}{}_{\tau}, \mathbf{\Gamma}^{U}{}_{\tau,}(\mathbf{c}^{T})^{U})$, such that the eigen values of $(\mathbf{\Phi}_{\tau} + \mathbf{G}\mathbf{c}^{T})$ lie inside the unit circle.

The output injection gain obtained for lower and upper bound system are

$$\mathbf{G} = \left[-0.8488 \;\; 1.5374 \right] \quad G1 = [-1.0128 \;\; 1.6111]$$

The open loop response and response with output injection gain for lower and upper bound system obtained in simulation are shown in Figure 2.



(a) (b)

Figure. 2 Response to excitation at first natural frequency with output injection gain

(a) --- Uncontrolled and — controlled for lower bound system

(b)--- Uncontrolled and — controlled for upper bound system

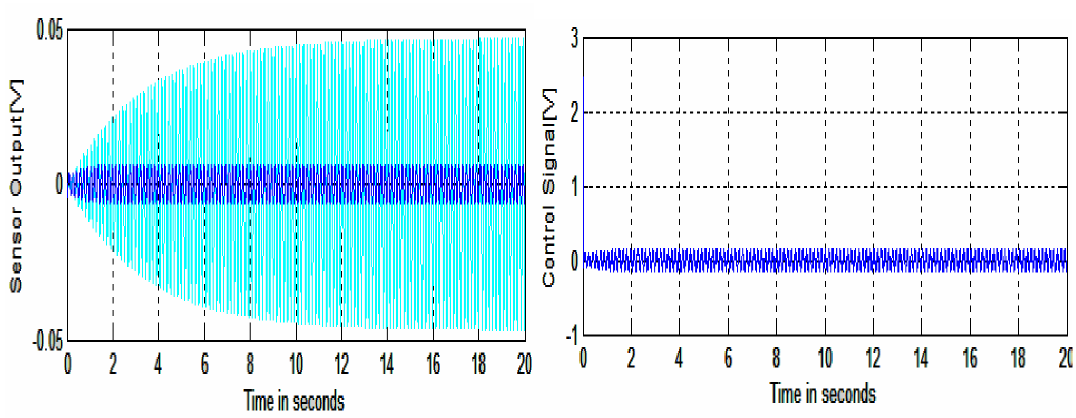
The controllability index of the system $(\mathbf{\Phi}_{\tau}, \mathbf{\Gamma}_{\tau,}\mathbf{c}^{T})$ is 2. Let $(\mathbf{\Phi}, \mathbf{\Gamma}, \mathbf{c}^{T})$ be the discrete time system obtained by sampling $(\mathbf{A}, \mathbf{b}, \mathbf{c}^{T})$ at a rate $\frac{1}{\Delta}$ where $\Delta = \frac{\tau}{N}$, number of subinterval N is chosen as 4. The periodic output feedback gain is obtained by solving $\mathbf{\Gamma K} = \mathbf{G}$ subject to that they minimize the performance index equation (7), so that amplitude of control signal required can be reduced.

Periodic output feedback gain obtained as explained above with the following performance index weight matrices are:

$\mathbf{R}=1$, $\bar{\mathbf{Q}}=4000\mathrm{I}_4$, $\bar{\mathbf{P}} = 1000\mathrm{I}_4$.

$$\mathbf{K} = \left[-20.3905 \;\; -14.6005 \;\; -9.6987 \;\; -5.7610 \right]^{T}$$

The open loop response, response with periodic output feedback gain and the control signal for Nominal, Lower and Upper bound are shown in figure 3, 4 and 5

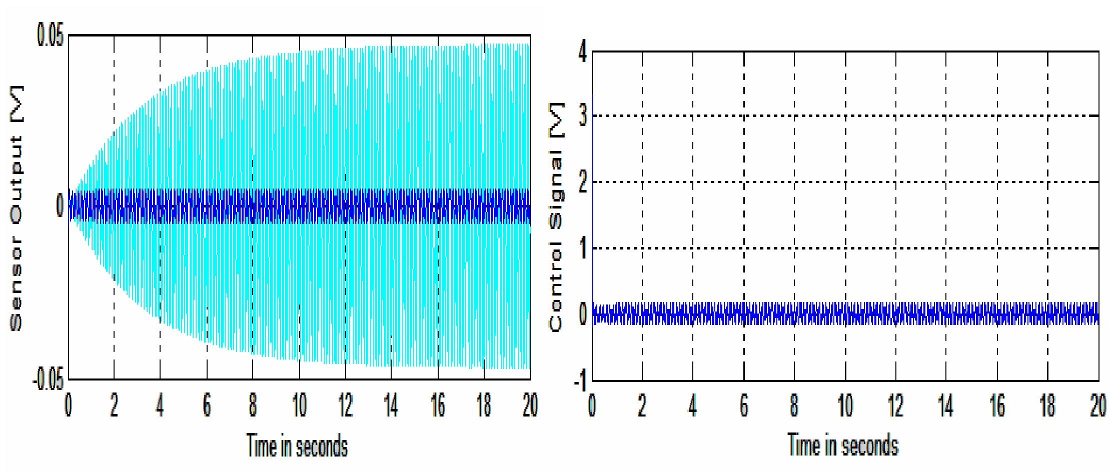


(a) **(b)**

Figure 3 Response to excitation at first natural frequency with simultaneous POF for lower bound system

(a) ---Uncontrolled and —— Controlled

(b) Control signal.

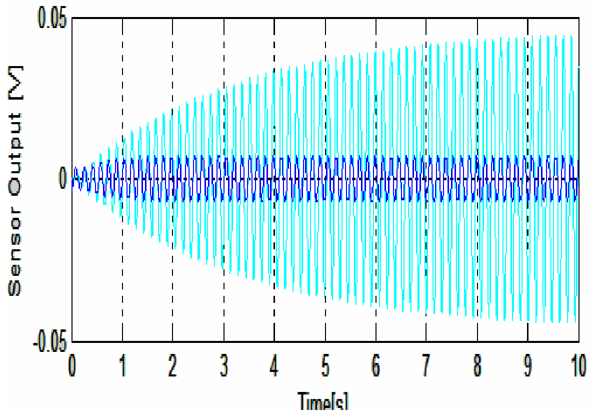


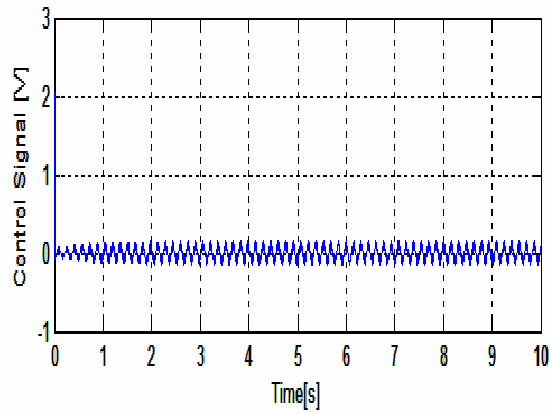
(a) **(b)**

Figure 4 Response to excitation at first natural frequency with simultaneous POF for upper bound system.:

(a) ---uncontrolled and —— controlled

(b) Control signal.





(a) (b)

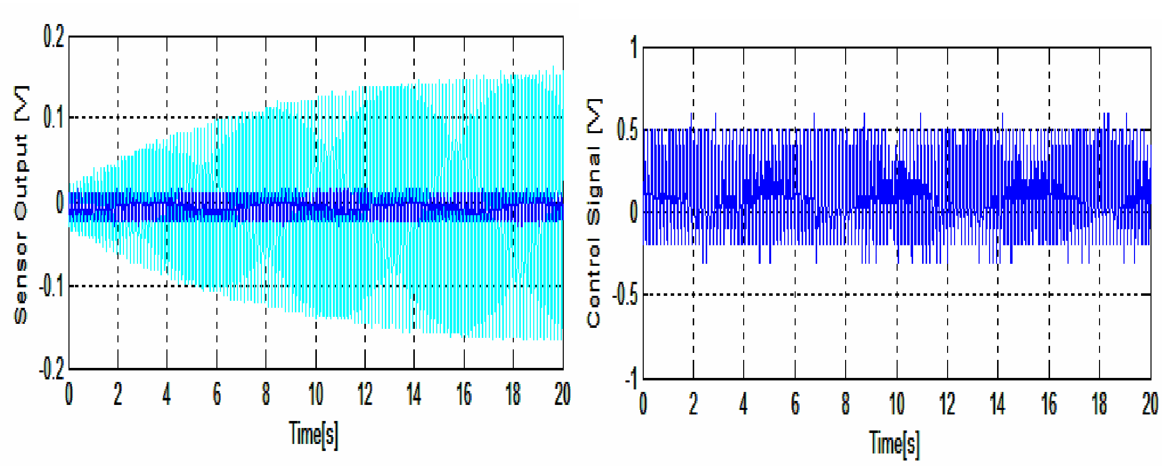
Figure 5 Response to excitation at first natural frequency with simultaneous POF for nominal system.:

(a)---uncontrolled and —— controlled

(b) Control signal.

B. Experimental Evaluation

The Periodic output feedback controller designed in section V(A) is applied for vibration suppression of smart structure. The sensor output was sampled at 0.0025 sec ($\Delta=\tau/N$) through ADC port of dSPACE and MATLAB/simulink. The control signal is applied to the control actuator at the sampling interval of 0.01 sec (τ) through DAC port of dSPACE system. The controller is implemented by developing a real time simulink model using MATLAB RTW in simulink. To demonstrate the robustness of the controller, the beam length was increased by 10mm and decreased by 5mm to get approximately $\pm 10\%$ variation in the natural frequency. Then the designed simultaneous periodic output sampling controller is experimentally evaluated by exciting the structure with their first natural frequency with amplitude of 10V peak-to-peak. The open loop response, closed loop response with simultaneous periodic output feedback control and control signal acquired from dSPACE control desk are shown in Figure 6, 7 and 8 respectively. From the results it is observed that the vibration reduction is approximately 96 % in all the three cases (actual plant, with increased length and decreased length of the beam).

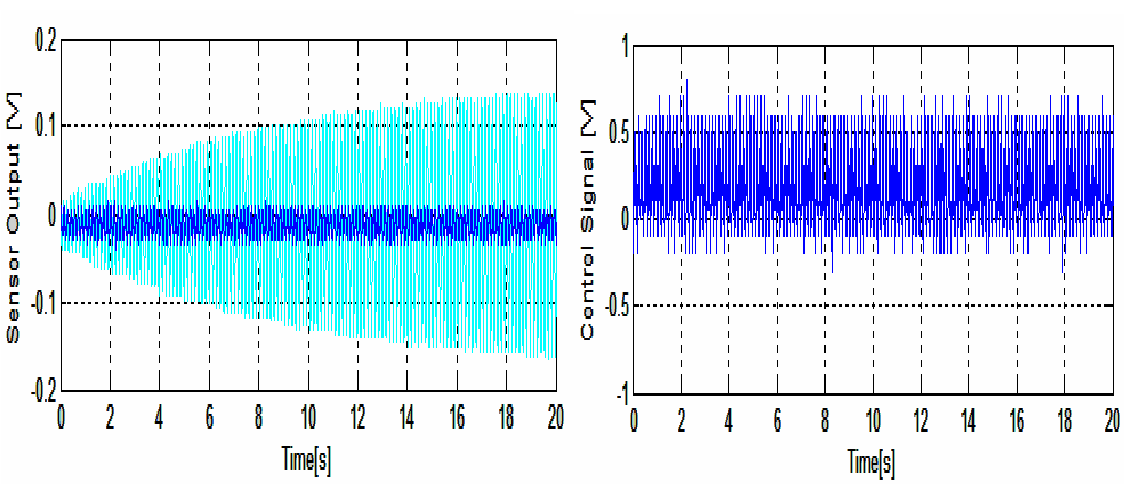


(a) (b)

Figure 6. Response to excitation at first natural frequency for the nominal system (Experimental)

(a) ---Uncontrolled and —— Controlled

(b) Control Signal.

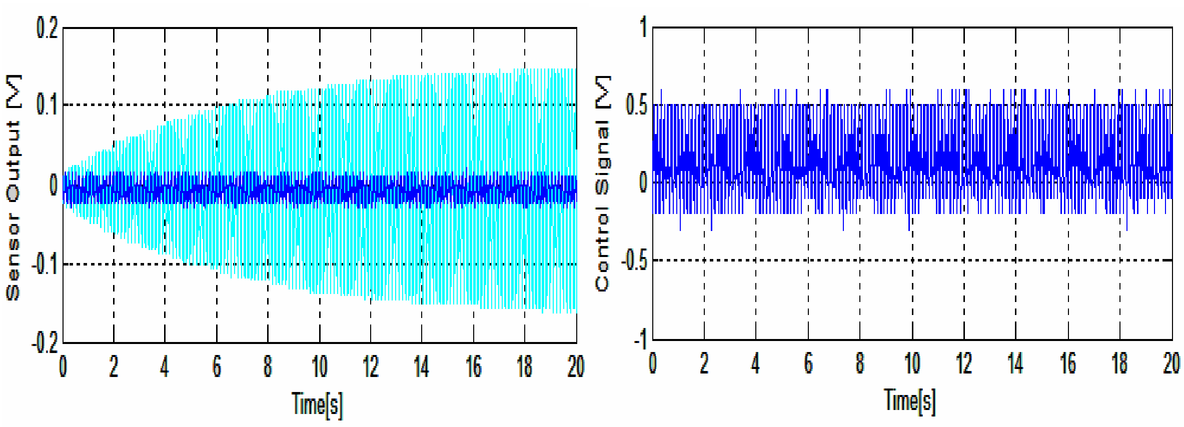


(a) (b)

Figure 7. Response to excitation at first natural frequency with increased in beam length (Experimental)

(a)---Uncontrolled and —— Controlled for the system

(b) Control Signal



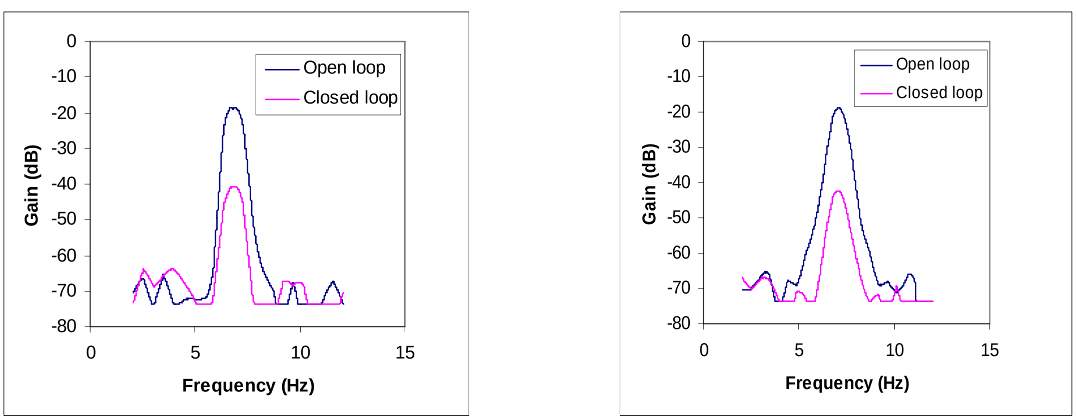
(a) (b)

Figure 8. Response to excitation at first natural frequency with decreased in beam length (Experimental)

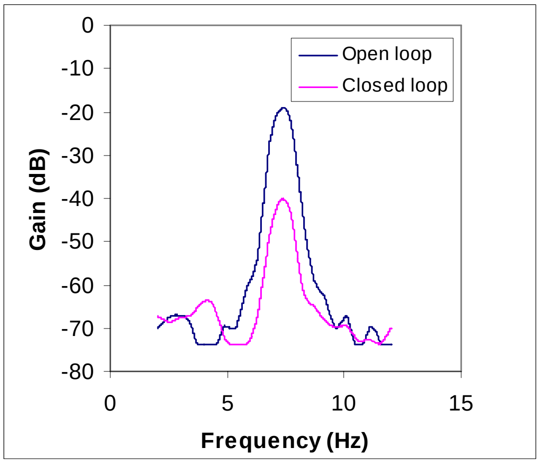
(a) ---uncontrolled and —— controlled for the system

(b) Control Signal.

The controlled and uncontrolled frequency responses captured using digital storage oscilloscopes (Agilent 54621A) are shown in Figure.9



(a) (b)



(c)

Figure 9. Frequency response ----uncontrolled; _____ controlled

(a) Nominal system

(b) Lower bound system

(c) Upper bound system

VI. CONCLUSION

An experimental evaluation of periodic output feedback controller has been designed for vibration suppression of smart cantilever beam. The accurate model for the beam structure incorporated with piezo ceramic patches is experimentally established using online ARX RLS system identification approach and the parameters converge to sensible values. The experimental evaluation of the control algorithms is performed. The R.M.S. vibration reduction is approximately 96% for periodic output feedback controller with sine wave excitation (at first natural frequency). The results of simulation and experimental study demonstrate very good closed loop performance and simplicity of the output feedback controllers.

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