CRACK IDENTIFICATION OF A ROTATING SHAFT WITH INTEGRATED WIRELESS SENSOR

F. Andrés Bejarano, Yi Jia*, Frederick Just

Department of Mechanical Engineering

University of Puerto Rico, Mayaguez Campus

PO Box 9045, Mayagüez, Puerto Rico 00681-9045, USA

Emails: jia0@me.uprm.edu

Abstract- This paper presents a novel real-time crack identification method to determine the position and depth of a transverse open crack on a rotating shaft. A newly developed wireless sensor capable of being mounted directly on the shaft is employed to monitor acceleration at different points of the shaft in a rotating coordinate system. The vibration parameters obtained from the wireless sensors and Finite Element Model provide operational data to perform Modal Analysis with different mock crack positions and depths, and an unique relation between the vibration parameters and crack characteristics is developed by Neural Networks Method working as function approximator to predict the crack size and location on the shaft. The method was experimentally validated and results have shown that the crack detection sensitivity parameters depend on the acceleration signals at different points of the shaft.

Index terms: rack detection, wireless sensor, vibration modal analysis, finite element method, neural Networks, rotating shaft.

I. INTRODUCTION

Rotating shafts carrying disks are broadly used in many mechanical applications like pumps, engines and turbines. It is observed that high speed and heavy duty shafts develop transverse cross-sectional cracks due to fatigue at some time during their life period. Cracks may be caused by mechanical stress raisers, such as sharp keyways, abrupt cross sectional changes, metallurgical factors, heavy shrink fits, grooves, and other stress concentration factors that promote the crack initiation. Once a crack is initiated it propagates and the stress required for propagation is smaller than that required for crack initiation. After many cycles operating stresses may be sufficient to propagate the crack. The crack propagation takes place over a certain depth when it is sufficient to create unstable conditions and fracture take place. It is important to develop new real-time based non destructive techniques to predict the behavior of a crack in order to avoid human and economical disasters.

Crack detection and diagnostic techniques can be classified into signal-based and model-based methods. Y. Narkis in 1992 [1] derived a continuous equation to determine the location of a crack on a homogeneous and symmetric rotor. The crack was simulated by an equivalent spring, connecting the two segments of the beam. Narkis used algebraic equations which relate natural frequencies to beam and crack characteristics. It was found that the only information required for accurate crack identification is the variation of the first two natural frequencies due to the crack. The reliability of the proposed method was evaluated with finite element method using ANSYS software giving acceptable results. K. Bikri, R. Benamar and M.M Bennouna. in 2006 [2] performed a theoretical investigation of the geometrically non-linear free vibrations of a clamped–clamped beam containing an open crack. The approach used a semi-analytical model based on an extension of the Rayleigh–Ritz method to non-linear vibrations. Emphasis was made on the backbone curves, i.e. amplitude-frequency dependence, obtained for various crack depth, and the effect of the vibration amplitudes upon the non-linear mode shapes of a cracked beam was examined. The work was restricted to the fundamental mode in order to concentrate on the study of the influence of the crack on the non-linear dynamic response near to the fundamental resonance.

G. M Owolabi. in 2002 [3] studied the vibration behavior of aluminum beams based on changes in the natural frequencies and amplitudes of the FRFs. Galerkin’s method was utilized to solve for the frequencies and vibration modes in a simulating model. Modal analysis was performed in the experimental work employing a dual channel signal analyzer and seven light accelerometers placed at different points of the beam. Results have shown that vibration behavior of the beams are very sensitive to the crack location, crack depth and mode number. This study illustrates that measured parameters of frequencies and response amplitudes are unique values. The unique values of the crack location and crack depth were obtained by plotting the contour lines of the first three modes of the fixed beam. Y. Fan and J. Li in 2000 [4] used an embedded modeling approach to identify the change in stiffness of a shaft as a result of a crack. They worked with a new methodology to identify multi-degree of freedom nonlinear systems from the system's operating data. The methodology includes a new nonlinear model architecture which embeds feedforward neural networks to represent unknown nonlinearities in a lumped parameter model, and a learning algorithm to train the embedded neural networks as well as model parameters to obtain model fidelity. The change in stiffness of the shaft was approximated using neural network, which is then, in turn, embedded in a lumped parameter model of the shaft. The neural network is refined iteratively with a solution method to minimize a cost function based on the difference in the response of an embedded model and the data collected from the cracked system.

Model based methods treated the crack identification in the static frame of coordinates to estimate the changes in natural frequencies, mode shapes and damping ratios of the shaft due to a reduction in local stiffness provoked by the presence of a crack. These methods require enormous amounts of computational time and effort. Furthermore, it is not easy to obtain an accurate measurement of the crack effects using an analytical approach. The simulating study presented in this research introduces the analysis of the dynamic system in the rotating coordinates based on the acceleration signals at different points of the shaft rotating at a constant driving frequency. An attempt has been made to detect the presence of a crack in a rotating shaft, and determine its location and size, based on experimental modal analysis. Previews experimental methods require high speeds to extract modal parameters like natural frequencies and frequency response amplitudes. Another advantage of the proposed method is that the system can be safety monitored online without the need to reach the fundamental frequency avoiding extreme operational conditions for diagnosis. Therefore, the crack can be identified and characterized in operational condition without the need of dismounting the shaft.

This paper presents a novel real time monitoring technology, which uses a set of acceleration frequency response of a shaft rotating at a fixed driving frequency, to determine position and depth of a transverse open crack on the shaft. A fault simulator machine was employed to simulate the system, where a shaft is supported on two bearings and a disk is attached at the midspan. The shaft is rotated by an electrical motor coupled to the shaft on one side. The rotating shaft vibrates in bending due to the harmonic excitation force induced by the unbalanced disk. Finite element method was used to calculate the modal parameters of the cracked shaft for known crack depths and positions. The artificial neural network trained with Backpropagation algorithm is found from a design of experiment to solve the inverse problem: from the modal parameters estimate the crack characteristics. Analog signal was conditioned and processed to obtain the acceleration signal in the radial direction of the shaft for three different points. Finally the diagnostic network is selected to associate the acceleration signal with the crack depth and position. Simulation results and experimental results are compared.

II. GENERAL DESCRIPTION OF THE SYSTEM

A complex system like a steam turbine can be represented by a Jeffcott Rotor in

Figure 1 (a), where the rotating shaft-disk system is modeled assuming that the bearings are rigid and provide no damping to the system. The shaft is assumed elastic and will deflect with respect to its equilibrium position in the presence of external mass unbalance due to the eccentricity of disk mounted at the midspan.

Figure 1(b) shows the section of the disk with the resulting acceleration components. The vectors of motion involving the mass center C are calculated as:

Position	*Velocity*	*Acceleration*	
$\vec{p}_c = \vec{p}_s + \vec{p}_{c/s}$	$\vec{v}_c = \vec{v}_s + \vec{v}_{c/s}$	$\vec{a}_c = \vec{a}_s + \vec{a}_{c/s}$	**(1)**

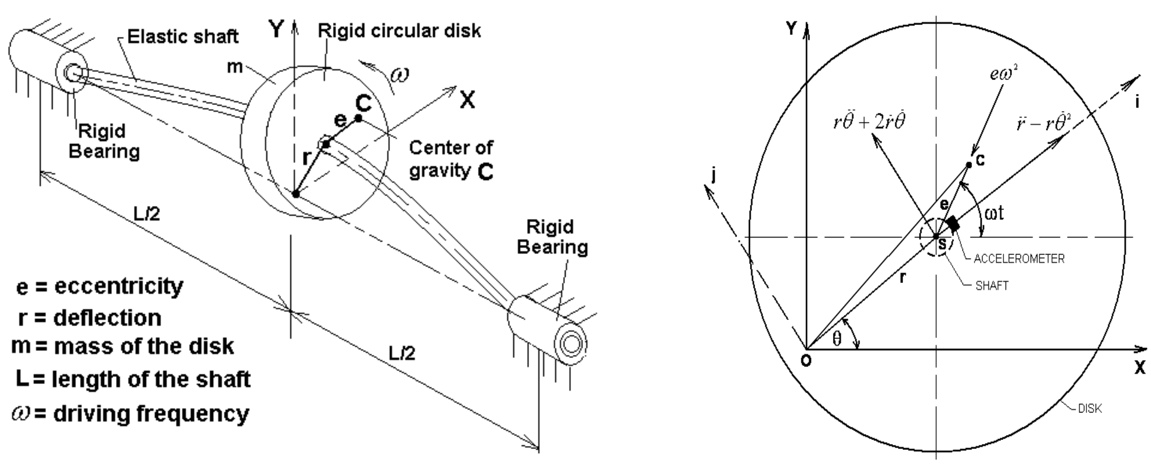


Figure 1. (a) The Jeffcott Rotor (b) Section of the unbalanced disk

From the vectorial analysis the acceleration components in the rotating coordinates we have:

$$a_{Ci} = \ddot{r} - r\dot{\theta}^2 - e\omega^2 \cos(\omega t - \theta) \qquad a_{Cj} = 2\dot{r}\dot{\theta} + r\ddot{\theta} - e\omega^2 \sin(\omega t - \theta) \tag{2}$$

Applying Newton's second law to equations of motion in the radial direction to find the harmonic force $F(\omega t)$ we get:

$$\ddot{r} + \frac{c}{m_d}\dot{r} + (\frac{k}{m_d} - \dot{\theta}^2)r = e\omega^2 \cos\left(\omega t - \theta\right) \quad where \quad F(\omega t) = m_d e\omega^2 \cos(\omega t - \theta) \tag{3}$$

III. FINITE ELEMENT MODEL OF THE ROTATING SHAFT

Finite Element Method was employed to model the cracked and uncracked shaft rotating with the unbalance disk, which is acting on the shaft as a constant harmonic force in order to verify theoretically the sensitivity and uniqueness of the acceleration response in the frequency domain for different crack positions and depths. These parameters were used to solve the inverse problem; knowing the frequency response amplitude of acceleration at three different points of the shaft, find the crack characteristics by means of Neural Networks.

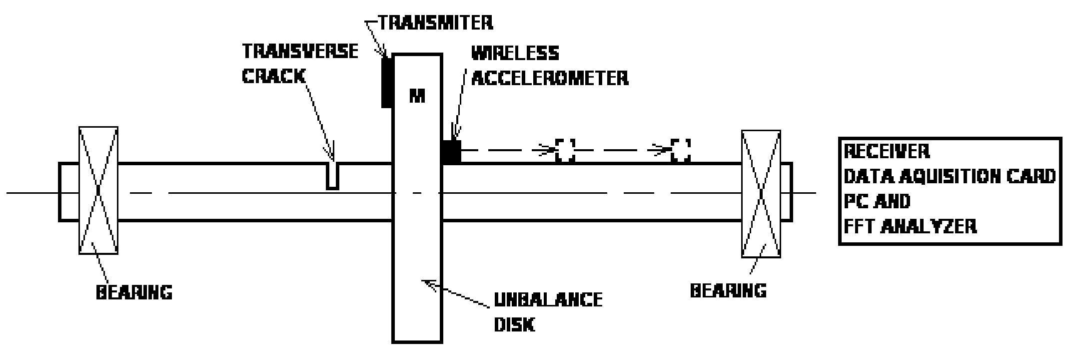


Figure 2. Cracked Shaft Model

The open crack was considered as a small element with reduced stiffness within the finite element model. Only one crack is assumed for this study and external damping was neglected. Wireless sensor directly mounted on the shaft allow to analyze the rotating system in a static frame relative to the sensor, therefore the model is assumed as a fixed beam carrying the disk and experiencing mainly flexural stresses due to the harmonic excitation of the disk at the midspan.

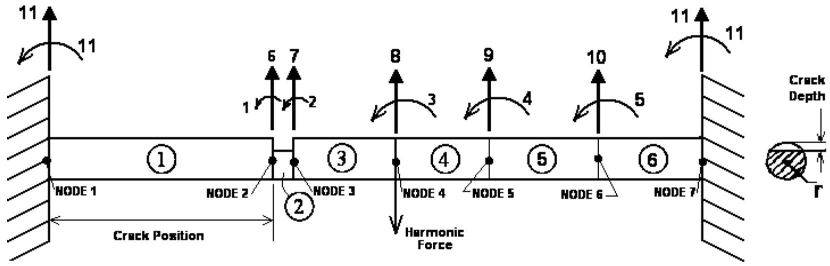


Figure 3. Fixed shaft divided in six elements and its nodal coordinates

Applying the dynamic condensation method proposed by M. Paz (1984) [**Error! Reference source not found.**] as an extension of the Static Condensation Method where the fixed degrees of freedom are reduced by expressing them in terms of the independent or primary degrees of freedom, the secondary degrees of freedom {Ys} (slopes 1 to 5) are condensed, and the primary degrees of freedom are retained (deflections 6 to 10 in

Figure 3). The secondary degrees of freedom are arranged at the first s coordinates and the primary degrees of freedom are the last p coordinates. After these arrangements the equations of free motion can be written in partitioned matrix form as

$$\left\{\frac{\{\ddot{Y}_s\}}{\{\ddot{Y}_p\}}\right\}+\left[\frac{[K_{ss}]}{[K_{ps}]}\Big|\frac{[K_{sp}]}{[K_{pp}]}\right]\left\{\frac{\{Y_s\}}{\{Y_p\}}\right\}=\left\{\frac{\{0\}}{\{0\}}\right\} \quad \textbf{(4)}$$

The reduced stiffness and mass matrices are calculated as

$$\left[\bar{K}\right]=\left[\bar{D}_i\right]+\omega_i^2\left[\bar{M}_i\right] \quad and \quad \left[\bar{M}_i\right]=\left[\bar{T}_i\right]^T\left[M\right]\left[\bar{T}_i\right] \quad \textbf{(5)}$$

Requiring for a nontrivial solution that

$$\left|[\bar{K}]-\omega^2[\bar{M}]\right|=0 \quad \textbf{(6)}$$

The force vector of the system and the modal matrix are employed to estimate the steady state response of the shaft subjected to the harmonic force $F_0\cos(\omega t-\theta)$ acting on the node 4 due to the unbalance disk. The force vector is assembled as follows.

$$F=\{0 \quad 0 \quad F_0\cos(\omega t-\theta) \quad 0 \quad 0\}' \quad \textbf{(7)}$$

Neglecting damping and assuming $\theta = 0$, the system can be represented with the normal equation

$$\ddot{Z}_n + \omega_n^2 Z_n = P_n \cos(\omega t) \quad where \quad P_n = \sum_{i=1}^{N} \Phi^{-1} F_{0i} \tag{8}$$

Solving the nonhomegeneous ordinary differential equation (8) employing the undetermined coefficient method and applying the initial conditions we have the deflection solution of the nodal coordinates in the time as follows:

$$z_n = \frac{P_n}{\omega_n^2 - \omega^2}[\cos(\omega t) - \cos(\omega_n t)] \tag{9}$$

The deflection of the nodal coordinates are found from the transformation

$$\{y\} = [\Phi]\{Z_n\} \quad where\,[\Phi]\,is\ the\ modal\ matrix \tag{10}$$

Substituting the values of $\{z_n\}$ into Equation (10) gives the amplitudes of deflection at the nodal coordinates. The second derivative corresponds to the amplitudes of acceleration at the nodal coordinates is.

$$\ddot{Z}_n = \frac{Pn}{(\omega_n^2 - \omega^2)}[\omega_n^2 \cos(\omega_n t) - \omega^2 \cos(\omega t)] \Rightarrow \{\ddot{y}\} = [\Phi]\{\ddot{Z}_n\} \tag{11}$$

IV. EXPERIMENTAL SETUP

An accelerometer ADXL202E with duty cycle output was conditioned to work wireless by the transmitter TXM-418-LC and the receiver RXM-418-LC-C. The accelerometer used is capable of measuring positive and negative absolute accelerations at least +/- 2g. The sensor is a surface micro-machined polysilicon structure built on top of silicon wafer. Polysilicon springs suspended the structure over the surface of the waffer and provoke a resistance against acceleration forces. Deflection of the structure is measured using a differential capacitor that consists of independent fixed plates and central plates attached to the moving mass. The resulting output is a square wave whose amplitude is proportional to acceleration. Data acquisition system consists of an acquisition board National Instruments CB-68LP, which connects the receiver circuit with the data acquisition board NI6024E. The data acquisition board is connected to the communication port of a laptop computer. Analog signal is sent by the transmitter as radio frequency waves at 418 KHz, which is launched by the receiver to the data acquisition board and data acquisition card. A set of this signal is recorded by the data acquisition card connected to the laptop computer. This card was configured to work with the software Matlab 7.1 and the signal processing toolbox. The signal was also monitored by an oscilloscope connected in parallel to the receiver circuit during the sampling. Figure 4 shows experimental setup and wireless sensor.

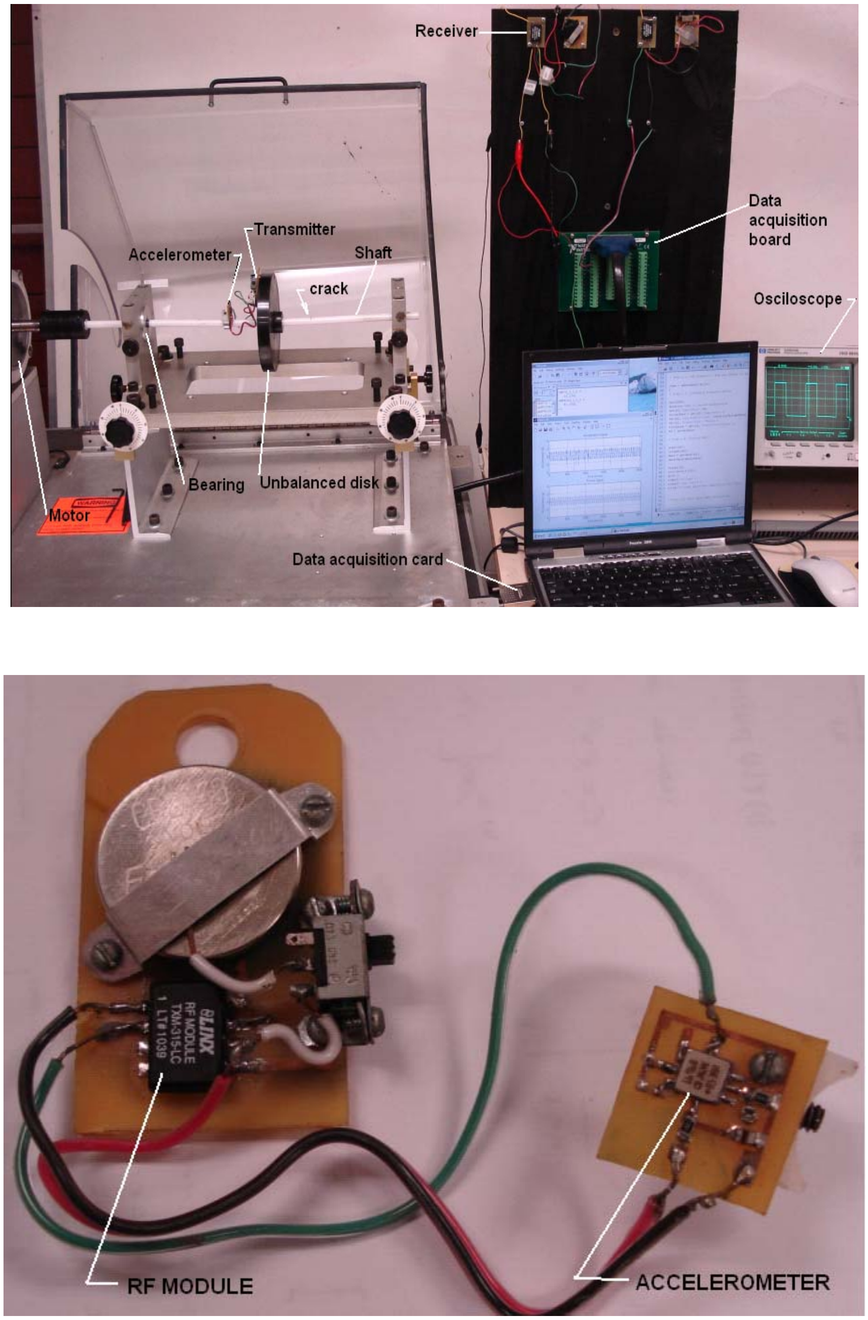
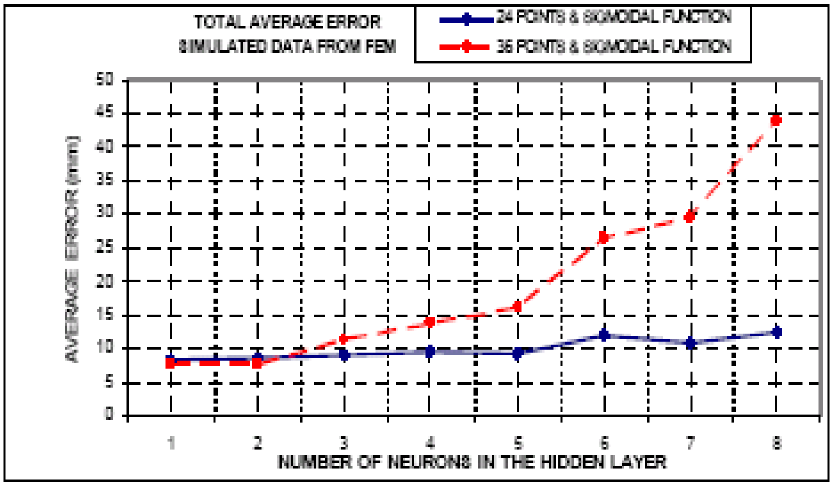


Figure 4. Data Acquisition System and Experimental Setup

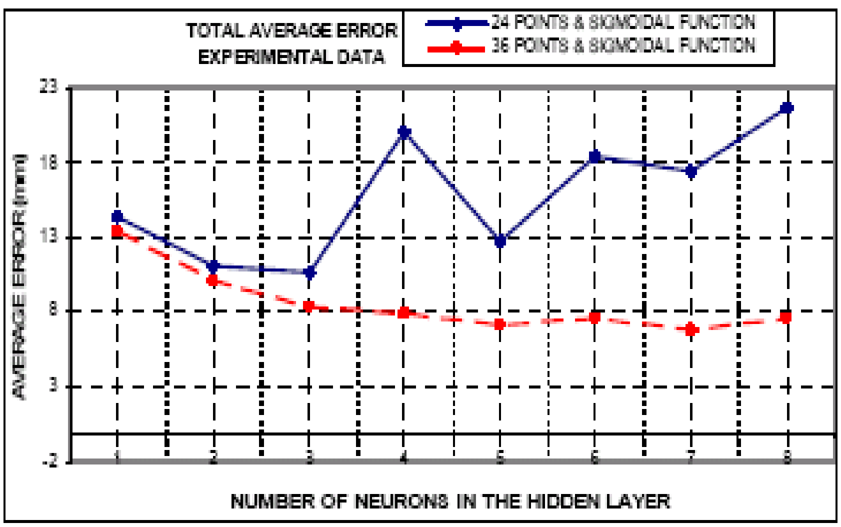
V. IDENTIFICATION OF CRACK SIZE AND LOCATION BASED ON ARTIFICIAL NEURAL NETWORKS

Multilayer feedforward neural networks with Backpropagation Levenberg Marquardt (LMBP) training algorithm was employed in this study as a non linear function approximator to establish a relationship between input vectors and target values. LMBP algorithm introduced by Hagan [5] was selected because it is one of the most efficient and fasted algorithms. The performance of the network is measured by comparing the output of the network with the corresponding target value. Known inputs P and targets T were obtained first theoretically by the simulation with finite element method and then experimentally employing the wireless accelerometer. The input vector (P) was created from the difference in the spectral amplitudes of acceleration response between cracked and uncracked shaft at nodes 4, 5 and 6. The target matrices were constructed by considering known locations and depths of the crack to be associated with the pattern of amplitudes of acceleration response differences. A neural network is trained when the learning algorithm find the weight matrices and bias vector in such way that the error between predicted and actual data is minimized.

There were considered two sizes for the input matrices and target vectors for training in order to determine the significance in the network performance: one with 6 positions and 4 depths for 24 total points, and another with 9 positions and 4 depths for 36 total points represented with blue and red respectively in Figures 5 (a) and (b). Simulated data obtained by the finite element model where used first for training and then for testing the network. Experimental data were obtained from the accelerometer. Deviations in crack depth and crack position were integrated in one variable named “Total Error” which was defined as the square root of the summation of the squared errors. The “Total Error” was assumed as the response variables of the system, which were treated for the analysis of variance. The experiment was conducted to establish statistically the rules to identify the structure of the network. Matlab 7.1 and the neural networks toolbox functions [**Error! Reference source not found.**] were employed to develop a code for training varying the sensitive parameters that affects the performance of the network. Next figures show the average error obtained from the predictive network for both simulating and experimental increasing the following parameters: number of neurons in the hidden layer with sigmoidal transfer functions and quantity of data employed for training.



(a)



(b)

Figure 5. Predicted average error (a) Simulated data (b) Experimental data

Figure 5 (a) corresponding to simulating results shows how increasing the number of data for training and reducing the number of neurons in the hidden layer the average error of the predictive network is reduced. Therefore the best structure selected was the one that reveals the lowest average error with the smallest number of neurons. A single neuron in the hidden layer was selected with a sigmoidal transfer function for the network. The icons and notation presented in Figure 6 were taken from Hagan [5], where W(m) represents the weight matrix of the mth layer, b(m) is the bias vector of the mth layer, n(m) is the net input vector to the network in the mth layer, a(m) is the net output of the network in the mth layer.

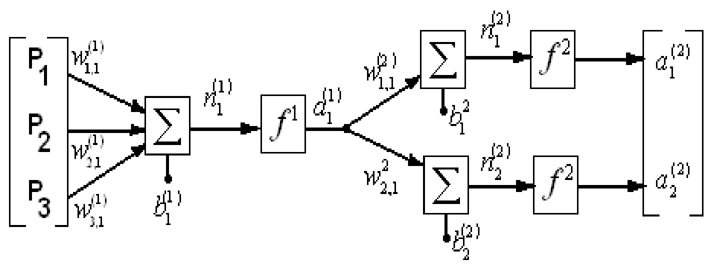


Figure 6. Neural network structure schematic of the predictive model for the simulated data

Figure 6 shows one input vector P of 3 x 1 with the three differences in amplitude of acceleration: P_1,P_2 and P_3 , and two neurons with linear transfer functions in the output layer that give the output vector a corresponding to crack depth and position . Figure 5 (b) corresponding to experimental results shows how increasing the number of data for training and the number of neurons in the hidden layer the average error of the predictive network is reduced. The structure revealing the lowest average error with the smallest number of neurons is shown in Figure 7. It was considered a network with five neurons in the hidden layer, a sigmoidal transfer function with one input vector P of 3 x 1, and two neurons with linear transfer functions in the output layer that give the output vector a corresponding to crack depth and position as shown in the next schematic.

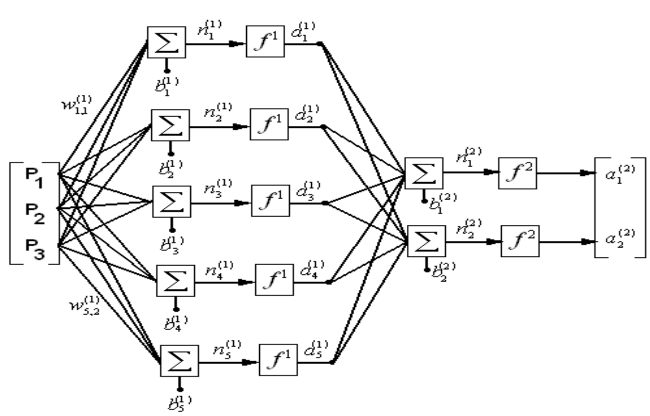
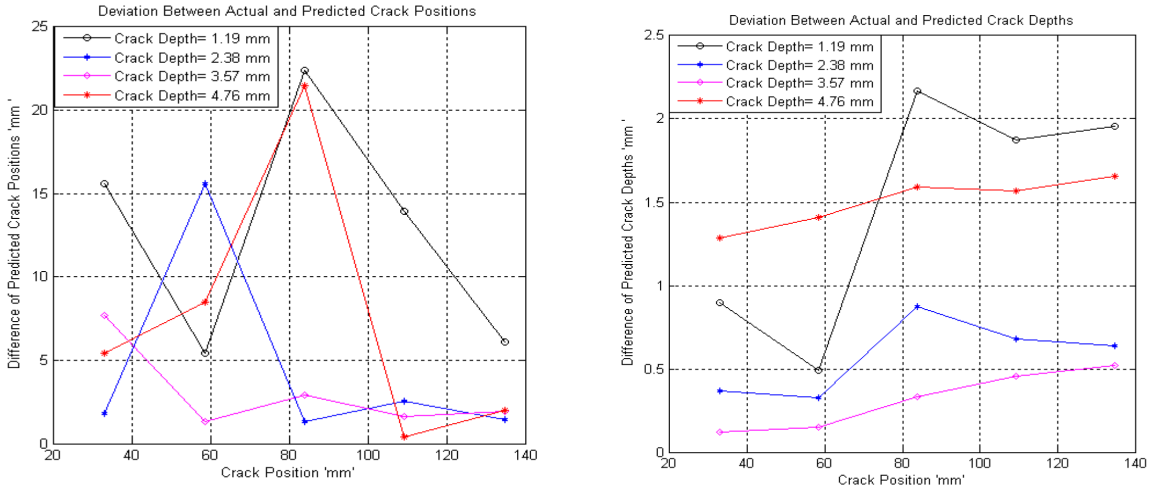


Figure 7. Neural network structure schematic of the predictive model for the experimental data

New cracked shafts were tested for the selected network obtained experimentally finding good results on prediction. Figure 8 shows a tendency of error reduction in prediction when the crack approaches to the middle of the shaft.



(a) (b)

Figure 8. Performance of the predictor network (a) Crack positions (b) Crack depths

This behavior can be attributed to the low sensitivity of the vibration parameters obtained from PSDAA differences obtained from at the nodes of analysis of the structure. Table 1 summarizes the performance of the neural networks:

Table 1. Comparison between theoretical and experimental predicted data

	NEURAL NETWORK PERFORMACE (IN MILIMETERS)					
	Average error in positon	Minimal error in position	Maximal error in position	Average error in depth	Minimal error in depth	Maximal error in depth
Theoretical	7.2119	0.0178	16.8989	0.6578	0.1257	2.0365
Experimental	7.7990	0.3636	26.5853	0.9678	0.1216	2.1658
Difference	0.5871	0.3458	9.6865	0.3099	-0.0042	0.1294

VI. RESULTS AND CONCLUSIONS

This research developed a method for online failure diagnosis on a shaft rotating with an unbalance disk at the midspan. A unique pattern of acceleration signals acquired from a new wireless sensor capable of being mounted at different positions of the shaft allow the development of neural network performing to predict crack characteristics from the vibration parameters. The selection of the predictor network architecture was based on its performance in pattern recognition. A simulating study was carried out employing the finite element method to model the problem before experimental application. The method was successfully implemented within a degree of accuracy to diagnose the crack depth and position in a rotating shaft demonstrating the effectiveness of the artificial neural networks predictor working in combination with the wireless sensor giving a signal in the rotating coordinates. Modal parameters such as mode shapes and natural frequencies obtained from the finite element model resulted to be very sensitive to crack depth and crack location. The differences of power spectral densities amplitudes of acceleration between the cracked and uncracked shaft at three different positions have shown a unique pattern of the failure scenario, except in crack positions close to the shaft support. This unique characteristic was successfully employed as the diagnostic input sensitive parameter to find a function approximator networks able to correlate this pattern with the failure characteristics employing the Back propagation training algorithm. The small error differences between actual and predicted data in the simulated and experimental studies demonstrated the consistency of the proposed method in the crack identification of a shaft employing wireless sensors and artificial neural networks method in a real system.

VII. ACKNOWLEDGMENTS

The authors would like to acknowledge National Science Foundation and US Department of Energy for the research support under grant NSF- CMMI- 0757486 and DE-FG26-07NT43061.

REFERENCES

[1] Y. Narkis, “Identification of Crack Location in Vibrating Simply Supported Beams’’, Journal of Sound and Vibration, 172(4), pp.549-558, 1994.

[2] K. Bikri, R. Benamar and M.M Bennouna., “Geometrically non-linear free vibrations of clamped-clamped beams with an edge crack’’, Elservier, Computers and Structures 84, pp. 485-502, 2006.

[3] G.M. Owolabi., A.S.J Swamidas and R. Seshadri, “Crack Detection in Beams Using Changes in Frequencies and Amplitudes of Frequency Response Functions’’, Journal of Sound and Vibration, 263 ,pp1-22,2003.

[4] Y. Fan and. J. Li, "Nonlinear System Identification Using Lumped Parameter Models With Embedded Feedforward Neural Networks", Rensselaer Polytechnic Institute, 2000.

[5] H. Demuth, M. Hagan and M. Beale, "Neural Network Design", Thompson Learning, Inc., 1996.

[6] M. Feldman & S. Seibold, "Damage Diagnosis of Rotors: Application of Hilbert Transform and Multi-Hypotesis Testing", Israel Institute of Technology, 1998.

[7] D. J. Inman., "Engineering Vibration", second edition, Prentice-Hall Inc., 2001.

[8] MATLAB Version 7.1.0.246 Release 14, Neural Network Toolbox, the Mathworks Inc., August 02, 2005.

[9] M. Paz, "Structural Dynamics Theory and Computation'', Van Nostrand Reinhold, Third Edition, 626 pages, 1991.

[10] Adewusi Surajudeen Adedotun, " Detection of a Transverse Crack in a Rotating Shaft Using Wavelet Transform", Thesis of Master of Science in Mechanical Engineering, King Fahd University of Petroleum & Minerals, 233 pages, May 2000.

[11] Analog Devices, Inc, ADXL202E Catalog, at http://www.analog.com,2000.

[12] Bachschmid N., Pennacchi P., Tanzi E. and Vania A., "Identification of Transverse Crack Position and Depth in Rotor Systems", Meccanica, vol 35, pp. 563-582, 2000.

[13] Ewins D.J., " Modal Testing Theory and Practice" , Research Studies Press Ltd, 269 pages, 1984.

[14] Grabowski B., "The Vibrational Behavior of a Turbine Rotor Containing a Transverse Crack", Transactions of the ASME, vol. 102, Jan. 1980.

[15] Haberman Richard ''Elementary Applied Partial Differential Equations'' whit Fourier Series and Boundary Value Problems, second edition by Prentice –Hall, 1987.

[16] Maharbiz Michael, "Introduction to MEMS – EECS 414 lectures", Center for Wireless Integrated MicroSystems, University of Michigan, September 2005.

[17] McKee Kristoffer K., "Rotor Transverse Crack Detection and Diagnosis Using Embedded Modeling", Thesis of Doctor of Philosophy in Mechanical Engineering, Rensselaer Polytechnic Institute, Troy, New York, 191 pages, November 2002.

[18] Sekhar A.S., "Identification of a Crack in a Rotor System Using a Model-based Wavelet Approach", Technische Universitat Darmstadt, Institute fur Mechanik, 2001.

[19] Sinha Jyoti K., “Bi-spectrum for identifying crack and misalignment in shaft of a rotating machine’’, Smart Structures and Systems, Vol. 2, No. 1, pp. 47-60, 2006.

[20] Tsai T. C. And. Wang Y. Z, “Vibration Analysis and Diagnosis of a Cracked Shaft”, Journal of Sound and Vibration, vol.192, No.3, pp.607-620, 1996.

[21] Xiao Yingming B, “An Experimental Study of Crack Detection in a Rotating Shaft”, Thesis for Master Engineering, Memorial University of Newfoundland-Canada, 109 pages, July 1995.