



## I. INTRODUCTION

Hydraulic actuator has been widely used in industrial equipments and processes due to its linear movements, fast response and accurate positioning of heavy load. This is principally due to its high-power density and system solution that it can provided [1,2]. The natural nonlinear property of hydraulic cylinder has challenged researchers in designing suitable controller for positioning control [3], motion control and tracking control [4]. With intention to improve the motion or tracking performance effectively, many researchers have used advanced control strategies to control hydraulic cylinder [5,6]. Classical feed-forward controller based on pole-zero cancellation for minimum phase system, makes the overall transfer function be unity thus perfect tracking control (PTC) is achieved [7]. Unfortunately, this controller cannot be implemented to non-minimum phase system as this would result an unstable tracking control. The zero-phase error- tracking control (ZPETC) was then proposed by Tomizuka [8] and has attracted attention many researchers as an effective and simple remedy to the problem due to unstable zeros. By eliminating phase error caused by non-cancelled zeros, ZPETC displays good tracking performance. The gain error, which cannot be eliminated by ZPETC becomes larger for fast tracking control and causes undesirable effect on the tracking performance. In resolving these problems, there has been many research works in this area [9-13].

Based on these scenarios, this paper discusses the implementation of real-time digital tracking control of electro-hydraulic cylinder using trajectory zero phase error tracking control without factorization of zeros polynomial where the controller parameters are determined using comparing coefficients method. Simulation and real-time experimental results will be compared and discusses on their tracking performances.

This paper was organized in the following manner: Section II describes ZPETC without factorization of zeros; Section III describes plant and model identification; Section IV describes controller design; Section V describes result and discussion; and Section VI is the conclusion.

## II. ZPETC WITHOUT FACTORIZATION OF ZEROS

The tracking control system with two-degrees-of-freedom that is consisting of feedback and feedforward controllers is given in figure 1. In tracking control system without feedforward



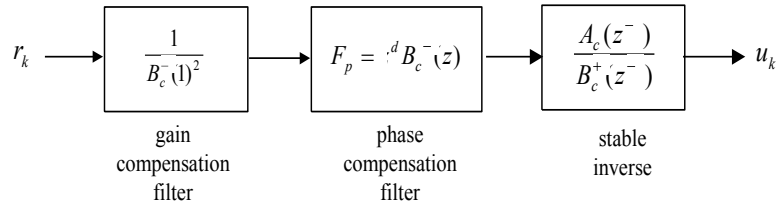


Figure 2. Conventional ZPETC structure block diagram

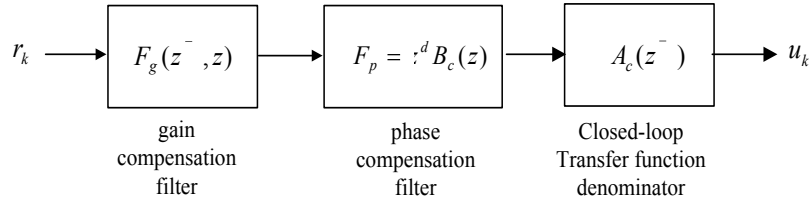


Figure 3. ZPETC without factorization of zeros structure block diagram

Similar to all others ZPETC, the design mainly focused on the selection of appropriate gains compensation filter to ensure the overall gain is unity within the frequency spectrum of reference trajectory. To ensure the gain compensation filter,  $F_g$  does not introduces any phase error, the same approach done by [15-16] will be followed. The FIR symmetric filter was used. The filter is represented by equation

$$F_g(z, z^{-1}) = \sum_{k=0}^{n_\alpha} \alpha_k (z^k + z^{-k}) \quad (3)$$

where  $n_\alpha$  is the order of the filter. A suitable cost function to represent the error between the desired and actual frequency response is given by Eq. (4).

$$J(\alpha_k) = \left\| \frac{1 - B_c^-(z^-) B_c(z)}{B_c^-(z^-) B_c(z)} \sum_{k=0}^{n_\alpha} \alpha_k (z^k + z^{-k}) \right\|_{l_2} \quad (4)$$

The design objective here is to find a set of  $\alpha_k$  such that the cost function given by Eq. (4) is minimized. For finite  $\alpha_k$ , Eq.(4) cannot be made zero for all frequencies. By minimizing the cost function of Eq. (4),

$$B_c^-(z^-) B_c(z) \sum_{k=0}^{n_\alpha} \alpha_k (z^k + z^{-k}) = \quad (5)$$

The optimal set of  $\alpha_k$  can be obtained by expanding Eq. (5) to polynomial of positive and negative power of  $z$ , and then compare the coefficients of the same power.



$$V_{in}(k) = 2 \cos 0.3 t_s k + 2 \cos 4 t_s k + \cos 6 t_s k \quad (7)$$

where  $a_i$  is the amplitude,  $\omega_i$  is the frequency (rad/sec),  $t_s$  is the sampling time (sec) and  $k$  is integer.

From Eq. (7), when using three different frequencies for input signal, the model that can be obtained is limited to second and third order only. Higher-order model may produce unstable output. In these studies, the third-order ARX331 model with input-output signals sampled at 50ms was selected to represent the nearest model of true plant.

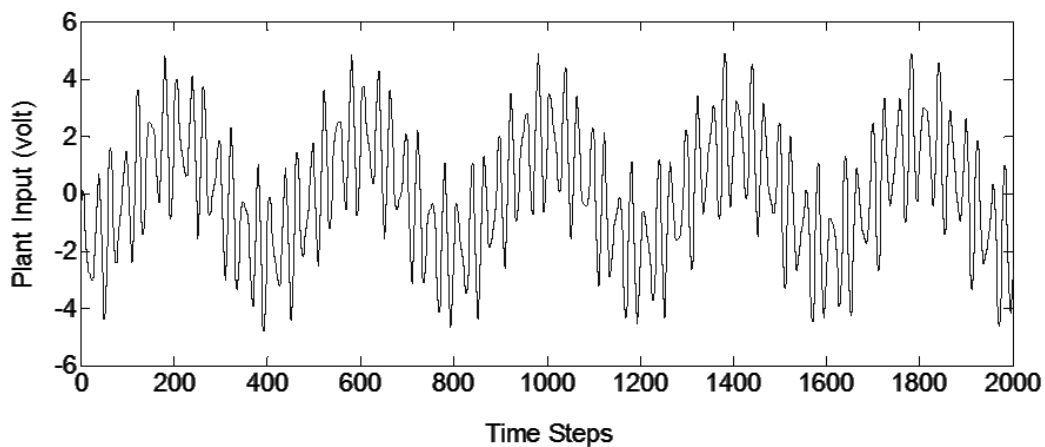


Figure 5. Input signals for model identification

The output signal of the plant obtained using the input signal of figure 5 and sampled at 50 ms, is given in figure 6. The input and output signals of Figure 5 and Figure 6 were divided into two parts, i.e. (500-1000) samples and (1001-1500) samples. The first part of the input – output signals were used to obtain the plant model and the second part of the input-output signals were used to validate the obtained model.



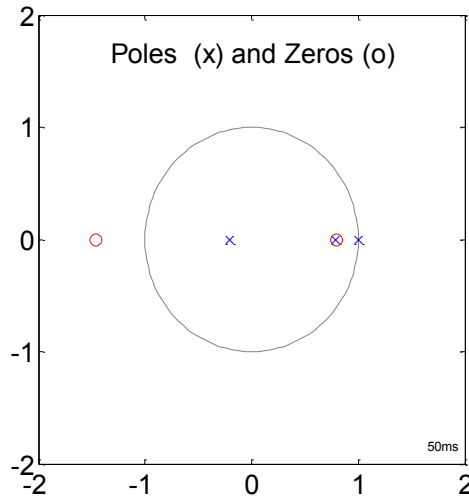


Figure 7. Pole-zero plot of Eq. (10)

The second part of input-output signals were used to validate the obtained model of Eq. (8). The second part of the input signal is used as an input to the model and the output from the model was compared with the second part of the output signal. The result can be seen from figure 8. Using model selection criteria, the following information were obtained:

Best Fit :	89 %
Loss Function :	$3.292 e^{-005}$
Akaike's Final Prediction Error, FPE:	$3.371 e^{-005}$

Based on the smallest values criteria of FPE and Best Fit of 89 %, this model can be accepted.

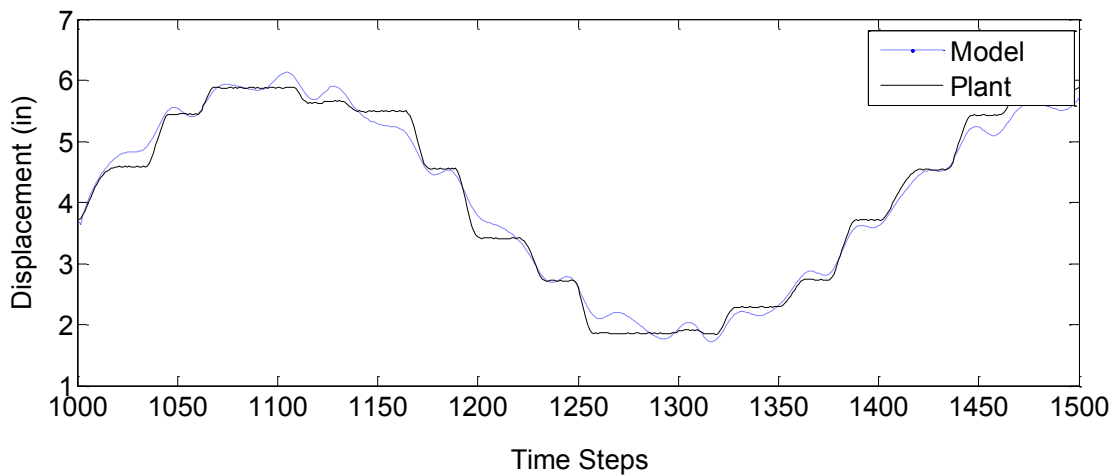


Figure 8. Comparison between the model and plant output signal





$$\begin{bmatrix} 2.2089 & -0.0125 & -2.0296 & 0 & 0 & 0 \\ -0.0063 & 1.1941 & -0.0063 & -1.0148 & 0 & 0 \\ -1.0148 & -0.0063 & 2.2089 & -0.0063 & -1.0148 & 0 \\ 0 & -1.0148 & -0.0063 & 2.2089 & -0.0063 & -1.0148 \\ 0 & 0 & -1.0148 & -0.0063 & 2.2089 & -0.0063 \\ 0 & 0 & 0 & -1.0148 & -0.0063 & 2.2089 \end{bmatrix} \begin{bmatrix} 2\alpha_0 \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_3 \\ \alpha_5 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (12)$$

Solving Eq.(12), the optimal set of  $\alpha_k$  is obtained. In these studies, we consider the order of  $F_g(z^{-1}, z)$  is  $n_a = 10, 20$  and  $30$ . The optimal  $\alpha_k$  obtained by minimizing the cost function is given in Table I. When the filter order is increase to  $20^{\text{th}}$  and  $30^{\text{th}}$  order, using the same technique, the obtained optimal set of  $\alpha_k$  is given in Table II and Table III. The results show that the values obtained are almost converging to zero, as the filter order increasing, as shown in figure 10.

Table I:  
Optimal  $\alpha_k$  for  $10^{\text{th}}$  order gain compensation  
filter of Eq. (8)

k	0	1	2	3	4	5	6	7	8	9	10
$\alpha_k$	0.7159	0.3738	0.4679	0.3064	0.3139	0.2258	0.2061	0.1479	0.1231	0.0743	0.0553

Table II:  
Optimal  $\alpha_k$  for  $20^{\text{th}}$  order gain compensation  
filter of Eq. (8)

k	0	1	2	3	4	5	6	7	8	9	10
$\alpha_k$	0.3785	0.4274	0.5151	0.3629	0.3691	0.2896	0.2716	0.2243	0.2020	0.1706	0.1501
k	11	12	13	14	15	16	17	18	19	20	
$\alpha_k$	0.1275	0.1102	0.0930	0.0789	0.0651	0.0536	0.0416	0.0326	0.0207	0.0147	

Table III:  
Optimal  $\alpha_k$  for  $30^{\text{th}}$  order gain compensation  
filter of Eq. (8)

k	0	1	2	3	4	5	6	7	8	9	10
$\alpha_k$	0.3834	0.4255	0.5208	0.3621	0.3724	0.2901	0.2743	0.2261	0.2051	0.1738	0.1546
k	11	12	13	14	15	16	17	18	19	20	21
$\alpha_k$	0.1328	0.1169	0.1011	0.0884	0.0766	0.0668	0.0579	0.0502	0.0434	0.0375	0.0322
k	22	23	24	25	26	27	28	29	30		
$\alpha_k$	0.0275	0.0234	0.0197	0.0163	0.0134	0.0104	0.0081	0.0052	0.0037		



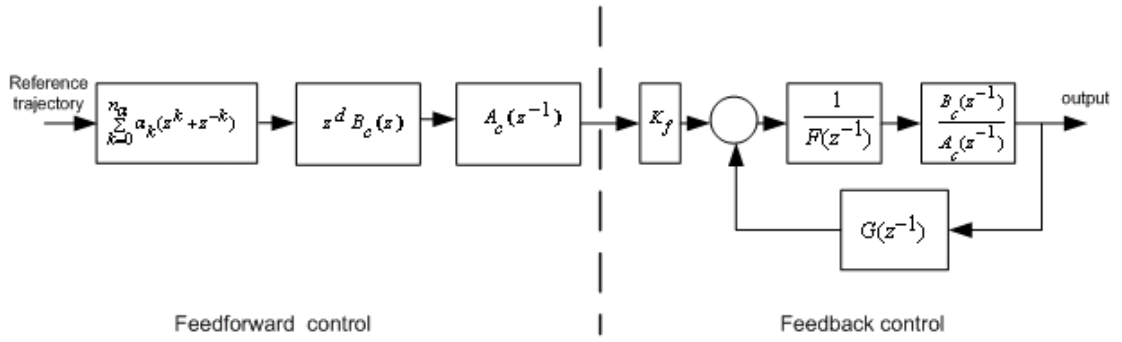


Figure 12. Tracking structure for real-time studies

#### D. Feedback Control System

The feedback control system for the proposed trajectory ZPETC system is given in figure 13. The controller was designed using pole-placement method.

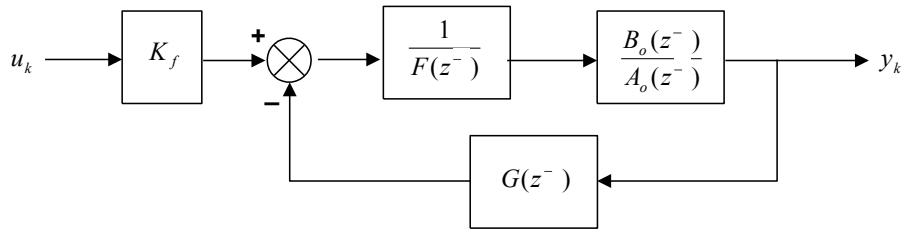


Figure 13. Feedback controller using pole-placement method.

This method enables all poles of the closed-loop to be placed at desired location and providing satisfactory and stable output performance. All controller parameters were obtained by solving the following Diophantine equation to solve for  $F(z^{-1})$  and  $G(z^{-1})$ .

The closed-loop transfer function of the system is given by:

$$\frac{Y(z^{-1})}{U(z^{-1})} = \frac{K_f B_o(z^{-1})}{A_o(z^{-1})F(z^{-1}) + B_o(z^{-1})G(z^{-1})} \quad (13)$$



## V. RESULTS AND DISCUSSION

In this section, the simulation and real-time results were analyzed to show the effectiveness of the designed controller. The simulation and real-time result of using controller parameters are given in Table I, II and III and applied to plant model which resulting RMSE that given in Table IV.

Table IV:  
RMSE performance (mm)

	Simulation			Real-Time		
$n_a$	10	20	30	10	20	30
RMSE	11.242	2.1692	0.7772	11.460	2.4435	1.3665

The tracking performances in terms of root mean squared error (RMSE) for all the simulated and real-time results are summarized in Table IV. The results show the 10<sup>th</sup>, 20<sup>th</sup> and 30<sup>th</sup> order filter tracking performances. It can be observed that the tracking error has been greatly reduced when the filter order was increased. As we can observe from the simulated results of Table IV, by introducing larger filter order to the system, the performance between 10<sup>th</sup> and 20<sup>th</sup> order filter has improved by 80%. For the real-time result, by introducing larger order filter, the performance between 10<sup>th</sup> and 20<sup>th</sup> has improved by 78%. However, the result between the simulated and real-time does not provide similar performance due to plant-model mismatch.

Figure 14, shows the frequency response for the overall system. The frequency response can be improved if the order of  $F_g(z^{-1}, z)$  is increased. Based on figure 14, it can be observed that by using 10<sup>th</sup> order filter, it will not able to produce a gain that near to unity. The gain is almost flat at unity when a higher order filter is used.

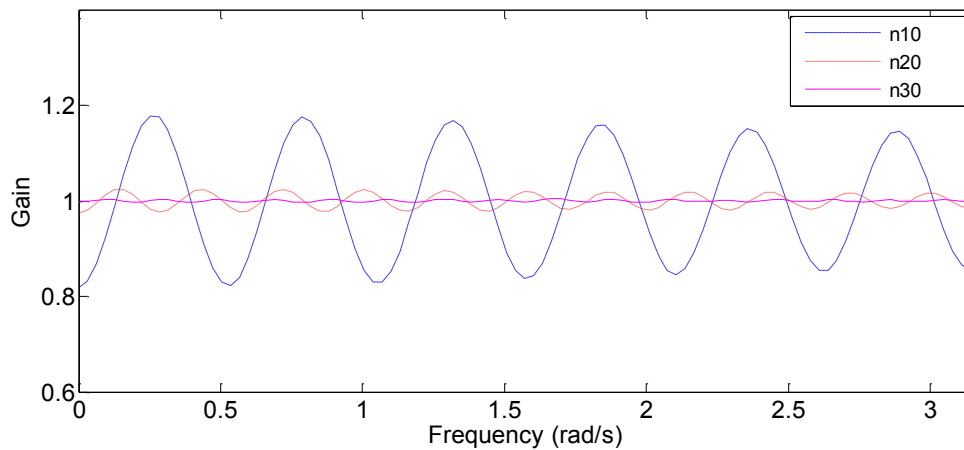


Figure 14. Frequency response of 10<sup>th</sup>, 20<sup>th</sup> and 30<sup>th</sup> order ZPETC



## VI. CONCLUSIONS

A controller design using trajectory ZPETC without factorization of zeros polynomial has been presented. The proposed feedforward controller design has been successfully tested by simulation and validated by real-time digital control of electro-hydraulic cylinder. Simulation and experimental results show good tracking performances when higher order filter was used in the design. A much smaller tracking error cannot be achieved due to plant-model mismatch and electronic valve open-close capability.

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