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STUDY ON THE CONGESTION CONTROLLER FOR TIME-DELAY NETWORKED CONTROL SYSTEMS WITH EXTERNAL DISTURBANCES

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Abstract- A successive approximation approach (SAA) is developed to obtain a new congestion controller for the singularly perturbed time-delay networked control systems affected by external disturbances. Based on the slow-fast decomposition theory of singular perturbations, the system is first decomposed into a fast non-delay subsystem and a slow time-delay subsystem with disturbances. Then, the perturbation approach is proposed to solve the slow-time scale time-delay optimal control problem, and the feedforward compensation technique is used to reject the external disturbances. We obtain the conditions of existence and uniqueness of the feedforward and feedback composite control (FFCC) law. The FFCC law consists of linear analytic functions and a time-delay compensation term which is a series sum of adjoint vectors. The linear analytic functions can be found by solving a Riccati matrix

equation and a Sylvester equation respectively. The compensation term can be approximately obtained by an iterative formula of adjoint vector equations. A reduced-order disturbance observer is constructed to make the FFCC law physically realizable. Numerical examples are presented to illustrate the effectiveness and robustness of the proposed design approach.

Index terms*:* Successive approximation approach, congestion controller, networked control systems, two-point boundary value problems, time-delay.

I. INTRODUCTION

It is well known that the insertion of the network in the feedback control loop makes the analysis and design of a networked control systems complex because the network imposes an undetermined communication delay [1-3]. The change of communication architecture from point-to-point to network, however, introduces different forms of time delay uncertainty between these devices. Delays are widely known to degrade the performance of a control system, so there has been a lot of research on networked control systems to reduce the performance degradation caused by delays. In Ref. [4], the author studied the effect of delays on the system modeling, and then a new optimal controller was designed to control the plant, however, the controller only considered the constant delay. In Ref. [5], the author utilized clock synchronization technology to evaluate the delays online, and then a LQR optimal controller based on the obtained delays was adopted to stabilize the plant, but the implementation of the controller caused some performance degradation. In Ref. [6], a fuzzy logic controller was used to control the networked control systems, which regretfully didn't use the communication information in design of controller. Ref. [7] proposes a new method to obtain a maximum allowable delay bound for a scheduling of networked control systems. The proposed method is formulated in terms of linear matrix inequalities and can give a much less conservative delay bound.

Since networked control systems are an integrated research area, which is not only concerned about control, but also relevant to communication, we must combine the knowledge of control and communication together to improve the system performance. Following this direction, in this paper, we address a novel scheme that integrates control technology with communication technology for a class of bilinear discrete-time networked control systems [8].

The rest of the paper is organized as follows. Section 2 presents the description of the problems and main assumptions. Section 3 will prove the existence and uniqueness of the two-point boundary value (TPBV) law. Section 4 discusses the detailed design scheme and algorithm of optimal control law for bilinear networked control systems. The validity of the laws will be illustrated by numerical examples in Sections 5. Conclusions are presented in Section 6.

II. PROBLEM FORMULATION

We consider the networked control systems consisting of a collection of bilinear plants whose feedback control loops are closed via a shared network link, as illustrated in Figure 1. All sample values of plant states are transmitted in one package [9-12].

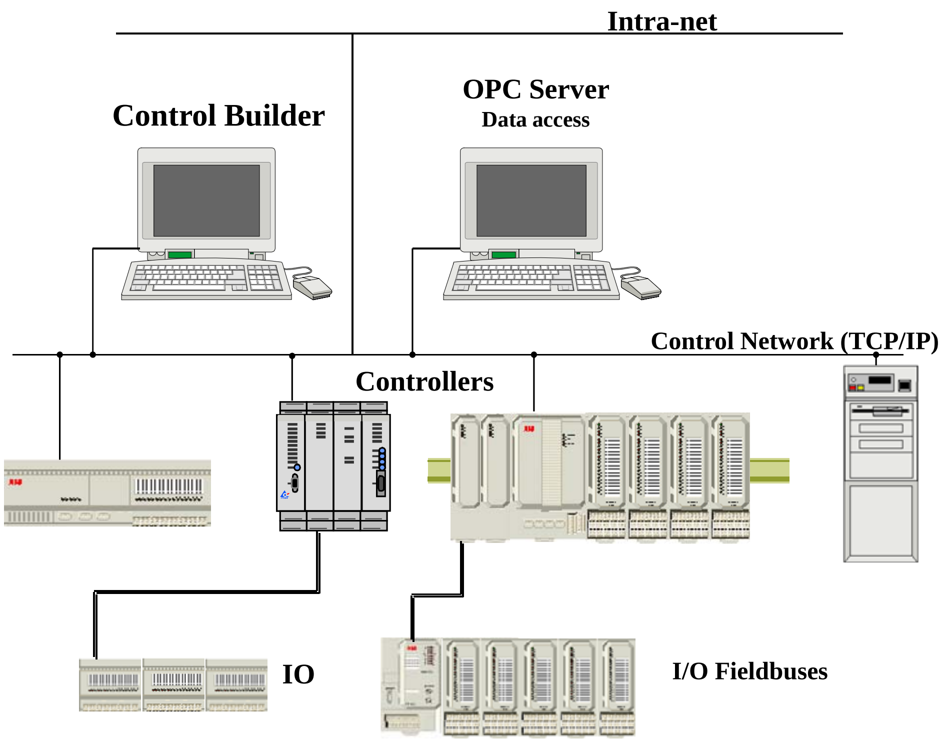

Figure 1. A collection of networked control systems shared by a communication link

The i-th plant ($i = 1, \ldots, N$) is given by

$$\begin{aligned} &\dot{x}(t) = A_{11}x(t) + A_{12}z(t) + A_{13}x(t-\tau) + B_1 u(t) + Dv(t) \\ &\varepsilon \dot{z}(t) = A_{21}x(t) + A_{22}z(t) + A_{23}x(t-\tau) + B_2 u(t), \quad t > 0 \\ &x(t) = \varphi(t), \quad -\tau \le t \le 0; \quad z(0) = z_0 \end{aligned} \tag{1}$$

where $x(t) \in R^n$ and $z(t) \in R^m$ are the state vectors, $u(t) \in R^r$ is the control input, τ is the positive time-delay, ε is the small positive perturbation parameter, and $\varphi(t)$ is the continuously differentiable initial function, A_{ij}, B_i $(i = 1, 2;\ j = 1, 2, 3)$ and D are the constant matrices of appropriate dimensions, $v \in R^p$ is the disturbances generated by the exosystem

$$\begin{aligned} &\dot{w}(t) = Gw(t) \\ &v(t) = Lw(t) \end{aligned} \tag{2}$$

where G and L are the constant matrices of appropriate dimensions.

Assumption 1. The pair (G, L) is observable completely.

Assumption 2. For the case of infinite-horizon, the real parts of all eigenvalues of G are non-positive. Moreover, the eigenvalues with zero real part are simple roots of minimum polynomial of the matrix G .

Remark 1. Exosystem (2) represents varieties of external disturbances. Assumption 2 ensures that the exosystem is critically stable or asymptotically stable.

The finite-horizon quadratic performance index is given by [13-14]

$$J = \frac{1}{2}\begin{bmatrix} x(t_f) \\ z(t_f) \end{bmatrix}^T F \begin{bmatrix} x(t_f) \\ z(t_f) \end{bmatrix} + \frac{1}{2}\int_0^{t_f} \left(\begin{bmatrix} x(t) \\ z(t) \end{bmatrix}^T Q \begin{bmatrix} x(t) \\ z(t) \end{bmatrix} + u^T(t) R u(t) \right) dt \tag{3}$$

where R is the positive definite matrix, F and Q are the positive semi-definite matrices with the block diagonal structures as $F = \begin{bmatrix} F_1 & F_2 \\ F_2^T & F_3 \end{bmatrix}$ and $Q = \begin{bmatrix} Q_1 & Q_2 \\ Q_2^T & Q_3 \end{bmatrix}$ with $F_1, Q_1 \in R^{n \times n}$, $F_2, Q_2 \in R^{n \times m}$ and $F_3,\ Q_3 \in R^{m \times m}$. Without loss of generality, we assume that $F_2 = 0$ and $Q_2 = 0$ in the following discussion. Our objective is to find the optimal control u^* such that the quadratic performance index (3) is minimized.

In view of the slow-fast decomposition theory of singular perturbation, the optimal problem (1) and (3) can be decomposed into the two optimal sub-problems. The slow time scale one is given by

$$\begin{aligned} &\dot{x}_s(t) = A_0 x_s(t) + A_3 x_s(t-\tau) + B_0 u_s(t) + Dv(t), \quad t > 0 \\ &x_s(t) = \varphi(t), \quad -\tau \le t \le 0 \end{aligned} \tag{4}$$

and

$$J_s = \frac{1}{2} x_s^T(t_f) F_0 x_s(t_f) + \frac{1}{2}\int_0^{t_f} \left[x_s^T(t) Q_0 x_s(t) + 2u_s^T(t) D_s x_s(t) + u_s^T(t) R_s u_s(t) \right] dt \tag{5}$$

where

$$\begin{aligned} A_0 &= A_{11} - A_{12} A_{22}^{-1} A_{21} \\ A_3 &= A_{13} - A_{12} A_{22}^{-1} A_{23} \\ B_0 &= B_1 - A_{12} A_{22}^{-1} B_2 \\ F_0 &= F_1 + A_{21}^T A_{22}^{-T} F_3 A_{22}^{-1} A_{21} \\ Q_0 &= Q_1 + A_{21}^T A_{22}^{-T} Q_3 A_{22}^{-1} A_{21} \\ D_s &= B_2^T A_{22}^{-T} Q_3 A_{22}^{-1} A_{21} \\ R_s &= R + B_2^T A_{22}^{-T} Q_3 A_{22}^{-1} B_2 \end{aligned} \tag{6}$$

The fast time scale optimal sub-problem is given by

$$\begin{aligned} &\varepsilon \dot{z}_f(t) = A_{22} z_f(t) + B_2 u_f(t), \quad t > 0 \\ &z_f(0) = z_0 - z_s(0) \end{aligned} \tag{7}$$

and

$$J_f = \frac{1}{2} z_f^T(t_f) F_3 z_f(t_f) + \frac{1}{2}\int_0^{t_f} \left[z_f^T(t) Q_3 z_f(t) + u_f^T(t) R u_f(t) \right] dt \tag{8}$$

where $z_s(t) = -A_{22}^{-1}[A_{21} x_s(t) + A_{23} x_s(t-\tau) + B_2 u_s(t)]$. The fast time scale optimal control law is given by

$$u_f^*(t) = -R^{-1} B_2^T P_f(t) z_f(t) \tag{9}$$

where $P_f(t)$ satisfies the *Riccati* differential equation

$$\begin{aligned} &\dot{P}_f(t) = P_f(t) S_2 P_f(t) - A_{22}^T P_f(t) - P_f(t) A_{22} - Q_3, \quad 0 \le t < t_f \\ &P_f(t_f) = F_3 \end{aligned} \tag{10}$$

with $S_2 = B_2 R^{-1} B_2^T$.

The slow time scale optimal control of system (4) with subject to quadratic performance index (5) is given by [15-19]

$$u_s^*(t) = -R_s^{-1}[D_s x_s(t) + B_0^T \lambda_s(t)] \tag{11}$$

where $\lambda_s(t) \in R^n$ satisfies TPBV problem[20-21]

$$\begin{aligned} &\dot{x}_s(t) = A_s x_s(t) + A_3 x_s(t-\tau) - S_0 \lambda_s(t) + Hw(t) \\ &-\dot{\lambda}_s(t) = Q_s x_s(t) + A_s^T \lambda_s(t) + d(t) \\ &x_s(t) = \varphi(t), \quad -\tau \le t \le 0 \\ &\lambda_s(t_f) = 0 \end{aligned} \tag{12}$$

with

$$\begin{aligned} &A_s = A_0 - B_0 R_s^{-1} D_s \\ &Q_s = Q_0 - D_s^T R_s^{-1} D_s \\ &S_0 = B_0 R_s^{-1} B_0^T, \quad H = DL \\ &d(t) = \begin{cases} A_3^T \lambda_s(t+\tau), & 0 < t \le t_f - \tau \\ 0, & t_f - \tau < t < t_f \end{cases} \end{aligned} \tag{13}$$

Note that TPBV problem (12) contains both delay and advance terms, obtaining the exact analytic solution of this problem is, in general, extremely difficult. It means that the optimal control $u_s^*(t)$ is difficult to obtain. Consequently, the composite control $u_c(t)$ of the original optimal problem (1) and (3) is impossible to obtain.

III. SIMPLIFICATION OF THE TPBV PROBLEM

To simplify TPBV problem(12), we introduce a perturbation parameter δ. Construct a new TPBV problem with δ as follows:

$$\begin{aligned} &\dot{x}_s(t,\delta) = A_s x_s(t,\delta) + \delta A_3 x_s(t-\tau,\delta) - S_0 \lambda_s(t,\delta) + Hw(t) \\ &-\dot{\lambda}_s(t,\delta) = Q_s x_s(t,\delta) + A_s^T \lambda_s(t,\delta) + \delta d(t,\delta) \\ &x_s(t,\delta) = \varphi(t), \quad -\tau \le t \le 0 \\ &\lambda_s(t_f,\delta) = 0 \end{aligned} \tag{14}$$

and a new optimal control law of the form

$$u_s(t,\delta) = -R_s^{-1}[D_s x_s(t,\delta) + B_0^T \lambda_s(t,\delta)] \tag{15}$$

Assume that the solution to the new TPBV problem (14) uniquely exists for any $|\delta| \le 1$. Note that for $\delta = 1$, TPBV problem (14) is equivalent to that in (12), and the control law (15) becomes (11). We also assume that $u_s(t,\delta)$, $x_s(t,\delta)$ and $\lambda_s(t,\delta)$ are infinitely differentiable with respect to δ at $\delta = 0$, and their *Maclaurin* series in δ can be written as

$$\alpha_s(t,\delta)=\sum_{i=0}^{\infty}\frac{\delta^i}{i!}\alpha_s^{(i)}(t),\quad \alpha_s\in\{x_s,u_s,\lambda_s\} \tag{16}$$

where $(\alpha_s)^{(i)}=\partial^i(\alpha_s)/\partial\delta^i\big|_{\zeta=0}$. We assume that the series sum in (16) is convergent at $\delta=1$ in the following discussion. When $\delta=1$, the optimal control law in (16) can be rewritten of the form

$$u_s(t)=\sum_{i=0}^{\infty}\frac{1}{i!}u_s^{(i)}(t) \tag{17}$$

Substituting (16) into (14) and (15) and comparing the coefficients of the same order terms with respect to δ, we obtain

$$\begin{aligned}
&\dot{x}_s^{(i)}(t)=A_s x_s^{(i)}(t)+\gamma^{(i)}(t)-S_0\lambda_s^{(i)}(t)+[1-\mathrm{sgn}(i)]Hw(t)\\
&-\dot{\lambda}_s^{(i)}(t)=Q_s x_s^{(i)}(t)+A_s^T\lambda_s^{(i)}(t)+\sigma^{(i)}(t)\\
&x_s^{(i)}(t)=[1-\mathrm{sgn}(i)]\varphi(t),\quad -\tau\le t\le 0\\
&\lambda_s^{(i)}(t_f)=0\\
&i=0,1,2,\cdots
\end{aligned} \tag{18}$$

and

$$u_s^{(i)}(t)=-R_s^{-1}[D_s x_s^{(i)}(t)+B_0^T\lambda_s^{(i)}(t)],\quad i=0,1,2,\cdots \tag{19}$$

where

$$\begin{aligned}
&\mathrm{sgn}(i)=\begin{cases}0, & i=0\\ 1, & i=1,2,\cdots\end{cases}\\
&\gamma^{(i)}(t)=\begin{cases}0, & i=0\\ iA_3x_s^{(i-1)}(t-\tau), & i=1,2,\cdots\end{cases}\\
&\sigma^{(i)}(t)=\begin{cases}0, & i=0,\quad 0\le t<t_f\\ iA_3^T\lambda_s^{(i-1)}(t+\tau), & i=1,2,\cdots;\quad 0\le t\le t_f-\tau\\ 0, & i=1,2,\cdots;\quad t_f-\tau<t<t_f\end{cases}
\end{aligned} \tag{20}$$

Through above transformation, the TPBV problem (12) is constructed into a series of TPBV problems(18). In the i-th TPBV problem, the delay and the advance terms are independent on the i-th variables. Therefore, the i-th TPBV problem is inhomogeneous, linear, without delay and advance terms. All the TPBV problems could be found by using an iterative process.

IV. APPROXIMATION PROCESS OF FINITE-HORIZON FFCC LAW

In this section, we will discuss in detail the design process of finite-horizon FFCC law via the perturbation method, and prove the existence and uniqueness of the infinite-horizon FFCC law.

Theorem 1: For the optimal control problem described by (1) and (3), there exists the unique FFCC law with the form as

$$u_c(t) = K_x(t)x(t) + K_z(t)z(t) + K_\tau(t)x(t-\tau) + K_c(t)[P_w(t)w(t) + g^{(\infty)}(t)] \tag{21}$$

with

$$\begin{aligned} & g^{(\infty)}(t) = \sum_{i=1}^{\infty} \frac{g^{(i)}(t)}{i!} \\ & K_z(t) = -R^{-1}B_2^T P_f(t) \\ & K_\tau(t) = K_z(t)A_{22}^{-1}A_{23} \\ & K_c(t) = -[I_r + K_z(t)A_{22}^{-1}B_2]R_s^{-1}B_0^T \\ & K_x(t) = K_z(t)A_{22}^{-1}A_{21} - [I_r + K_z(t)A_{22}^{-1}B_2]R_s^{-1}[D_s + B_0^T P_s(t)] \end{aligned} \tag{22}$$

where I_r is the identity matrix of dimension r, $P_f(t)$ and $P_s(t)$ are the unique positive semi-definite solutions of *Riccati* differential equations (10) and

$$\begin{aligned} & \dot{P}_s(t) = P_s(t)S_0P_s(t) - A_s^T P_s(t) - P_s(t)A_s - Q_s, \quad 0 \le t < t_f \\ & P_s(t_f) = F_0 \end{aligned} \tag{23}$$

$P_w(t)$ is the unique solution of matrix differential equation

$$\begin{aligned} & \dot{P}_w(t) = [P_s(t)S_0 - A_s^T]P_w(t) - P_s(t)H - P_w(t)G, \quad 0 \le t < t_f \\ & P_w(t_f) = 0 \end{aligned} \tag{24}$$

and $g^{(i)}(t)$ is the unique solution of vector differential equation end-point problem

$$\begin{aligned} & \dot{g}^{(i)}(t) = [S_0P_s(t) - A_s]^T g^{(i)}(t) - iP_s(t)A_3x^{(i-1)}(t-\tau) - \sigma_1^{(i)}(t) \\ & g^{(i)}(t_f) = 0, \quad i = 1, 2, \cdots \\ & g^{(0)}(t) = 0, \quad 0 \le t < t_f \end{aligned} \tag{25}$$

with

$$\sigma_1^{(i)}(t) = \begin{cases} iA_3^T[P_s(t+\tau)x^{(i-1)}(t+\tau) + g^{(i-1)}(t+\tau)], & 0 \le t \le t_f - \tau \\ 0, & t_f - \tau < t < t_f \end{cases} \tag{26}$$

and $x^{(i)}(t)$ satisfies

$$\begin{aligned}&\dot{x}^{(i)}(t)=[A_s-S_0P_s(t)]x^{(i)}(t)+\gamma_1^{(i)}(t)-S_0g^{(i)}(t)\\&\qquad+[1-\mathrm{sgn}(i)][H-S_0P_w(t)]w(t),\quad i=0,1,2,\cdots;\quad 0<t\le t_f\\&x^{(i)}(t)=0,\quad i=1,2,\cdots;\quad -\tau\le t\le 0\\&x^{(0)}(t)=\varphi(t),\quad -\tau\le t\le 0\end{aligned}\tag{27}$$

with

$$\gamma_1^{(i)}(t)=\begin{cases}0, & i=0\\ iA_3x^{(i-1)}(t-\tau), & i=1,2,\cdots\end{cases}\tag{28}$$

Proof. Let

$$\lambda_s^{(i)}(t)=P_s(t)x_s^{(i)}(t)+[1-\mathrm{sgn}(i)]P_w(t)w(t)+g^{(i)}(t),\quad i=0,1,2,\cdots\tag{29}$$

where $g^{(i)}(t)\in R^n$ is the i-th adjoint vector, and $g^{(0)}(t)\equiv 0$.

Substituting (29) into (18) and(19), we easily obtain the i-th state equation of the slow subsystem:

$$\begin{aligned}&\dot{x}_s^{(i)}(t)=[A_s-S_0P_s(t)]x_s^{(i)}(t)+\gamma^{(i)}(t)-S_0g^{(i)}(t)\\&\qquad+[1-\mathrm{sgn}(i)][H-S_0P_w(t)]w(t),\quad i=0,1,2,\cdots;\quad 0<t\le t_f\\&x_s^{(i)}(t)=0,\quad i=1,2,\cdots;\quad -\tau\le t\le 0\\&x_s^{(0)}(t)=\varphi(t),\quad -\tau\le t\le 0\end{aligned}\tag{30}$$

and the corresponding optimal control

$$\begin{aligned}u_s^{(i)}(t)=&-R_s^{-1}[D_s+B_0^TP_s(t)]x_s^{(i)}(t)-R_s^{-1}B_0^Tg^{(i)}(t)\\&-[1-\mathrm{sgn}(i)]R_s^{-1}B_0^TP_w(t)w(t),\quad i=0,1,2,\cdots\end{aligned}\tag{31}$$

Taking the derivative to the both sides of (29), together with (30), we get

$$\begin{aligned}\dot{\lambda}_s^{(i)}(t)=&\{\dot{P}_s(t)+P_s(t)[A_s-S_0P_s(t)]\}x_s^{(i)}(t)\\&+P_s(t)\gamma^{(i)}(t)-P_s(t)S_0g^{(i)}(t)+\dot{g}^{(i)}(t)\\&+[1-\mathrm{sgn}(i)]\{P_s(t)[H-S_0P_w(t)]+P_w(t)G+\dot{P}_w(t)\}w(t)\end{aligned}\tag{32}$$

Substituting (29) into the second equation of (18) and comparing with (32), one can obtain the *Riccati* differential equation (23), *Sylvester* matrix differential equation (24) and adjoint vector differential equations:

$$\begin{aligned}&\dot{g}^{(i)}(t)=[S_0P_s(t)-A_s]^Tg^{(i)}(t)-iP_s(t)A_3x_s^{(i-1)}(t-\tau)-\sigma^{(i)}(t)\\&g^{(i)}(t_f)=0,\quad i=1,2,\cdots\\&g^{(0)}(t)=0,\quad 0\le t<t_f\end{aligned}\tag{33}$$

By solving matrix differential equation end-point problems(23), (24), we get $P_s(t)$ and $P_w(t)$, respectively. Further, we obtain $g^{(i)}(t)$ from (33). Consequently, the optimal control of the i-th slow subsystem is determined uniquely by (31). Hence, from (16), (17) and (31), we obtain

$$u_s^*(t) = -R_s^{-1}[D_s + B_0^T P_s(t)]x_s(t) - R_s^{-1}B_0^T P_w(t)w(t) - R_s^{-1}B_0^T g^{(\infty)}(t) \tag{34}$$

On the other hand, from (9) and (10), we get the optimal control law of the fast subsystem. Further, by replacing $x_s(t)$ with $x(t)$ and $z_f(t)$ with $z(t) - z_s(t)$ in (9) and (34), we can directly obtain the composite control law $u_c(t)$ expressed by (21). Also, by replacing $x_s(t)$ with $x(t)$ in (33) and (30), we obtain (25) and (27). The proof is complete.

V. INFINITE-HORIZON FFCC LAW

In this section, we will discuss in detail the design process of infinite-horizon FFCC law via the perturbation method, and prove the existence and uniqueness of the infinite-horizon FFCC law.

We know that if the exosystem is critically stable as $t_f \to \infty$, then the disturbances v will tend to oscillation with constant amplitudes, and the state vector x and the control vector u are impossible to tend to zero contemporaneously. Therefore, the quadratic performance index

$$J = \int_0^\infty \left\{ \begin{bmatrix} x(t) \\ z(t) \end{bmatrix}^T Q \begin{bmatrix} x(t) \\ z(t) \end{bmatrix} + u(t)^T R u(t) \right\} dt \tag{35}$$

may not be convergent. In this case, we can choose quadratic average performance index as

$$J = \lim_{t_f \to \infty} \frac{1}{t_f} \int_0^{t_f} \left\{ \begin{bmatrix} x(t) \\ z(t) \end{bmatrix}^T Q \begin{bmatrix} x(t) \\ z(t) \end{bmatrix} + u(t)^T R u(t) \right\} dt \tag{36}$$

It is necessary to emphasize here that when the exosystem is asymptotically stable, quadratic performance index (35) is available, and the analysis process of corresponding FFCC law is similar to the case of choosing the quadratic average performance index.

In order to prove the existence and uniqueness of the FFCC law, the following Lemma is useful.

Lemma 1. Assume that $\tilde{H} \in R^{m\times m}$, $\tilde{E} \in R^{n\times n}$, $\tilde{L} \in R^{n\times m}$, then *Sylvester* matrix equation

$$\tilde{E}X + X\tilde{H} + \tilde{L} = 0 \tag{37}$$

has unique solution X if and only if $\lambda+\mu\neq 0$ for any $\lambda\in\sigma(\tilde{E})$ and $\mu\in\sigma(\tilde{H})$ with $\sigma(\cdot)$ denoting spectra of matrix.

Theorem 2: Consider the optimal control problem of the standard linear time-delay singularly perturbed system (1) with subject to infinite-horizon quadratic average performance index (36). Assume that:

The triples $(A_{22}, B_2, Q_3^{1/2})$ and $(A_0, B_0, Q_s^{1/2})$ are controllable-observable completely;

The Assumption 2 holds.

Then there exists the unique FFCC law $u_c(t)$ formulated as

$$u_c(t)=K_x x(t)+K_z z(t)+K_\tau x(t-\tau)+K_c[P_w w(t)+g^{(\infty)}(t)] \tag{38}$$

with

$$\begin{aligned} K_z &= -R^{-1}B_2^T P_f, \quad K_\tau = K_z A_{22}^{-1}A_{23} \\ K_c &= -(I_r + K_z A_{22}^{-1}B_2)R_s^{-1}B_0^T \\ K_x &= K_z A_{22}^{-1}A_{21} - (I_r + K_z A_{22}^{-1}B_2)R_s^{-1}(D_s + B_0^T P_s) \end{aligned} \tag{39}$$

P_s and P_f are the unique positive definite solutions of the algebraic *Riccati* equations

$$A_s^T P_s + P_s A_s - P_s S_0 P_s + Q_s = 0 \tag{40}$$

and

$$A_{22}^T P_f + P_f A_{22} - P_f S_2 P_f + Q_3 = 0 \tag{41}$$

P_w is the unique solution of the *Sylvester* matrix equation

$$(A_s^T - P_s S_0)P_w + P_w G + P_s H = 0 \tag{42}$$

$g^{(i)}(t)$ satisfies the adjoint state vector differential equation

$$\begin{aligned} &\dot{g}^{(i)}(t) = (S_0 P_s - A_s)^T g^{(i)}(t) - iP_s A_3 x^{(i-1)}(t-\tau) - \sigma_0^{(i)}(t) \\ &\lim_{t_f\to\infty} g^{(i)}(t_f) = 0, \quad i = 1, 2, \cdots \\ &g^{(0)}(t) = 0, \quad t \ge 0 \end{aligned} \tag{43}$$

with

$$\sigma_0^{(i)}(t) = \begin{cases} iA_3^T[P_s x^{(i-1)}(t+\tau) + g^{(i-1)}(t+\tau)], & 0 \le t \le t_f - \tau \\ 0, \quad t_f - \tau < t < t_f \end{cases} \tag{44}$$

and

$$\begin{aligned}\dot{x}^{(i)}(t) &= (A_s - S_0 P_s) x^{(i)}(t) + \gamma_1^{(i)}(t) - S_0 g^{(i)}(t) \\ &\quad + [1-\mathrm{sgn}(i)](H - S_0 P_w) w(t), \quad i = 0,1,2,\cdots; \quad t > 0 \\ x^{(i)}(t) &= 0, \quad i = 1,2,\cdots; \quad -\tau \le t \le 0 \\ x^{(0)}(t) &= \varphi(t), \quad -\tau \le t \le 0\end{aligned} \tag{45}$$

Proof. Let

$$\lambda_s^{(i)}(t) = P_s x_s^{(i)}(t) + [1-\mathrm{sgn}(i)] P_w w(t) + g^{(i)}(t), \quad i = 0,1,2,\cdots \tag{46}$$

Proceeding in a manner similar to that of the case of finite-horizon in section 3.2, we obtain (38), (40), (41), (42), (43) and (45), which are analogs of (21), (23), (10), (24), (25) and (27), respectively.

In the following, we only need to prove the existence and uniqueness of FFCC law (38). Obviously, it is equivalent to the existence and uniqueness of the solutions to algebraic *Riccati* matrix equations (40), (41) and *Sylvester* matrix equation (42).

Note that the triples $(A_0, B_0, Q_s^{1/2})$ and $(A_{22}, B_2, Q_3^{1/2})$ are controllable-observable completely, then algebraic *Riccati* matrix equations (40) and (41) have unique positive definite solutions P_s and P_f. From the regulator theory of linear system, it follows that for any $\zeta \in \sigma(A_s - S_0 P_s)$, the inequality $\mathrm{Re}\,\zeta < 0$ holds. In view of Assumption 2, $\mathrm{Re}\,\mu \le 0$ holds for any $\mu \in \sigma(G)$. Thus, by Lemma 1, *Sylvester* matrix equation (42) has the unique solution P_w. The proof is complete.

VI. PHYSICALLY REALIZABLE PROBLEM OF THE FFCC LAW

In this section, we construct a reduced-order disturbances observer to make the FFCC laws physically realizable, and the design algorithm is also presented in the sense of the practical engineering.

The optimal control law $u_c(t)$ in (38) contains the unknown state variable $w(t)$ of exosystem (2), which is physically unrealizable. In the practical engineering, we can introduce a disturbances observer to make it physically realizable.

We now construct a reduced-order observer for the state of the exosystem. It is well known that for the full rank matrix L in exosystem (2), there exists a constant matrix $\overline{L} \in R^{(q-p)\times q}$ such that the matrix $\begin{bmatrix} L^T & \overline{L}^T \end{bmatrix} \in R^{q\times q}$ is nonsingular. Let

$$T = \begin{bmatrix} L \\ \bar{L} \end{bmatrix}^{-1} = \begin{bmatrix} T_1 & T_2 \end{bmatrix}, \quad TGT^{-1} = \begin{bmatrix} G_1 & G_{12} \\ G_{21} & G_2 \end{bmatrix} \tag{47}$$

where $T_1 \in R^{q\times p}$, $T_2 \in R^{q\times(q-p)}$, $G_1 \in R^{p\times p}$, $G_{12} \in R^{p\times(q-p)}$, $G_{21} \in R^{(q-p)\times p}$ and $G_2 \in R^{(q-p)\times(q-p)}$ are constant matrices. In order to construct a disturbances observer, we make the equivalent linear transformation $w = T\bar{w}$. Denote that $\bar{w}^T = [\bar{w}_1^T \quad \bar{w}_2^T]$, where $\bar{w}_1 \in R^p$, and $\bar{w}_2 \in R^{(q-p)}$. An equivalent system of the exosystem is obtained as follows

$$\begin{aligned} \dot{\bar{w}}_1(t) &= G_1\bar{w}_1(t) + G_{12}\bar{w}_2(t) \\ \dot{\bar{w}}_2(t) &= G_{21}\bar{w}_1(t) + G_2\bar{w}_2(t) \\ v(t) &= \bar{w}_1(t) \end{aligned} \tag{48}$$

where $\bar{w}_1(t)$ is just the external disturbance vector $v(t)$. We need only construct a reduced-order observer with respect to $\bar{w}_2(t)$. Note that $LT = \begin{bmatrix} I_p & 0 \end{bmatrix}$ and the pair (G, L) is completely observable, obviously the pair (G_2, G_{12}) is also completely observable. Construct the reduced-order observer as follows

$$\begin{aligned} \dot{\eta}(t) &= \hat{G}\eta(t) + \hat{L}v(t) \\ w_2(t) &= \eta(t) + Kv(t) \end{aligned} \tag{49}$$

where $\eta \in R^{q-p}$ is a constructed variable, $w_2(t)$ is the observing value of $\bar{w}_2(t)$, K is an undetermined coefficient matrix, and

$$\begin{aligned} \hat{G} &= G_2 - KG_{12} \\ \hat{L} &= G_2K - KG_{12}K + G_{21} - KG_1 \end{aligned} \tag{50}$$

In order to guarantee the speediness and nicety of observer (49), we can select matrix K such that all the eigenvalues of matrix $G_2 - KG_{12}$ are assigned to appointed places. From (2), (47) and (48), we can get the observing value of $w(t)$ as follows

$$\hat{w}(t) = T_2\eta(t) + (T_1 + T_2K)v(t) \tag{51}$$

By above reconstruction of $w(t)$, the FFCC law in (38) can be expressed as

$$u_c(t) = K_x x(t) + K_z z(t) + K_\tau x(t-\tau) + K_c[P_w\hat{w}(t) + g^{(\infty)}(t)] \tag{52}$$

where $\hat{w}(t)$ is determined by (49) and (51).

Remark 2. In (52), the action of the term $K_c g^{(\infty)}(t)$ is to compensate the unfavorable effect derived from the time-delay, while $K_c P_w \hat{w}(t)$ compensates the effect caused by the external disturbances. Especially, if $P_w = 0$, then we can obtain feedback composite control (FCC) law of the form

$$u_c(t) = K_x x(t) + K_z z(t) + K_\tau x(t-\tau) + K_c g^{(\infty)}(t) \tag{53}$$

If system (1) is in the absence of external disturbances, namely, $v(t) \equiv 0$, then (53) also represents the composite suboptimal control (CSC) law of the singularly perturbed time-delay system.

Remark 3. In practical applications, by replacing ∞ with suitable integer M in (52) and (53), we obtain respectively the approximate FFCC law and FCC (or CSC) law as follows:

$$u_M(t) = K_x x(t) + K_z z(t) + K_\tau x(t-\tau) + K_c [P_w \hat{w}(t) + g_M(t)] \tag{54}$$

$$u_M(t) = K_x x(t) + K_z z(t) + K_\tau x(t-\tau) + K_c g_M(t) \tag{55}$$

where $g_M(t) = \sum_{i=1}^{M} (g^{(i)}(t) / i!)$. Further, from (54) we can get the recursion formula of dynamic approximate FFCC law:

$$\begin{aligned} u_i(t) &= u_{i-1}(t) + \frac{K_c}{i!} g^{(i)}(t), \quad i = 1, 2, \cdots, M \\ u_0(t) &= K_x x(t) + K_z z(t) + K_\tau x(t-\tau) + K_c P_w \hat{w}(t) \end{aligned} \tag{56}$$

Similarly, for FCC (or CSC) law (55) we have

$$\begin{aligned} u_i(t) &= u_{i-1}(t) + \frac{K_c}{i!} g^{(i)}(t), \quad i = 1, 2, \cdots, M \\ u_0(t) &= K_x x(t) + K_z z(t) + K_\tau x(t-\tau) \end{aligned} \tag{57}$$

Remark 4. For finite-horizon FFCC law (21), there are similar results to that of (52)-(57).

In fact, the iteration times M in (56) and (57) can be determined by the given tolerance error bound α, $0 < \alpha < 1$. We now give the design algorithm as follows.

Algorithm 1. FFCC law of system (1)

Step 1: Calculate the matrices P_s, P_f and P_w from (40), (41)and (42) respectively, $u_0(t)$ from the second equation in (56).

Step 2: Give the tolerance error bound α. Let $M = 0$, $i = 1$. Obtain $x(t)$ by substituting $u_0(t)$ into system (1), and then calculate J_0 from the following equation

$$J_M = \lim_{t_f \to \infty} \frac{1}{t_f} \int_0^{t_f} \left\{ \begin{bmatrix} x(t) \\ z(t) \end{bmatrix}^T Q \begin{bmatrix} x(t) \\ z(t) \end{bmatrix} + (u_M(t))^T R u_M(t) \right\} dt \tag{58}$$

Step 3: Obtain $g^{(i)}(t)$ from (43), $u_i(t)$ from the first equation in (56), then obtain $x(t)$ by substituting $u_i(t)$ into system (1).

Step 4: Let $M = i$, and then calculate J_M from (58).

Step 5: If $|(J_M - J_{M-1})/J_M| < \alpha$, then stop and output $u_M(t)$. Or else, find the state vector $x^{(i)}(t)$ from (45).

Step 6: Letting $i = i+1$, go to Step 3.

VII. NETWORKED CONTROL LOOP

To demonstrate the feasibility and effectiveness of the proposed method, numerical examples are carried out in figure 2.

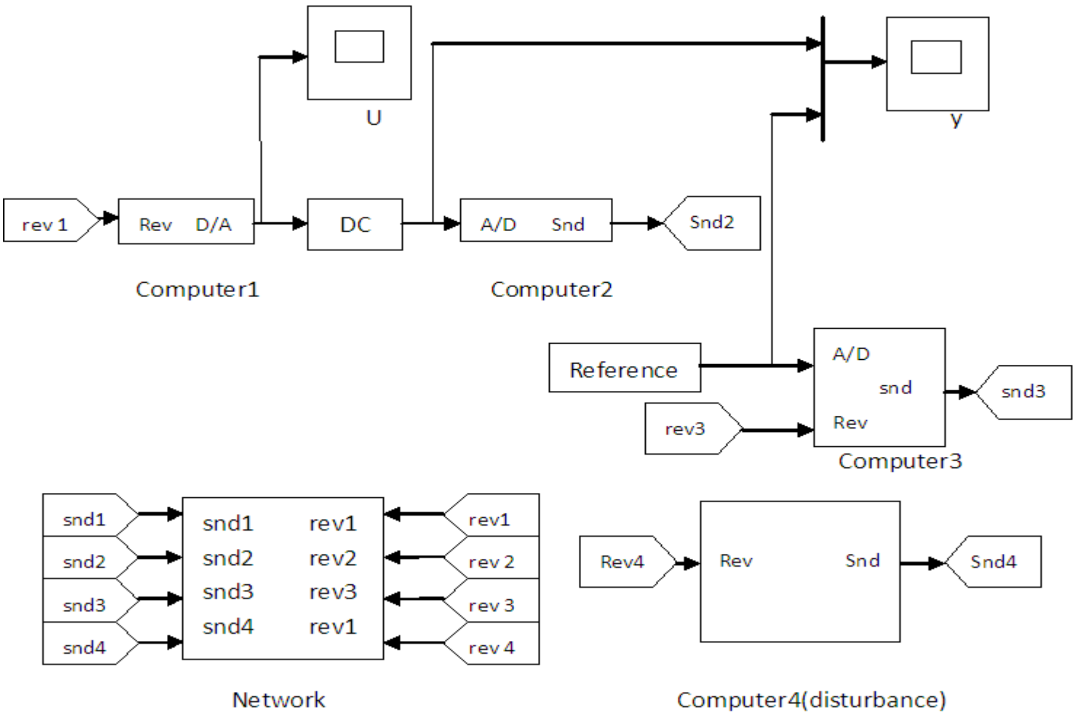

Figure 2. True time model of networked control loop

We consider linear singularly perturbed time-delay system with the specific matrices:

$$A_{11}=\begin{bmatrix}-0.5 & 1\\ 0 & -0.5\end{bmatrix},\quad A_{12}=\begin{bmatrix}-2 & 0.1\\ -1 & -2\end{bmatrix}, A_{13}=\begin{bmatrix}1 & -0.3\\ 0 & -1\end{bmatrix},$$
$$A_{21}=\begin{bmatrix}-0.4 & 0.5\\ 0 & -0.4\end{bmatrix}, A_{22}=\begin{bmatrix}0.5 & 0\\ 0.1 & 0.5\end{bmatrix},\quad A_{23}=\begin{bmatrix}0 & 0.2\\ -0.1 & 0.05\end{bmatrix} \tag{59}$$
$$D=\begin{bmatrix}0.1 & 1.5\\ 0 & -1\end{bmatrix},\quad B_1=\begin{bmatrix}0\\ 2\end{bmatrix},\quad B_2=\begin{bmatrix}0\\ 1\end{bmatrix}$$

The initial conditions are given by

$$\varphi(t)=\begin{bmatrix}1 & 0.5\end{bmatrix}^T,\quad -0.2\le t\le 0$$
$$z(0)=\begin{bmatrix}0 & -2\end{bmatrix}^T \tag{60}$$

Exosystem (2) is critically stable. Let

$$G_c=\begin{bmatrix}0 & 3 & 0 & 0\\ -1 & 0 & 0 & 0\\ 0 & 0 & 0 & -1\\ 0 & 0 & 3 & 0\end{bmatrix} \tag{61}$$

The quadratic performance index is chosen as

$$J=\int_0^{t_f}\left\{\begin{bmatrix}x(t)\\ z(t)\end{bmatrix}^T\begin{bmatrix}Q_1 & 0\\ 0 & Q_3\end{bmatrix}\begin{bmatrix}x(t)\\ z(t)\end{bmatrix}+u^T(t)Ru(t)\right\}dt \tag{62}$$

The corresponding slow state variables x_1, x_2, and the fast state variables z_1, z_2 are presented in Figs.3-4, respectively, where the solid lines for the suboptimal trajectories of the FFCC, while the dash-dotted lines for the FCC.

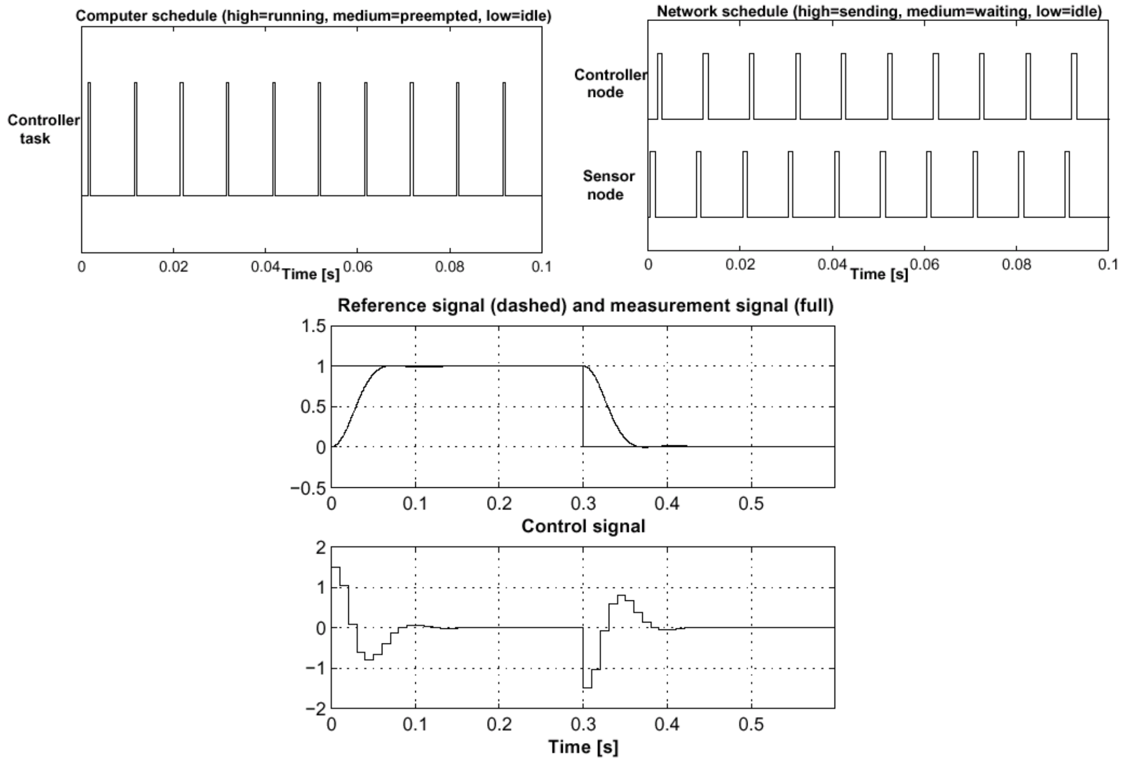

Figure 3. Results without interference

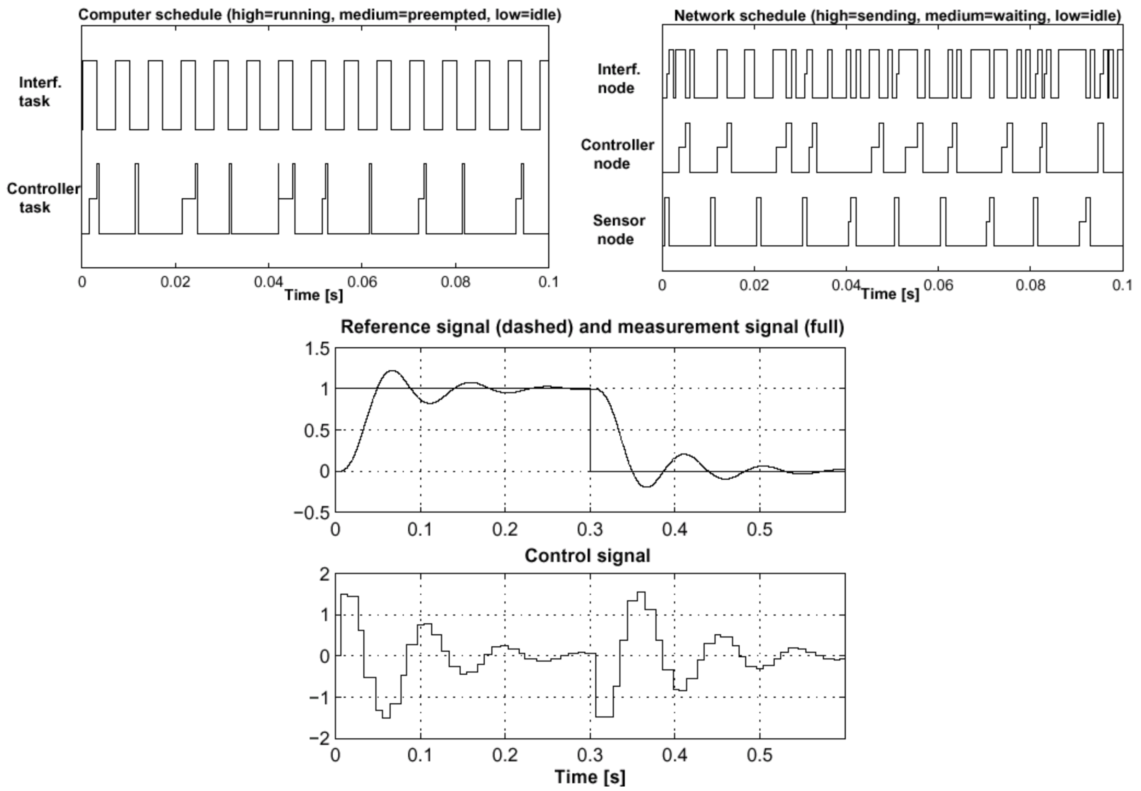

Figure 4. Results with interference

From Fig. 3 and Fig. 4, we can see clearly that the perturbation method proposed in this paper is valid for the optimal control problem for the singularly perturbed time-delay systems, and preserves very good convergence.

VIII. CONCLUSIONS

In this paper, the decision-making law has been studied for the singularly perturbed time-delay networked control systems affected by external disturbances. The optimization problem of the linear time-delay singularly perturbed systems is replaced by a non-delay sequence of the singularly perturbed optimization problems via the perturbation method, and the feedforward and feedback optimal control technique is used to reject the external disturbances. This method avoids ill-defined numerical TPBV problem and reduces the size of computations. On the other hand, it is shown that the FFCC laws proposed are effective and easy to implement, and more robust with respect to external disturbances.

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