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DE-NOISING SIGNAL OF THE QUARTZ FLEXURAL ACCELEROMETER BY MULTIWAVELET SHRINKAGE

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Abstract- Methods of de-noising the output signal of the JSD-I/A quartz flexural accelerometer based on five types of multiwavelets are comparatively investigated in this paper. Firstly, the theory of multiwavelet transform and the generalized cross validation criterion are analyzed. Secondly, because the JSD-I/A quartz flexural accelerometer which is fixed in SCT-1 two-axis rotation platform by the appropriative clamp has a start-up procedure of 3 minutes, the output signal of the quartz flexural accelerometer are sampled after applying the voltage for 5 minutes. Thirdly, based on the soft threshold function and the universal threshold, GHM orthogonal multiwavelet, SA4 orthogonal multiwavelet, CL orthogonal multiwavelet, Cardbal2 balanced multiwavelet and BIGHM biorthogonal multiwavelet are

applied to de-noise the sampled signal of the JSD-I/A quartz flexural accelerometer with 4 decomposition level, respectively. Lastly, the generalized cross validation criterion is used to evaluate the de-noising effects of the above five multiwavelets. The experimental results shows that the generalized cross validation value of BIGHM biorthogonal multiwavelet is effective in de-nosing the the output signal of the JSD-I/A quartz flexural accelerometer, and offer the best performance than GHM orthogonal multiwavelet, SA4 orthogonal multiwavelet, CL orthogonal multiwavelet and Cardbal2 balanced multiwavelet.

Index terms*:* GHM orthogonal multiwavelet, SA4 orthogonal multiwavelet, CL orthogonal multiwavelet, Cardbal2 balanced multiwavelet, BIGHM biorthogonal multiwavelet, generalized cross validation, JSD-I/A quartz flexural accelerometer.

I. INTRODUCTION

Since the 20th century, marine gravimeters are widely used in the fields such as the offshore oil and gas exploration, the marine geothermal resources survey, gravity passive navigation, etc. Many countries in the world have developed various marine gravimeters that are used for measuring the marine gravity changes. For example, S-marine gravimeter is developed by American LaCoste & Romberg Company and BGM-3 gravimeter is developed by American Bell Company. Many kinds of the metal zeros-length spring, the quartz zeros-length spring and the inertial accelerometer are adopted for the gravitational sensors of marine gravimeter. In the development of the high precision marine gravimeter, the high precision quartz flexural gravitational sensor has been adopted as the gravitational sensor in our research project. Because the signal de-noising of the quartz flexural gravitational sensor is one of the key technologies of the high precision marine gravimeter, the output signal processing of the quartz flexural gravitation sensor based on multiwavelet analysis is researched in the paper.

II. MULTIWAVELET THEORY

Multiresolution analysis is researched firstly in this section; on the basis we analyze the multiscaling function and multiwavelet function and the orthogonal multiwavelet transform [1 ~ 4].

a. Multiwavelet analysis

Let function $\Phi(t)=[\phi_1(t),\cdots,\phi_r(t)]^{\mathrm{T}}\in L^2(R)^r$ (r = 2 in the paper) be the vector function with $\phi_i(t)\in L^2(R), i=1,2,\cdots,r$. For every $j\in Z$ the following subspace is defined:

$$V_j=\overline{span\{2^{j/2}\phi_i(2^j t\text{-}k):1\le i\le r,k\in Z\}} \tag{1}$$

Vector function Φ(t) is called a multiscaling function if the subspace V_j, defined in the formula (1), satisfies the following properties:

(1) $\cdots\subset V_{-1}\subset V_0\subset V_1\subset\cdots$;

(2) $\overline{\bigcup_{j\in Z}V_j}=L^2(R)$ and $\bigcap_{j\in Z}V_j=\{0\}$;

(3) $f(t)\in V_j\Leftrightarrow f(2t)\in V_{j+1},\forall j\in Z$;

(4)function family $\{\phi_i(t-k):1\le i\le r,k\in Z\}$ is a Riesz basis of V_0.

and saying that the multiwavelet generates a r multiplicity multiresolution analysis of space $L^2(R)$.

Because the multiscaling function $\Phi(t)\in V_0\subset V_1$ and the subspace V_1 is generated by $\left\{\sqrt{2}\phi_i(2t-k):1\le i\le r,k\in Z\right\}$, the multiscaling function satisfies the two-scale equation:

$$\Phi(t)=\sqrt{2}\sum_k G_k\Phi(2t-k) \tag{2}$$

where $G_{k\in Z}\in\ell^2(Z)^{r\times r}$ is $r\times r$ dimension low-pass filter coefficient matrix.

If the scaling function set $\{\phi_{i,j,k}(t)=2^{j/2}\phi_i(2^j t-k):1\le i\le r,k\in Z\}$ is a Riesz basis of subspace V_j, the multiscaling function $\Phi(t)$ can be denoted:

$$\Phi_{j,k}(t)=\left[\phi_{1,j,k}(t),\cdots,\phi_{r,j,k}(t)\right]^T \tag{3}$$

A multiscaling function $\Phi(t)$ is orthogonal if the basis $\{\phi_i(t-k):1\le i\le r,k\in Z\}$ is not only a Riesz basis of V_0, but also is orthogonal:

$$\left\langle\Phi(t),\Phi(t-k)\right\rangle=\int\Phi(t)\Phi^T(t-k)dt=\mathrm{I}_r\delta_{k,0}(k\in Z) \tag{4}$$

b. Multiwavelet Function

Let W_j represents the complementary space of V_j in the space V_{j+1}, so we have

$$V_{j+1} = V_j \dot{+} W_j \text{ and } V_j \cap W_j = \{0\} \quad (5)$$

It can be known from the properties (1) and (2) of the define of multiresolution analysis that $\dot{+}_{j \in Z} W_j = L^2(R)$.

Let $\Psi(t) = [\psi_1(t), \cdots, \psi_r(t)]^T \in L^2(R)^r$, for every $j \in Z$ the following subspace is defined:

$$W_j = \overline{span\{2^{j/2}\psi_i(2^j t - k) : 1 \le i \le r, k \in Z\}} \quad (6)$$

The function $\Psi(t)$ is called a multiwavelet function if $\{\psi_i(t-k) \colon 1 \le i \le r, k \in \mathrm{Z}\}$ is a Riesz basis of W_0. The wavelet function set $\{\psi_{i,j,k}(t) = 2^{j/2}\psi_i(2^j t - k) \colon 1 \le i \le r, k \in \mathrm{Z}\}$ is a Riesz basis of subspace W_j, and $\{\psi_{i,j,k}(t) : 1 \le i \le r, j, k \in \mathrm{Z}\}$ is a Riesz basis of space $L_2(R)$. Similar with the multiscaling function, we can denote $\Psi_{j,k}(t) = [\psi_{1,j,k}(t), \ldots, \psi_{r,j,k}(t)]^{\mathrm{T}}$.

Because the wavelet functions $\psi_1(t), \cdots, \psi_r(t)$ belong to $W_0 \subset V_1$ and the subspace V_1 has the basis $\left\{\sqrt{2}\phi_i(2t - k) : 1 \le i \le r, k \in Z\right\}$, the multiwavelet function satisfies the two-scale equation:

$$\Psi(t) = \sqrt{2}\sum_k H_k \Phi(2t - k) \quad (7)$$

where $\mathrm{H}_{k \in Z} \in \ell^2(Z)^{r \times r}$ is $r \times r$ dimension high-pass filter coefficient matrix. The multiresolution analysis is called a orthogonal multiresolution analysis, if the multiwavelet space W_j, defined in formula (6), is a orthogonal complementary space of V_j in the space V_{j+1}, namely,

$$V_{j+1} = V_j \oplus W_j \text{ and } V_j \perp W_j$$

A multiwavelet function $\Psi(t)$ is called a semi-orthogonal multiwavelet, if it generates a orthogonal multiresolution analysis. If the multiwavelet function satisfies $\left\langle \Psi(2^j t - k), \Psi(2^i t - n) \right\rangle = \delta_{j,i}\delta_{k,n} I_r (j, i, k, n \in Z)$, it is a orthogonal multiwavelet.

c. Orthogonal discrete multiwavelet transform

First let $f(t) \in V_0$ and $V_0 \subset V_1$, then define

$$v_{0,k}^T = \left\langle f(t), \Phi_{0,k}(t) \right\rangle \text{ and } v_{1,k}^T = \left\langle f(t), \Phi_{1,k}(t) \right\rangle \quad (8)$$

Let $v_{0,k}$ and $v_{1,k}$ are $r \times 1$ dimension vector. It can be obtained from formula (2) that $\Phi_{0,k}(t) = \sum_m G_m \Phi_{1,2k+m}(t)$. Substituting the above formula into the left formula of formulas (8):

$$\begin{aligned} v_{0,k}^T &= \int f(t)\Phi_{0,k}^T(t)dt = \int f(t)\sum_m \Phi_{1,2k+m}^T(t)G_m^T dt \\ &= \sum_m \left[\int f(t)\Phi_{1,2k+m}^T(t)dt \right] G_m^T \\ &= \sum_m \left\langle f(t), \Phi_{1,2k+m}(t) \right\rangle G_m^T = \sum_m v_{1,2k+m}^T G_m^T \end{aligned} \tag{9}$$

After transforming the formula (9) we can obtain:

$$v_{0,k} = \sum_m G_{m-2k} v_{1,m} \tag{10}$$

Without loss of generality, formula (10) can be extended:

$$v_{j,k} = \sum_m G_{m-2k} v_{j+1,m} = < f(t), \Phi_{j,k}(t) >^T \tag{11}$$

Let $w_{0,k}^T = \left\langle x(t), \Psi_{0,k}(t) \right\rangle$, and $w_{0,k}$ be $r\times 1$ dimension vector. It can be obtained from formula (7) that

$$\Psi_{0,k}(t) = \sum_m H_m \Phi_{1,2k+m}(t) \tag{12}$$

Similarly,

$$\begin{aligned} w_{0,k}^T &= \int f(t)\sum_m \Phi_{1,2k+m}^T(t)H_m^T dt \\ &= \sum_m \left[\int f(t)\Phi_{1,2k+m}^T(t)dt \right] H_m^T \\ &= \sum_m \left\langle f(t), \Phi_{1,2k+m}(t) \right\rangle H_m^T = \sum_m v_{1,2k+m}^T H_m^T \\ \Rightarrow w_{0,k} &= \sum_m H_{m-2k} v_{1,m} \end{aligned} \tag{13}$$

Without loss of generality, formula (13) can be extended:

$$w_{j,k} = \sum_m H_{m-2k} v_{j+1,m} \tag{14}$$

Formula (11) and (14) are the decomposition formulas of Mallat fast algorithm. For the orthogonal multiscaling function and multiwavelet function, $\{\sqrt{2}\phi_i(2t-k): 1 \le i \le r, k \in Z\}$ is a orthogonal basis of V_0, and $\{\sqrt{2}\psi_i(2t-k): 1 \le i \le r, k \in Z\}$ is a orthogonal basis of W_0. So $f(t) \in V_1 = V_0 \oplus W_0$ can be expressed as

$$f(t) = \sum_m v_{0,m}^T \Phi_{0,m}(t) + \sum_m w_{0,m}^T \Psi_{0,m}(t) \tag{15}$$

And because $v_{1,k}^T = \left\langle f(t), \Phi_{1,k}(t) \right\rangle$, we can have

$$\begin{aligned} v_{1,k}^T &= \left\langle \sum_m v_{0,m}^T \Phi_{0,m}(t) + \sum_m w_{0,m}^T \Psi_{0,m}(t), \Phi_{1,k}(t) \right\rangle \\ &= \sum_m v_{0,m}^T \left\langle \Phi_{0,m}(t), \Phi_{1,k}(t) \right\rangle + \sum_m w_{0,m}^T \left\langle \Psi_{0,m}(t), \Phi_{1,k}(t) \right\rangle \\ &= \sum_m v_{0,m}^T G_{k-2m} + \sum_m w_{0,m}^T H_{k-2m} \\ \Rightarrow v_{1,k} &= \sum_m G_{k-2m}^T v_{0,m} + \sum_m H_{k-2m}^T w_{0,m} \end{aligned} \tag{16}$$

Without loss of generality, the above can be extended:

$$v_{j+1,k} = \sum_m G_{k-2m}^T v_{j,m} + \sum_m H_{k-2m}^T w_{j,m} \tag{17}$$

Formula (17) is the reconstruction formulas of Mallat fast algorithm.

d. Multiwavelet prefiltering

As the above shows that the multiwavelet is a multiple input multiple output system. Before transforming with multiwavelet, the 1-D signal have to be prefiltered to obtain the multidimensional vector signal, and after reconstructing with multiwavelet, the multidimensional vector signal should be postfiltered for obtaining the 1-D signal[5, 6]. The commonly used pretreatment methods are oversampled method and critically-sampled method. The oversampled method gets second input row by repeating the first one to obtain the initial vector. In the paper critically-sampled method is taken.

Assume P is prefilter and Q is postfilter, P and Q satisfy $PQ = I$. We can obtain the initial vector by filtering $\left[x_{2(m+k)} \; x_{2(m+k)+1} \right]^T (k, m \in N)$ with the following formula

$$v_{0,k} = \sum_{m=0}^{M} P_m \begin{bmatrix} x_{2(m+k)} \\ x_{2(m+k)+1} \end{bmatrix} \tag{18}$$

where M is the number of prefilter coefficients.

III. GENERALIZED CROSS VALIDATION

Assume that there is a discrete signal

$$y_t = f_t + \varepsilon_t (t = 1, 2, \cdots, N) \tag{19}$$

or the above formula can be denoted

$$y = f + \varepsilon \tag{20}$$

where vector y is sample signal, f is unknown really signal, ε is independent and identically distributed signal with *0* mean and σ standard deviation, namely, $\forall t = 1, 2, \cdots, N$, $\mathrm{E}\left[\varepsilon_t\right] = 0$, $\mathrm{E}\left[\varepsilon_t^2\right] = \sigma^2$ and $\mathrm{E}\left[\varepsilon_t \varepsilon_j\right] = \delta_{t,j}\sigma^2 (j = 1, 2, \cdots, N)$.

After de-noising signal y with wavelet, mean square error (abbreviated MSE) is used to evaluating de-noising effect of wavelet[7, 8]. Mean square error defined in the wavelet fields is shown in the following formula

$$MSE(\lambda) = \frac{1}{N}\sum_{t=1}^{N}(y_{\lambda,t} - f_t)^2 \tag{21}$$

where λ is threshold, $y_{\lambda,t}$ is de-noised signal, ft is real signal. We can know from formula (21) that MSE(λ) measures the de-noising effect of wavelet based on threshold λ in the wavelet field. Threshold λ approximates the optimal threshold more, the de-noised signal $y_{\lambda,t}$ will approximate the real signal f_t, so the MSE will be smaller and the de-noising effect will be better. But in the actual project, the real signal is unknown; we are unable to get the MSE, and analyze de-noising effect of different wavelets with formula (13).

Generalized Cross Validation (abbreviated GCV) is a threshold function, which only bases on sample signal. Its minimum value is an asymptotically optimal threshold. In the field of wavelet, Generalized Cross Validation is defined as

$$GCV(\lambda) = \frac{N\left\|y - y_\lambda\right\|^2}{C^2} = \frac{N\left\|w - w_\lambda\right\|^2}{C^2} \tag{22}$$

where y is sample signal, w is coefficient of multiwavelet transformation, w_λ is thresholded coefficient, C is the number that is set to zero in the process of thresholding with soft thresholding function.

Based on soft thresholding:

$$\hat{w}_{j,m} = sign(w_{j,m})(|\,w_{j,m}\,| - \lambda)_+ \tag{23}$$

where $sign(w_{j,m}) = \begin{cases} +1, & w_{j,m} > 0 \\ 0, & w_{j,m} = 0 \\ -1, & w_{j,m} < 0 \end{cases}$, $(x)_+ = \begin{cases} x, & x \geq 0 \\ 0, & x < 0 \end{cases}$. Maarten Jansen et al proved that when N tends to infinity, the following conclusion can be got

$$\mathrm{E}\left[GCV(\lambda)\right] \approx \mathrm{E}\left[MSE(\lambda)\right] + \sigma^2 \tag{24}$$

Therefore, in the practical applications, GCV can replace the MSE to evaluate the de-noising effect of wavelet. The form of multiwavelet decomposition result is $2\times N$

$$w=[\cdots,w_{j,k},\cdots] \tag{25}$$

where $w_{j,k}$ is k*th* 2×1 multiwavelet vector coefficient of jth level. So Tai-Chiu Hsung and Maarten Jansen et al improved the formula (25) as following and applied it to the field of multiwavelet[10 ~ 12]

$$w=[\cdots,w_{j,k}^T,\cdots] \tag{26}$$

In the paper, we take use of the universal threshold[13], which is proposed by Donoho et al and bases on the noise level

$$\lambda=\sigma\sqrt{2\ln(N)} \tag{27}$$

where σ is the standard deviation of noise, and its estimation formula is

$$\sigma=\frac{median\left(\left|w_{1,0}\right|,\left|w_{1,1}\right|,\cdots,\left|w_{1,N/2-1}\right|\right)}{0.6745} \tag{28}$$

where median indicates the median calculation. The effects of different multiwavelets separating useful signal and noise are different. So the thresholds calculated from same threshold will be different. Under condition of same thresholding function, the de-noising effects are different. In the paper, GCV is used to evaluate the de-noising effects of different multiwavelets.

IV. EXPERIMENT

The JSD-I/A quartz flexural accelerometer is shown in Figure 1, and the signals of JSD-I/A quartz flexural accelerometer are studied in the paper. Firstly, the JSD-I/A quartz flexural accelerometer is fixed in SCT-1 two-axis rotation platform by appropriative clamp, and connected to Lenovo E43A computer by Agilent 34401A digital multimeter. Because the JSD-I/A quartz flexural accelerometer has a start-up procedure, therefore, one set of the output signal are gathered after applying voltage on the JSD-I/A quartz flexural accelerometer after 5 minutes, shown in Figure 2.

Figure 1. JSD-I/A quartz flexural accelerometer

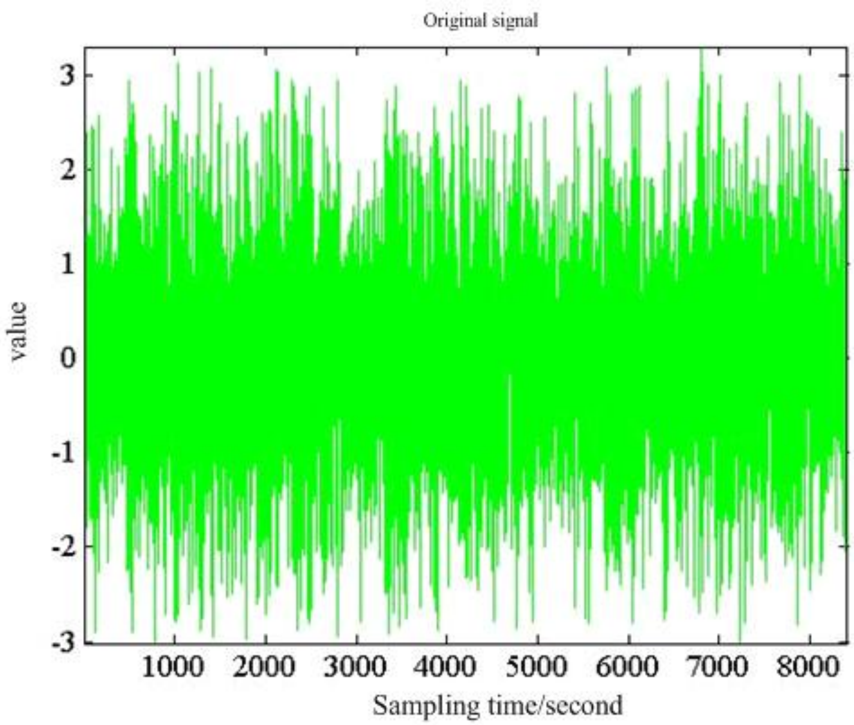
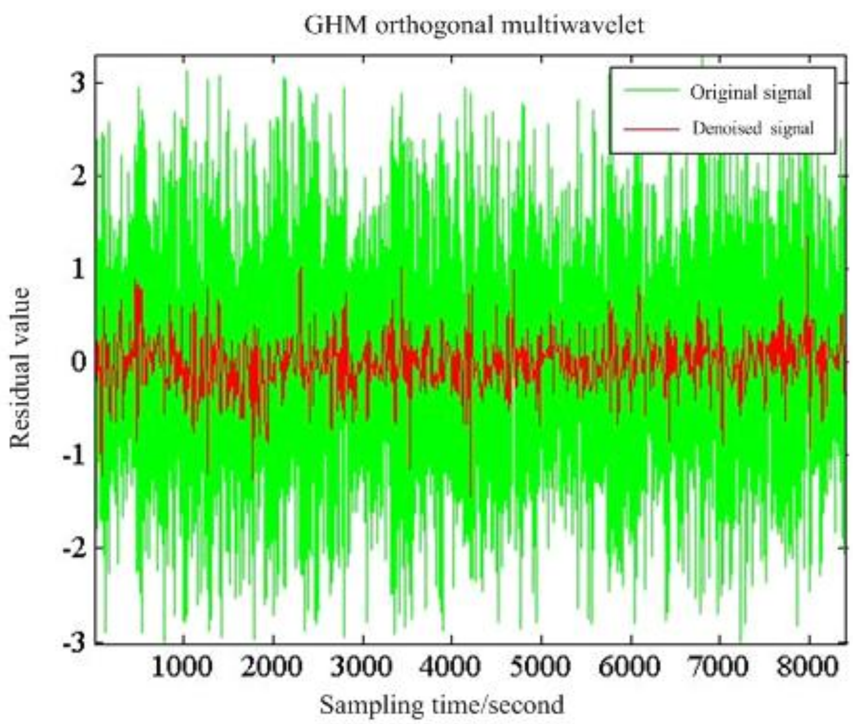


Figure 2. the signal of the quartz flexural accelerometer

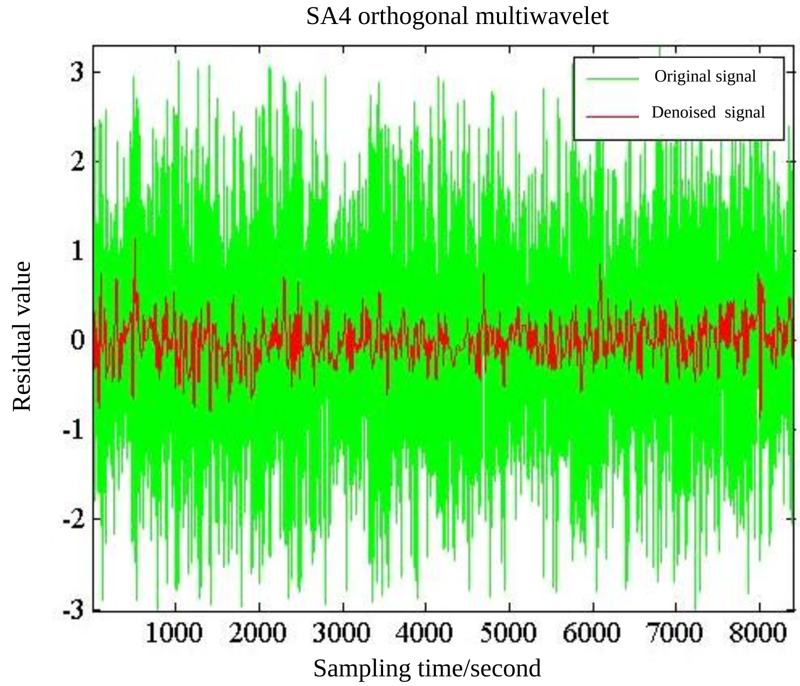
Firstly, signal $\{x_t\}$ is transformed with GHM orthogonal multiwavelet, SA4 orthogonal multiwavelet, CL orthogonal multiwavelet, Cardbal2 balanced multiwavelet and BIGHM biorthogonal multiwavelet respectively, and the decomposition level is 4. Then based on universal threshold and soft thresholding function, we thresh the five groups of multiwavlet coefficients respectively and calculate GCV values of the five groups of multiwavelets coefficients. Lastly, we inverse transform the threshed coefficients to get the de-noised signal. Please refer to the appendix for five multiwavelets filters coefficients and coefficients of prefilter and postfilter. The GCV values of five multiwavelets are show in Table 1 and the de-noising effects are show in Figure 3.

Table 1: The GCV values of five multiwavelets

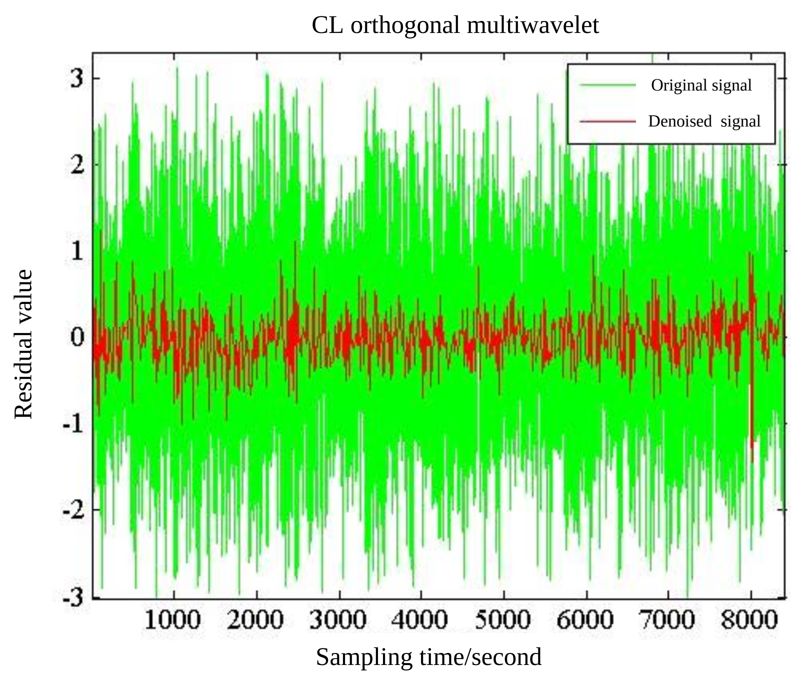
	GHM	**SA4**	**CL**	**Cardbal2**	**BIGHM**
GCV	0.83707596	0.92077635	0.12754789	0.919984728	0.00716926



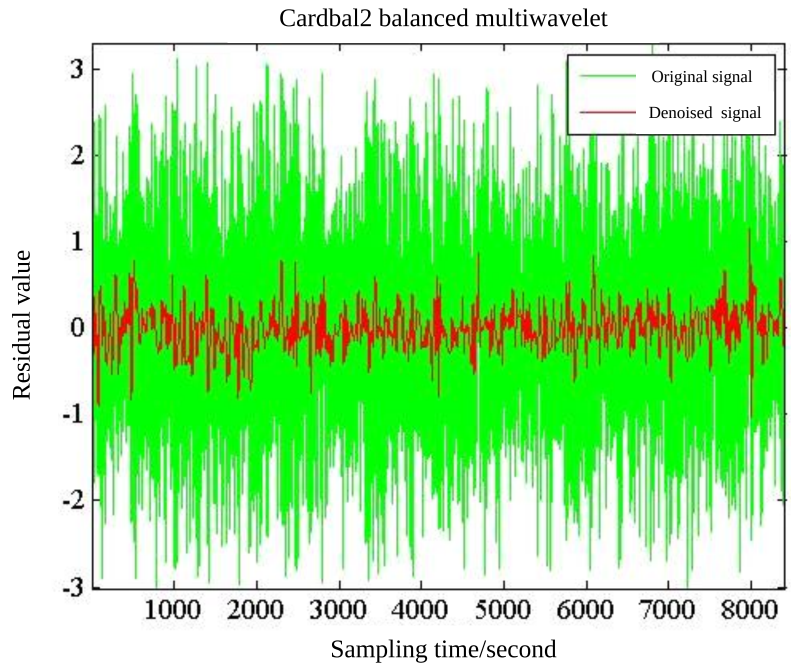
(a) GHM orthogonal multiwavelet



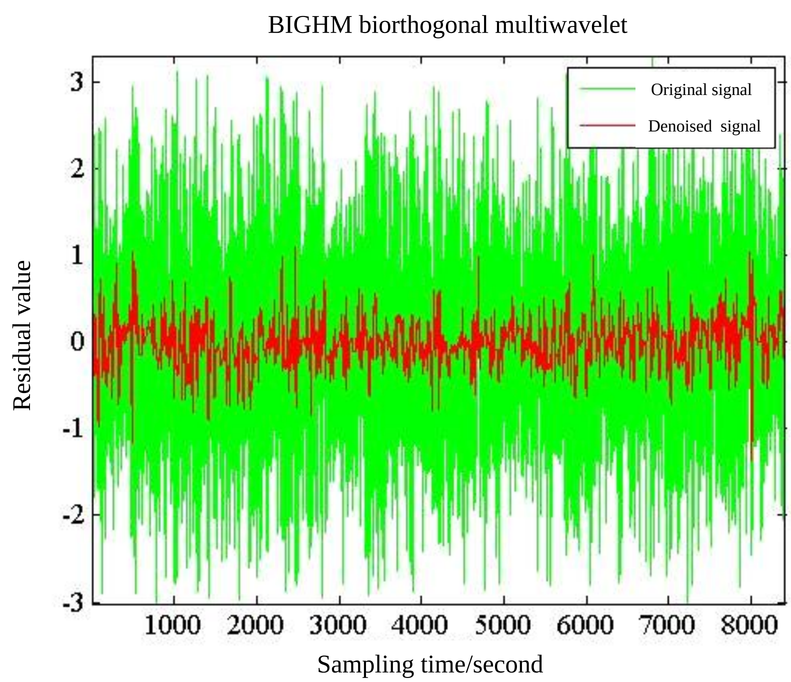
(b) SA4 orthogonal multiwavelet



(c) CL orthogonal multiwavelet



(d) Cardbal2 balanced multiwavelet



(e) BIGHM biorthogonal multiwavelet

Figure 3. The de-noising effects of five multiwavlets

The following conclusion can be obtained from the Table 1. The difference between GCV value of SA4 orthogonal multiwavelet and Cardbal2 balanced multiwavelet is small, so their de-noising effects are close. The GCV value of GHM orthogonal multiwavelet is slightly less than the former two, so that its de-noising effect is slightly better than former two. The GCV value of CL orthogonal multiwavelet is much less than the GCV values of GHM orthogonal multiwavelet, SA4 orthogonal multiwavelet and Carbal2 balanced multiwavelet, so its de-noising effect is much better than the latter three. The GCV value of BIGHM biorthogonal multiwavelet is two orders of magnitude less than the GCV value of CL orthogonal multiwavelet, so the de-noising effect of BIGHM biorthogonal multiwavelet is far better CL orthogonal multiwavelet, and it has the best de-noising effect in the five multiwavelets.

V. CONCLUSION

In the paper basis content of multiwavelet theory and generalized cross-validation criteria used to evaluate the de-noising effects of different multiwavelets are firstly researched. Then based on the universal threshold and soft thresholding function, GHM orthogonal multiwavelet, SA4 orthogonal multiwavelet, CL orthogonal multiwavelet, Cardbal2 balanced multiwavelet and BIGHM biorthogonal multiwavelet are applied to de-noise the output signal of JSD-I/A quartz flexural accelerometer, and the de-noising effects of the multiwavelets are evaluated by GCV criteria. The experiment shows that the GCV value of BIGHM biorthogonal multiwavelet is far less than the four others', so BIGHM biorthogonal multiwavelet has the best de-noising effect.

APPENDIX

1. The filter coefficients of multiwavelets

(1) The coefficients of GHM multiwavelet.

$$G_0 = \begin{bmatrix} \frac{3}{5\sqrt{2}} & \frac{4}{5} \\ -\frac{1}{20} & -\frac{3}{10\sqrt{2}} \end{bmatrix},\ G_1 = \begin{bmatrix} \frac{3}{5\sqrt{2}} & 0 \\ \frac{9}{20} & \frac{1}{\sqrt{2}} \end{bmatrix};$$

$$G_2=\begin{bmatrix}0 & 0\\ \frac{9}{20} & -\frac{3}{10\sqrt{2}}\end{bmatrix},\quad G_3=\begin{bmatrix}0 & 0\\ -\frac{1}{20} & 0\end{bmatrix};$$

$$H_0=\frac{1}{10}\begin{bmatrix}-\frac{1}{2} & -\frac{3}{\sqrt{2}}\\ \frac{1}{\sqrt{2}} & 3\end{bmatrix},\ H_1=\frac{1}{10}\begin{bmatrix}\frac{9}{2} & -\frac{10}{\sqrt{2}}\\ -\frac{9}{\sqrt{2}} & 0\end{bmatrix};$$

$$H_2=\frac{1}{10}\begin{bmatrix}\frac{9}{2} & -\frac{3}{\sqrt{2}}\\ \frac{9}{\sqrt{2}} & -3\end{bmatrix},\ H_3=\frac{1}{10}\begin{bmatrix}-\frac{1}{2} & 0\\ -\frac{1}{\sqrt{2}} & 0\end{bmatrix}.$$

(2) The coefficients of SA4 multiwavelet.

$$G_0=\frac{1}{\sqrt{2}}\begin{bmatrix}\frac{1}{32+8\sqrt{15}} & \frac{1}{8}\\ \frac{1}{32+8\sqrt{15}} & -\frac{1}{8}\end{bmatrix};\ G_1=\frac{1}{\sqrt{2}}\begin{bmatrix}\frac{31+8\sqrt{2}}{32+8\sqrt{15}} & \frac{1}{8}\\ -\frac{31+8\sqrt{2}}{32+8\sqrt{15}} & \frac{1}{8}\end{bmatrix};$$

$$G_2=\frac{1}{\sqrt{2}}\begin{bmatrix}\frac{31+8\sqrt{2}}{32+8\sqrt{15}} & -\frac{1}{8}\\ \frac{31+8\sqrt{2}}{32+8\sqrt{15}} & \frac{1}{8}\end{bmatrix};\ G_3=\frac{1}{\sqrt{2}}\begin{bmatrix}\frac{1}{32+8\sqrt{15}} & -\frac{1}{8}\\ -\frac{1}{32+8\sqrt{15}} & -\frac{1}{8}\end{bmatrix};$$

$$H_0=\frac{1}{\sqrt{2}}\begin{bmatrix}-\frac{1}{8} & \frac{1}{32+8\sqrt{15}}\\ -\frac{1}{8} & -\frac{1}{32+8\sqrt{15}}\end{bmatrix};\ H_1=\frac{1}{\sqrt{2}}\begin{bmatrix}\frac{1}{8} & -\frac{31+8\sqrt{15}}{32+8\sqrt{15}}\\ -\frac{1}{8} & -\frac{31+8\sqrt{2}}{32+8\sqrt{15}}\end{bmatrix};$$

$$H_2=\frac{1}{\sqrt{2}}\begin{bmatrix}\frac{1}{8} & \frac{31+8\sqrt{15}}{32+8\sqrt{15}}\\ \frac{1}{8} & -\frac{31+8\sqrt{2}}{32+8\sqrt{15}}\end{bmatrix};\ H_3=\frac{1}{\sqrt{2}}\begin{bmatrix}-\frac{1}{8} & -\frac{1}{32+8\sqrt{15}}\\ \frac{1}{8} & -\frac{1}{32+8\sqrt{15}}\end{bmatrix}.$$

(3) The coefficients of CL multiwavelet.

$$\mathrm{G}_0=\frac{1}{2\sqrt{2}}\begin{bmatrix}1 & -1\\ \frac{\sqrt{7}}{2} & -\frac{\sqrt{7}}{2}\end{bmatrix};\ \mathrm{G}_1=\frac{1}{2\sqrt{2}}\begin{bmatrix}2 & 0\\ 0 & 1\end{bmatrix};\ G_2=\frac{1}{2\sqrt{2}}\begin{bmatrix}1 & 1\\ -\frac{\sqrt{7}}{2} & \frac{\sqrt{7}}{2}\end{bmatrix};$$

$$H_0=\frac{1}{4\sqrt{2}}\begin{bmatrix}2 & -2\\ -1 & 1\end{bmatrix};\ H_1=\frac{1}{4\sqrt{2}}\begin{bmatrix}-4 & 0\\ 0 & 2\sqrt{7}\end{bmatrix};\ H_2=\frac{1}{4\sqrt{2}}\begin{bmatrix}2 & 2\\ 1 & 1\end{bmatrix}.$$

(4) The coefficients of Cardbal2 multiwavelet.

$$\mathrm{G}_0=\begin{bmatrix}0.0221 & 0\\ 0.6629 & 0\end{bmatrix},\ \mathrm{G}_1=\begin{bmatrix}0.1740 & 0.7071\\ -0.1712 & 0\end{bmatrix};$$

$$\mathrm{G}_2=\begin{bmatrix}0.0221 & 0\\ 0.0028 & 0\end{bmatrix},\ \mathrm{G}_3=\begin{bmatrix}-0.0028 & 0\\ 0.0221 & 0\end{bmatrix};$$

$$\mathrm{G}_4=\begin{bmatrix}0.1712 & 0.7071\\ -0.1740 & 0\end{bmatrix},\ \mathrm{G}_5=\begin{bmatrix}0.6629 & 0\\ 0.0221 & 0\end{bmatrix};$$

$$\mathrm{H}_0=\begin{bmatrix}-0.0221 & 0\\ -0.6629 & 0\end{bmatrix},\ \mathrm{H}_1=\begin{bmatrix}-0.1740 & 0.7071\\ 0.1712 & 0\end{bmatrix};$$

$$\mathrm{H}_2=\begin{bmatrix}-0.0221 & 0\\ -0.0028 & 0\end{bmatrix},\ \mathrm{H}_3=\begin{bmatrix}0.0028 & 0\\ -0.0221 & 0\end{bmatrix};$$

$$\mathrm{H}_4=\begin{bmatrix}-0.1712 & 0.7071\\ 0.1740 & 0\end{bmatrix},\ \mathrm{H}_5=\begin{bmatrix}-0.6629 & 0\\ -0.0221 & 0\end{bmatrix}.$$

(5) The decomposition coefficients of BIGHM multiwavelet.

$$\mathrm{G}_0=\frac{1}{\sqrt{2}}\begin{bmatrix}-\frac{1}{40} & \frac{1}{40}\\ -\frac{1}{40} & \frac{1}{40}\end{bmatrix},\ \mathrm{G}_1=\frac{1}{\sqrt{2}}\begin{bmatrix}\frac{1}{40} & -\frac{9}{40}\\ \frac{1}{40} & -\frac{9}{40}\end{bmatrix};$$

$$G_2 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -\frac{1}{4} \\ \frac{13}{20} & \frac{1}{10} \end{bmatrix},\ G_3 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & \frac{1}{4} \\ -\frac{13}{20} & \frac{1}{10} \end{bmatrix};$$

$$G_4 = \frac{1}{\sqrt{2}}\begin{bmatrix} \frac{1}{40} & \frac{9}{40} \\ -\frac{1}{40} & -\frac{9}{40} \end{bmatrix},\ G_5 = \frac{1}{\sqrt{2}}\begin{bmatrix} -\frac{1}{40} & -\frac{1}{40} \\ \frac{1}{40} & \frac{1}{40} \end{bmatrix};$$

$$H_0 = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & \frac{13}{40} \\ \frac{1}{100} & \frac{1}{25} \end{bmatrix},\ H_1 = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & -\frac{13}{40} \\ -\frac{1}{100} & \frac{1}{25} \end{bmatrix}.$$

(6) The reconstruction coefficients of BIGHM multiwavelet

$$G_0 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 \\ \frac{8}{5} & -\frac{2}{5} \end{bmatrix},\ G_1 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 \\ -\frac{8}{5} & -\frac{2}{5} \end{bmatrix};$$

$$H_0 = \frac{1}{\sqrt{2}}\begin{bmatrix} \frac{3}{13} & -\frac{2}{13} \\ \frac{3}{2} & -1 \end{bmatrix},\ H_1 = \frac{1}{\sqrt{2}}\begin{bmatrix} -1 & -\frac{2}{13} \\ -\frac{13}{2} & -1 \end{bmatrix};$$

$$H_2 = \frac{1}{\sqrt{2}}\begin{bmatrix} \frac{10}{13} & \frac{40}{13} \\ -4 & 26 \end{bmatrix},\ H_3 = \frac{1}{\sqrt{2}}\begin{bmatrix} \frac{10}{13} & -\frac{40}{13} \\ 4 & 26 \end{bmatrix};$$

$$H_4 = \frac{1}{\sqrt{2}}\begin{bmatrix} -1 & \frac{2}{13} \\ \frac{13}{2} & -1 \end{bmatrix},\ H_5 = \frac{1}{\sqrt{2}}\begin{bmatrix} \frac{3}{13} & \frac{2}{13} \\ -\frac{3}{2} & -1 \end{bmatrix}.$$

2. The coefficients of prefilter and postfilter

(1) The coefficients of prefilter and postfilter of GHM multiwavelet

$$P_0=\begin{bmatrix}\frac{3}{8\sqrt{2}} & \frac{10}{8\sqrt{2}}\\ 0 & 0\end{bmatrix},\ P_1=\begin{bmatrix}\frac{3}{8\sqrt{2}} & 0\\ 1 & 0\end{bmatrix};$$

$$Q_0=\begin{bmatrix}0 & 1\\ 0 & \frac{-3}{10}\end{bmatrix},\ Q_1=\begin{bmatrix}0 & 0\\ \frac{4\sqrt{2}}{5} & \frac{-3}{10}\end{bmatrix}.$$

(2) The coefficients of prefilter and postfilter of SA4 multiwavelet

$$P_0=\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ -1 & 1\end{bmatrix},\ Q_0=\frac{1}{\sqrt{2}}\begin{bmatrix}1 & -1\\ 1 & 1\end{bmatrix}.$$

(3) The coefficients of prefilter and postfilter of CL multiwavelet

$$P_0=\begin{bmatrix}\frac{1}{4} & \frac{1}{4}\\ \frac{1}{1+\sqrt{7}} & -\frac{1}{1+\sqrt{7}}\end{bmatrix},\ Q_0=\begin{bmatrix}2 & \frac{1+\sqrt{7}}{2}\\ 2 & -\frac{1+\sqrt{7}}{2}\end{bmatrix}.$$

(4) The coefficients of prefilter and postfilter of Cardbal2 multiwavelet

$$P_0=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},\ Q_0=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}.$$

(5) The coefficients of prefilter and postfilter of BIGHM multiwavelet

$$P_0=\begin{bmatrix}\frac{1}{4} & \frac{1}{4}\\ \frac{1}{3} & -\frac{1}{3}\end{bmatrix},\ Q_0=\begin{bmatrix}2 & \frac{3}{2}\\ 2 & -\frac{3}{2}\end{bmatrix}.$$

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