www.s2is.org
International Journal on Smart Sensing and Intelligent Systems
ISSN 1178-5608

DESIGN OF BACKING-UP FUZZY CONTROLLERS BASED ON VARIABLE UNIVERSE OF DISCOURSE

Zuqiang Long[1*], Wen Long[2], Yan Yuan[3], Xiaobo Yi[4]

[*1]Department of Physics and Electronics Information Science，Hengyang Normal University，Hengyang 421008, China

[2]Guizhou Key Laboratory of Economics System Simulation, Guizhou College of Finance and Economics, Guiyang 550004，China

[3]School of Information Science and Engineering, Central South University, Changsha 410083, China

[4]Department of Computer Science，Hengyang Normal University，Hengyang 421008, China

* Corresponding Author: Zuqiang Long, dragon51@yahoo.cn

Submitted: Jan. 27, 2013 Accepted: Mar. 15, 2013 Published: Apr. 10, 2013

Abstract- Fuzzy controllers with variable universe of discourse (VUD) have been applied in many fields of intelligent controlling because of their high-accuracy performance. This paper provides a lookup table method to design backing-up fuzzy controllers based on VUD. By setting a set of random start points, input–output data pairs are obtained using test-driving method. One data pair defines one fuzzy rule and also assigns the strength of every fuzzy rule. Conflicting fuzzy rule groups are integrated into

one fuzzy rule by selecting the one with the maximum strength. A fuzzy rule table is built by the fuzzy rules deduced from input–output data pairs. Simulation experiments show that the VUD fuzzy controller outperforms the general fuzzy controller in accuracy at the final position of the parking lot.

Index terms: Fuzzy controller, fuzzy system, variable universe of discourse, backing-up.

I. INTRODUCTION

Fuzzy controllers with variable universe of discourse (VUD) are a hot area of research in artificial intelligent control theory. The VUD fuzzy controllers can tune online universes of discourse using a set of nonlinear scale factors. They greatly improve the accuracy of control and, thus, have received extensive attention in recent years. Earlier in 1995, Oh Seok-Yong et al. introduced the VUD method, which was applied to a nonlinear control system with a dead-zone and obtained fine control effects [1]. Later, Hongxing Li et al. proposed two VUD fuzzy control algorithms, respectively, for single input–single output (SISO) controller and double input–single output (DISO) controller [2, 3]. The VUD fuzzy controller is one whose universe of discourse (UD) tunes online and becomes very fine as the system goes into steady state. In November 2001, Hongxing Li et al. succeeded in controlling the simulation model of quadruple inverted pendulum by applying the method of VUD [4]. In August 2002, they again succeeded in controlling a real quadruple inverted pendulum [5]. These two developments have surprised many control experts. Since then, an increasing number of researchers have been devoted to the development of VUD fuzzy controllers. Many fuzzy control algorithms concerning VUD have been published, and a great number of application examples have been successful in various fields of engineering [6-15]. For example, For example, some schemes for chaotic control systems were proposed in terms of VUD fuzzy control theory in [6-8], an analog circuit of a VUD fuzzy controller was implemented in [9], some approximation properties of a VUD fuzzy controller were presented in [10, 11, 15], an intelligent approach for improving the functionality of conventional PID controller was provided by Yazdani [12], type-2 fuzzy logic system was used to handle exhibited uncertainty in [13], and a fuzzy correlation method for target tracking and data fusion was discussed in [14]. However, little work has been done concerning the method of constructing VUD fuzzy controllers-based input–output data. At the same time, it is an important way to design VUD fuzzy controllers using input–output data. This paper investigates the lookup table method of using input–output data.

The organization of the paper is as follows. After a general introduction of the general fuzzy controller, the contraction–expansion factors and construction method (lookup table method) of VUD fuzzy controllers are presented in section II. An example using the lookup table method has been given in section III to demonstrate the effectiveness of backing-up fuzzy controllers. The paper provides a conclusion in section VI.

II. LOOKUP TABLE METHOD

Denote sample steps to be $k = 0, 1, 2, \cdots$, input variables to be $\boldsymbol{x}^k = (x_1^k, x_2^k, \cdots, x_n^k)^{\mathrm{T}}$, output variable to be y^{k+1}, the UD of input variables to be $U = U_1 \times U_2 \times \cdots \times U_n \subset R^n$, and the UD of the output variable to be $V \subset R$. For the sake of simplicity, we take $U_i = [-E_i,\ E_i]$ $(i = 1, 2, \cdots, n)$, $V = [-Y,\ Y]$. Suppose that U_i is partitioned to be n_i fuzzy sets $A_{ir_i}(r_i = 1, 2, \cdots, n_i)$, whose central points are denoted by $\bar{x}_{ir_i}$. Also, suppose that V is partitioned to be n_o fuzzy sets B_{r_o} $(r_o = 1, 2, \cdots, n_o)$, whose central points are denoted by $\bar{y}_{r_o}$. A rule base is listed as follows:

$$R_j: \text{If } x_1 \text{ is } A_{1j} \text{ and } x_2 \text{ is } A_{2j} \cdots \text{ and } x_n \text{ is } A_{nj}, \text{then } y \text{ is } B_j,\ j = 1, 2, \cdots, m. \tag{1}$$

where $m = \prod_{i=1}^{n} n_i$,

$$\left\{A_{ij} \,\middle|\, j = 1, 2, \cdots, m\right\} = \left\{A_{ir_i} \,\middle|\, r_i = 1, 2, \cdots, n_i\right\}, \tag{2}$$

$$\left\{B_j \,\middle|\, j = 1, 2, \cdots, m\right\} = \left\{B_{r_o} \,\middle|\, r_o = 1, 2, \cdots, n_o\right\}. \tag{3}$$

An example of fuzzy partition and its rule base is given as follows:

For a DISO fuzzy controller, the UD of input variable x_1 is partitioned to be 2 fuzzy sets (A_{11} and A_{12}), the UD of input variable x_2 is partitioned to be three fuzzy sets (A_{21}, A_{22} and A_{23}), and the UD of output variable y is partitioned to be two fuzzy sets (B_1 and B_2). Here, the size m of rule base should be $m = 2 \times 3 = 6$, and a rule base should be written as

R_1: If x_1 is A_{11} and x_2 is A_{21}, then y is B_1,

R_2: If x_1 is A_{11} and x_2 is A_{22}, then y is B_1,

R_3: If x_1 is A_{11} and x_2 is A_{23}, then y is B_1,

R_4: If x_1 is A_{12} and x_2 is A_{21}, then y is B_2,

R_5: If x_1 is A_{12} and x_2 is A_{22}, then y is B_2,

R_6: If x_1 is A_{12} and x_2 is A_{23}, then y is B_2.

With a singleton fuzzifier, product inference, a central average defuzzifier[16-17], for the rule base given by (1), a general fuzzy controller, $\boldsymbol{x}^k \in U \subset R^n \to y^{k+1} \in V \subset R$, can be written as

$$y^{k+1} = f(\boldsymbol{x}^k) = \frac{\sum_{j=1}^{m} \bar{y}_j \prod_{i=1}^{n} A_{ij}(x_i^k)}{\sum_{j=1}^{m} \prod_{i=1}^{n} A_{ij}(x_i^k)}, \tag{4}$$

where $\bar{y}_j$ is the central point of B_j.

During the control process of $k = 0, 1, 2, \cdots$, the general fuzzy controller (4) will be improved to be a VUD adaptive fuzzy controller when its input–output UD are tuned online by a given law, which is referred to as contraction–expansion factor [2, 3]. The method of tuning UD will be illustrated as follows.

Next, denote input UD to be $U_i^k = [-E_i^k, E_i^k]$ and output UD to be $V^k = [-Y^k, Y^k]$. U_i^0 is called the initial input UD, and V^0 is called the initial output UD. We take an input contraction–expansion factor $\alpha(x_i^k)$ and an output contraction–expansion factor $\beta(y^k)$, respectively, as follows:

$$\alpha(x_i^k) = \begin{cases} 1, & k = 0, \\ \left|\dfrac{x_i^k}{E_i}\right|^{\tau} + \delta, & k = 1, 2, 3, \cdots. \end{cases} \tag{5}$$

$$\beta(y^k) = \begin{cases} 1, & k = 0, \\ \left|\dfrac{y^k}{Y}\right|^{\tau} + \delta, & k = 1, 2, 3, \cdots, \end{cases} \tag{6}$$

where τ is a positive parameter satisfying $\tau \in (0,\ 1)$ and δ is an infinitesimal positive parameter to guarantee that (5) and (6) are meaningful as the divisor in the following parts. Thus, input UD U_i^k and output UD V^k, respectively, can be expressed as:

$$U_i^k = [-\alpha(x_i^k),\ \alpha(x_i^k)]\,. \tag{7}$$

$$V^k = [-\beta(y^k),\ \beta(y^k)]\,. \tag{8}$$

Thus, a VUD fuzzy controller: $\boldsymbol{x}^k \in U \subset R^n \to y^{k+1} \in V \subset R$ can be written as

$$y^{k+1} = f(\boldsymbol{x}^k) = \beta(y^k)\frac{\sum_{j=1}^{m}\overline{y}_j^0\prod_{i=1}^{n}\mu_{A_{ij}^0}\left(\frac{x_i^k}{\alpha_i(x_i^k)}\right)}{\sum_{j=1}^{m}\prod_{i=1}^{n}\mu_{A_{ij}^0}\left(\frac{x_i^k}{\alpha_i(x_i^k)}\right)}\,. \tag{9}$$

where $\mu_{A_{ij}^0}\left(x_i^k\right)$ are the membership functions of initial input fuzzy sets A_{ij}^0, and $\overline{y}_j^0$ are the central points of initial output fuzzy sets B_j^0. Based on fuzzification, fuzzy inference, and defuzzification, the construction of VUD fuzzy controllers should include issues such as the definition and partition of initial UD, the design of membership function, the calculation of the total of fuzzy rules, the selection of the expansion–contraction, the definition of the central points of initial output fuzzy sets, and the determination of any fuzzy rule. In this paper, we shall focus on the determination of fuzzy rule using the method of lookup table, which is listed as follows:

Step 1. One data pair defines one fuzzy rule.

First, we use one input–output data pair $(x_{01}^p, x_{02}^p, \ldots, x_{0n}^p; y_0^p)$, $p = 1, 2, \cdots, N$, to determine the value of membership function $A_{ir_i}(r_i = 1, 2, \cdots,\ n_i)$ for the data $x_{0i}^p(i = 1, 2, \cdots, n)$ and the value of membership function $B_{r_o}(r_o = 1, 2, \cdots, n_o)$ for the datum y_0^p. That is to say, we need to calculate the expression $\mu_{A_{ir_i}}(x_{0i}^p)(r_i = 1, 2, \cdots, n_i,\ i = 1, 2, \cdots, n)$ and $\mu_{B_{r_o}}(y_0^p)(r_o = 1, 2, \cdots, n_o)$. For Figure 1, the value of x_{01}^1 on fuzzy sets A_{12}, A_{13} and others can be considered to be approximately 0.8, 0.2, and 0, respectively; the value of x_{02}^1 on fuzzy sets A_{26}, A_{27}, and others can be considered to be approximately 0.9, 0.1, and 0; and the value of y_0^1 on fuzzy sets B_2, B_3, and others can be considered to be approximately 0.75, 0.25, and 0.

Then, we choose a fuzzy set to maximize the value of membership function, that is, choose a fuzzy set $A_{ir_i^*}$ to satisfy $\mu_{A_{ir_i^*}}(x_{0i}^p) \geq \mu_{A_{ir_i}}(x_{0i}^p)(r_i = 1, 2, \cdots, n_i)$. Similarly, we determine the fuzzy set $B_{r_o^*}$, which can get the maximum of membership functions. That is to say, we need to calculate $\mu_{B_{r_o^*}}(y_0^p) \geq \mu_{B_{r_o}}(y_0^p)(r_o = 1, 2, \cdots, n_o)$. For Figure 1, based on the input–output data pair $(x_{01}^1, x_{02}^1; y_0^1)$, we can get the following: $A_{1r_1^*} = A_{12}$, $A_{r_2^*} = A_{26}$, $B_{r_o^*} = B_2$. Thus, an if–then rule can be written as

$$\text{If } x_1 \text{ is } A_{12} \text{ and } x_2 \text{ is } A_{26}, \text{then } y \text{ is } B_2 .$$

Similarly, for input–output data pair $(x_{01}^2, x_{02}^2; y_0^2)$, another if–then rule can be written as

$$\text{If } x_1 \text{ is } A_{13} \text{ and } x_2 \text{ is } A_{23}, \quad \text{then } y \text{ is } B_5 .$$

Therefore, for the data pair $(x_{01}^p, x_{02}^p, \ldots, x_{0n}^p; y_0^p)$, an if–then rule can be written as

$$\text{If } x_1 \text{ is } A_{1r_1} \text{ and } x_2 \text{ is } A_{2r_2} \cdots \text{and } x_n \text{ is } A_{nr_n}, \text{then } y \text{ is } B_{r_o}; \tag{10}$$

Step 2. Assign a value for the strength of every fuzzy rule.

When a data pair deduces a fuzzy rule, the collision of fuzzy rules maybe appears because a large number of data pairs are needed. For this problem, we assign a value for the strength of any fuzzy rule so that there is only one fuzzy rule with maximum strength. Thus, not only the collision of fuzzy rules does not occur, but also the number of fuzzy rules decreases greatly. For the input-output data pair $(x_{01}^p, x_{02}^p, \ldots, x_{0n}^p; y_0^p)$, the strength of fuzzy rule is defined as follows:

$$D = \prod_{i=1}^{n} \mu_{A_{ir_i^*}}(x_{0i}^p)\mu_{B_{r_o^*}}(y_0^p) \tag{11}$$

For the data pair $(x_{01}^1, x_{02}^1; y_0^1)$ in Figure 1, the strength of the relevant fuzzy rule can be calculated by

$$D = \mu_{A_{12}}(x_{01}^1)\mu_{A_{26}}(x_{02}^1)\mu_{B_2}(y_0^1)$$
$$= 0.8 \times 0.9 \times 0.75 = 0.54 .$$

For the data pair $(x_{01}^2, x_{02}^2; y_0^2)$ in Figure 1, the strength of the relevant fuzzy rule can be calculated by

$$D = \mu_{A_{12}}(x_{01}^1)\mu_{A_{26}}(x_{02}^1)\mu_{B_2}(y_0^1)$$
$$= 0.6 \times 0.6 \times 0.6 = 0.216$$

If we can use data to depict the reliability of input–output data pairs, then the reliability can be integrated into the strength of fuzzy rules. To be specific, suppose that the reliability of input–output data pair $(x_{01}^p, x_{02}^p, \ldots, x_{0n}^p; y_0^p)$ is denoted as $\mu^p (\in [0,1])$, the strength of fuzzy rules can be defined as

$$D = \prod_{i=1}^{n} \mu_{A_{ir_i^*}}(x_{0i}^p)\mu_{B_{r_o^*}}(y_0^p)\mu^p \qquad (12)$$

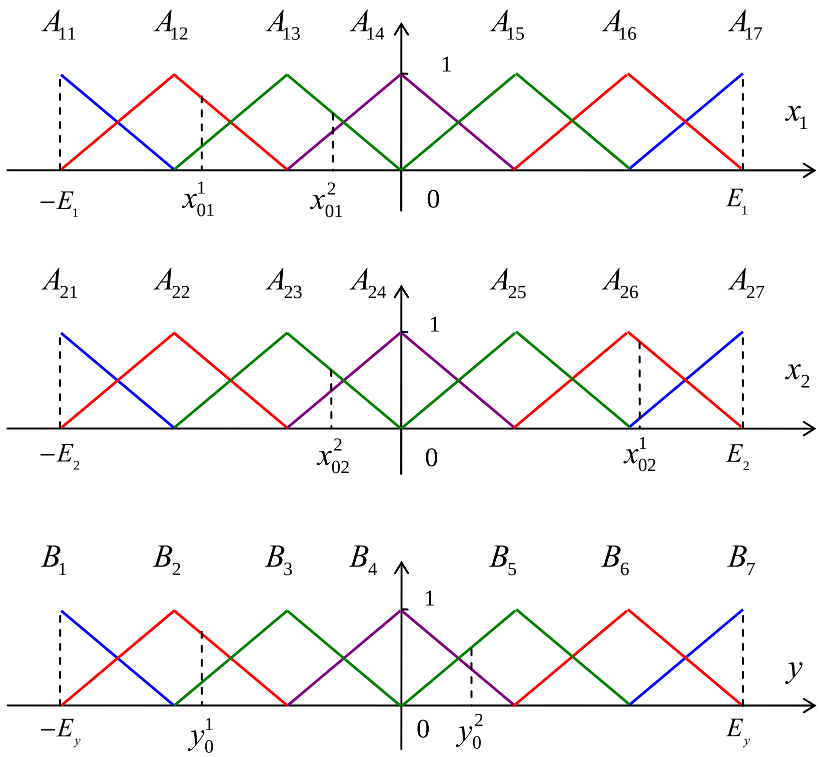

Figure 1. A sample of input-output data pair on two-dimension

In practice, when the number of input–output data pairs is small, it is suitable for experts to check data and evaluate the reliability μ^p. Moreover, when the noise characteristic of input–output data pairs is known, the reliability μ^p can be used to reflect the strength of noise. When the difference among data pairs is unknown, we can simply define $\mu^p = 1$, $p = 1, 2, \cdots, N$. Thus, (12) is degenerated to (11).

Step 3. Build a fuzzy rule base.

Fuzzy rules in their base can be partitioned into two parts: ① nonconflicting rule groups; and ② conflicting rule groups. The conflicting rule groups mean that they have the same antecedent "if." To be simple, a two-dimensional (2D) example in Figure 2 is considered to illustrate the method of the construction of fuzzy rule base.

Intuitively, a fuzzy rule base can be depicted as a 2D table. Figure 2 shows the schematic diagram of the fuzzy rule base, which is derived from the fuzzy sets in Figure 1. In Figure 2, any grid represents a combination of the fuzzy sets between $[-E_1, E_1]$ and $[-E_2, E_2]$. Thus, a fuzzy rule comes into being. A conflicting rule group is formed by several fuzzy rules in the same grid. This method can be seen to use proper rules to fill in the blanks in the table. Therefore, the solution is called as the lookup table method.

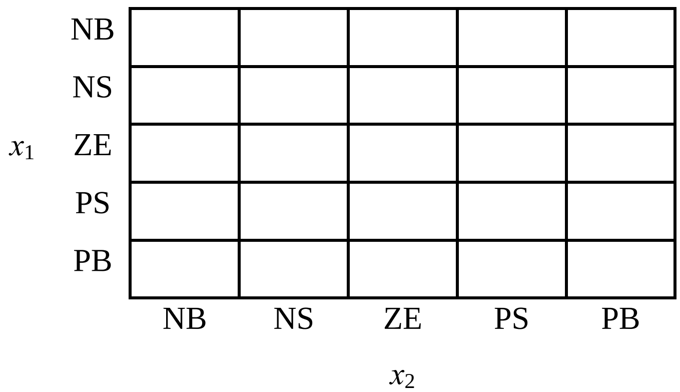

Figure 2. The framework of fuzzy rule base

In the fuzzy rule table, the number of fuzzy rules is determined by two variables, which are the number of input–output data pairs and the number of $\prod_{i=1}^{n} n_i$ (the maximum of the combination of input fuzzy sets). If the dimensionality is large, then $\prod_{i=1}^{n} n_i$ becomes very large. It is possible for the maximum of the combination of input fuzzy sets to exceed the number of input–output data pairs. Moreover, some input–output data pairs may direct the same grid in Figure 2, and only a fuzzy rule is used. This equals that the number of input–output data pairs is reduced. Thus, the fuzzy rule base may become incomplete, and it needs experts to interpolate the blanks in the table based on their experience and knowledge. Although the dimensionality is small, it is easy to construct a complete rule base using input–output data pairs. Therefore, the lookup table method is suitable for the cases of three dimensions and those of less than three dimensions.

Ⅲ. EXAMPLE

How to back a car without stopping and to park it at the prearranged place is generally a difficult process. Suppose that there is a rectangle stopping lot with the width of 20 m and unknown length, shown as Figure 3. It is required to design a backing-up fuzzy controller with VUD to imitate the behavior of an experienced driver and park the central position of the car at the dashed line.

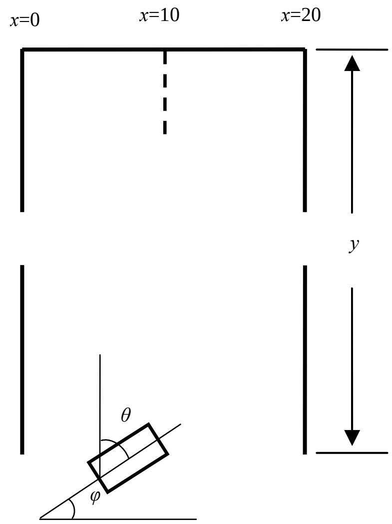

Figure 3. The diagram of parking lot

In Figure 3, it can be seen that the position of the car is determined by three variables of φ, x, and y, where φ represents the intersection angle of the car and the horizontal line, x represents the horizontal distance between the central point of car rear and the left line of stopping lot, y represents the vertical distance between the central point of car rear and the bottom of stopping lot, and θ represents the turn angle of the steering wheel in the range of $[-40^\circ, 40^\circ]$. It is easily seen that φ is in the range of $[0^\circ, 180^\circ]$ and x is in the range of $[0, 20]$. To be simple, it is supposed that the length of stopping lot is sufficient to park a car and y is an intermediate variable.

To design the backing-off controller that satisfied the above-mentioned requirements, a heuristic method is considered to obtain a set of input–output data pairs $(x^p, \varphi^p; \theta^p)$. A veteran driver is required to test-drive to obtain the data $(x^p, \varphi^p; \theta^p)$ according to fourteen typical initial states (x_0, φ_0) shown as follows:

$$(x_0, \varphi_0) = (1, 0^\circ), (1, 90^\circ), (1, 180^\circ); (7, 0^\circ), (7, 75^\circ), (7, 105^\circ), (7, 180^\circ);$$
$$(13, 0^\circ), (13, 75^\circ), (13, 105^\circ), (13, 180^\circ); (19, 0^\circ), (19, 90^\circ), (19, 180^\circ)$$

For the initial state $(x_0,\ \varphi_0) = (1,\ 0^\circ)$, when the car is backward running, the input-output data pairs from information processing box (CASKA D105) are recorded in Table 1. For the other 13 initial states, the relevant data can be obtained. Thus, 14 sets of input–output data pairs are obtained. Then, we use the lookup table method to design a VUD fuzzy controller.

Table 1. The input-output data $(x^p, \varphi^p; \theta^p)$ at the initial state $(x_0, \varphi_0) = (1, 0^\circ)$ when the car is controlled by a veteran driver

k (s)	x_k (m)	φ_k (°)	θ_k (°)	k (s)	x_k (m)	φ_k (°)	θ_k (°)
0	1.00	0.00	-19.00	9	8.72	65.99	-9.55
1	1.95	9.37	-17.95	10	9.01	70.75	-8.50
2	2.88	18.23	-16.90	11	9.28	74.98	-7.45
3	3.79	26.57	-15.85	12	9.46	78.70	-6.40
4	4.65	34.44	-14.80	13	9.59	81.90	-5.34
5	5.45	41.78	-13.75	14	9.72	84.57	-4.30
6	6.18	48.60	-12.70	15	9.81	86.72	-3.25
7	7.48	54.91	-11.65	16	9.88	88.34	2.20
8	7.99	60.71	-10.60	17	9.91	89.44	0

Next, the VUD fuzzy controller is designed by the following steps:

① A range transformation method is used to transform the three ranges $[0^\circ, 180^\circ]$, $[0, 20]$, and $[-40^\circ, 40^\circ]$ to $[-6, 6]$ for $\varphi,\ x$, and θ.

② The triangle membership functions are considered for the VUD fuzzy controller.

③ The range $[-6, 6]$ is partitioned to seven fuzzy sets shown as Figure 4, which are denoted as NB, NM, NS, ZE, PS, PM, and PB, respectively. Thus, the total of fuzzy rules can be written as $M^{'} = 49$.

④ The contraction–expansion factors of the variables $\varphi,\ x$ are set as, respectively,

$$\alpha_x(x^k) = \begin{cases} 1, & k = 0, \\ \left|\dfrac{x^k}{6}\right|^{0.82} + 0.001, & k = 1, 2, 3, \cdots \end{cases} \tag{13}$$

$$\alpha_\varphi(\varphi^k)=\begin{cases}1, & k=0,\\ \left|\dfrac{\varphi^k}{6}\right|^{0.82}+0.001, & k=1,2,3,\cdots\end{cases} \tag{14}$$

$$\beta_\theta(\theta^k)=\begin{cases}1, & k=0,\\ \left|\dfrac{\theta^k}{6}\right|^{0.82}+0.001, & k=1,2,3,\cdots\end{cases} \tag{15}$$

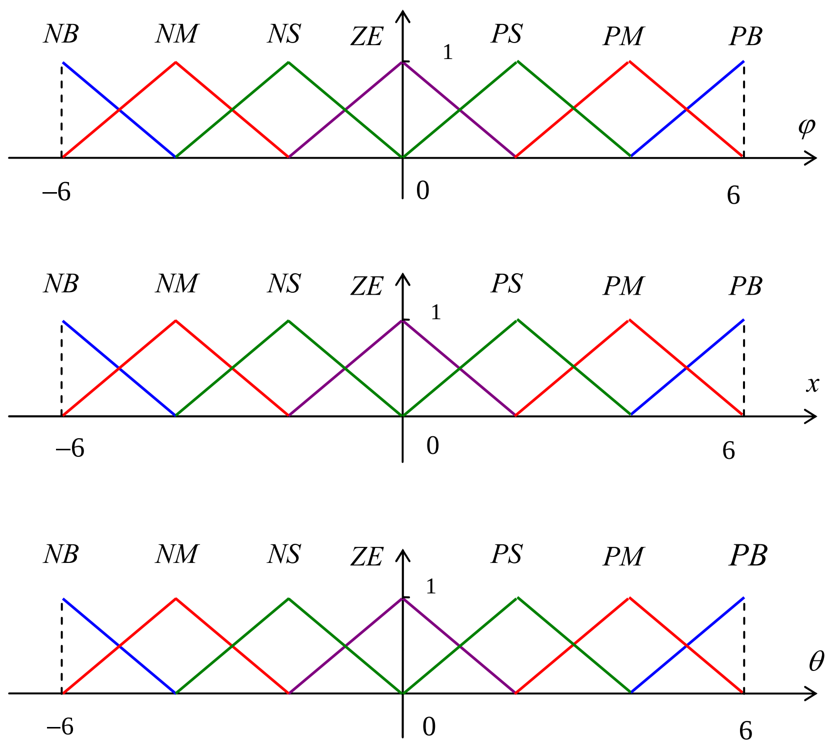

Figure 4. Membership functions of backing-up fuzzy controller

⑤ Every input–output data pair is used to produce a fuzzy rule and to calculate its strength. Then, the data in Table 1 leads to an intermediate form of a fuzzy rule table, shown as Table 2. Thus, the fourteen initial states produce fourteen intermediate forms. In a conflicting–rule group, the rule with the maximum value of strength *D* is assigned to represent the group. Finally, a complete fuzzy rule table is given as Table 3.

⑥ Figure 5 indicates the VUD fuzzy controller is set up by twelve boxes. In Figure 5, the box Fuzzy Inference Engine plays the role defined by the equation (5). The box Fuzzy Rule Base is established using the rules listed in Table 3. The box Car is the mathematical model defined by (16), (17) and (18), and it is used to execute instructions given by the front box Gain_3. The three boxes Gain_1, Gain_2 and Gain_3 are amplifiers, and their enlargement factors are set as 1.5, 4, and 2, respectively. The three boxes α_x, α_φ and β_θ represent the equation (13), (14) and (15), respectively. The functions of the three boxes Fuzzifier_1, Fuzzifier_2 and Defuzzifier have been shown in Figure 4.

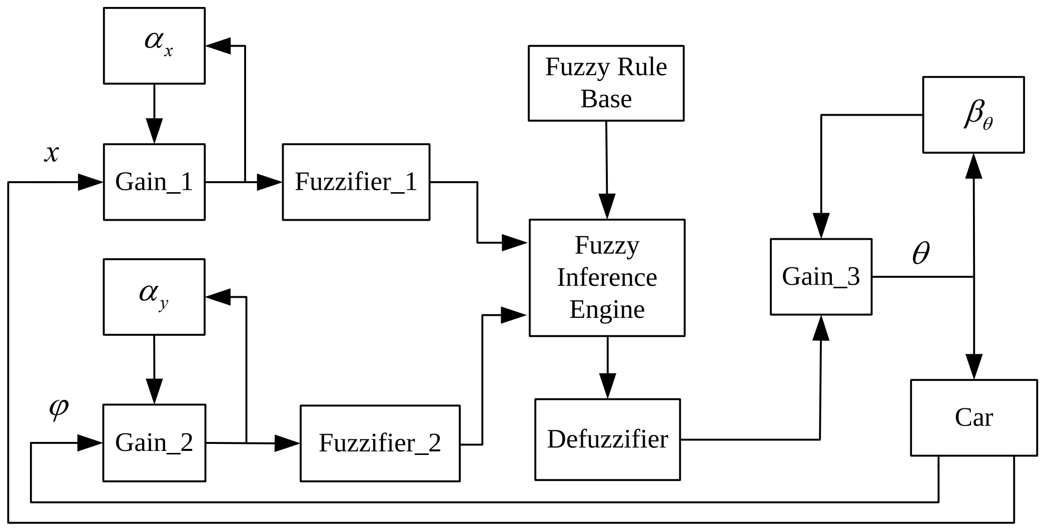

Figure 5. Simulation structure of backing-up VUD fuzzy control system

Table 2. Intermediate fuzzy rule and its strength at the initial state $(x_0, \varphi_0) = (1, 0^\circ)$

x	φ	θ	D	x	φ	θ	D
NB	*NB*	*NB*	1.0	*NS*	*NS*	*NS*	0.37
NB	*NB*	*NB*	0.94	*ZE*	*NS*	*NS*	0.45
NB	*NB*	*NB*	0.37	*ZE*	*NS*	*NS*	0.88
NB	*NM*	*NB*	0.18	*ZE*	*NS*	*NS*	0.07
NM	*NM*	*NM*	0.45	*ZE*	*ZE*	*NS*	0.31
NM	*NM*	*NM*	0.56	*ZE*	*ZE*	*ZE*	0.23
NM	*NM*	*NM*	0.26	*ZE*	*ZE*	*ZE*	0.95
NS	*NM*	*NM*	0.78	*ZE*	*ZE*	*ZE*	0.56
NS	*NS*	*NS*	0.12	*ZE*	*ZE*	*ZE*	0.94

At the same time, a simulation mathematical model of the car is obtained as follows:

$$x^{k+1} = x^k + \cos(\varphi^k + \theta^k) + \sin\theta^k \sin\varphi^k \tag{16}$$

$$y^{k+1} = y^k + \sin(\varphi^k + \theta^k) - \sin\theta^k \cos\varphi^k \tag{17}$$

$$\varphi^{k+1} = \varphi^k - \sin^{-1}\left(\frac{2\sin\theta^k}{b}\right) \tag{18}$$

where b is the lengthy of the car and is set as $b = 4$ and the end sample step is set as $k = 23$.

Table 3. Fuzzy rules of a VUD backing-up controller

φ	x						
	NB	*NM*	*NS*	*ZE*	*PS*	*PM*	*PB*
NB	*NM*	*NB*	*NB*	*NB*	*NM*	*NS*	*NS*
NM	*NM*	*NB*	*NB*	*NB*	*NM*	*NM*	*NS*
NS	*PS*	*ZE*	*NS*	*NB*	*NB*	*NM*	*NM*
ZE	*PM*	*PM*	*PS*	*ZE*	*NS*	*NM*	*NM*
PS	*NM*	*NM*	*NB*	*NB*	*NS*	*ZE*	*PS*
PM	*PS*	*PM*	*PM*	*PB*	*PB*	*PB*	*PM*
PB	*PS*	*PS*	*PM*	*PB*	*PB*	*PB*	*PM*

Figure 6 shows two backing-up trajectory starting at two initial points of $(x_0, \varphi_0) = (3, 121^\circ)$ and $(x_0, \varphi_0) = (12, 5^\circ)$. It can be easily seen in Figure 6 that the VUD fuzzy controller achieves satisfactory effect.

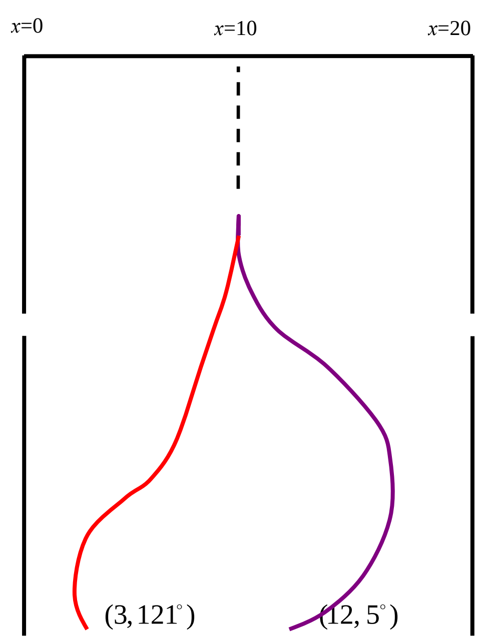

Figure 6. Two backing-up trajectory by the VUD fuzzy controller

When the contraction–expansion factors are set as 1, a general fuzzy controller noted by $g(\boldsymbol{x}^k)$ is obtained using the design method given previously. To contrast the control effect, 16 random initial states are considered to be the starting points, which are denoted as $\rho = 1, 2, \cdots, 16$. Then, simulation data are listed in Table 4, where x_{23} and $x_{23}^{'}$ represent the horizontal position of the car at the sample step of $k = 23$, using the VUD fuzzy controller and the general fuzzy controller, respectively. Similarly, φ_{23} and $\varphi_{23}^{'}$ represent the intersection-angle at the sample step $k = 23$, also using the VUD fuzzy controller and the general fuzzy controller, respectively.

Table 4. Contrast of data of simulation experiment at the final position

n	Initial position		VUD fuzzy controller		General fuzzy controller	
	x_0 (m)	φ_0 (°)	x_{23} (m)	φ_{23} (°)	$x_{23}^{'}$ (m)	$\varphi_{23}^{'}$ (°)
1	1.00	0.00	9.99	90.01	9.91	90.05
2	1.85	6.37	10.00	89.98	10.10	89.90
3	2.68	48.63	10.00	90.02	10.02	89.95
4	3.00	121.00	9.98	90.01	9.93	90.07
5	4.14	34.44	10.01	89.98	9.95	90.11
6	5.05	41.78	10.01	90.01	10.03	90.01
7	6.98	48.60	9.99	90.00	10.05	89.92
8	7.65	54.91	9.98	90.01	9.99	90.07
9	8.89	60.71	10.00	90.00	9.89	89.89
10	9.86	12.84	10.00	90.01	9.90	90.04
11	12.00	5.00	10.01	89.99	10.11	89.99
12	13.06	165.88	10.00	89.99	10.03	89.86
13	14.88	87.99	9.99	89.98	9.92	90.08
14	15.81	156.34	10.01	90.00	10.06	90.00
15	17.87	101.33	10.01	89.98	10.04	89.95
16	19.08	111.55	10.00	89.99	10.09	89.96

For a clear view of the experimental effect of the VUD fuzzy controller at the final position, two horizontal-position-error variables are defined as $\varepsilon_{x1} = \left| x_{23} - 10 \right|$ and $\varepsilon_{x2} = \left| x_{23}^{'} - 10 \right|$, respectively.

Next, two intersection-angle-error variables are defined as $\varepsilon_{\varphi 1} = \left|\varphi_{23} - 90°\right|$ and $\varepsilon_{\varphi 2} = \left|\varphi_{23}^{'} - 90°\right|$, respectively. Then, for the sixteen initial state $\rho = 1,\ 2,\ \cdots,\ 16$, the relevant horizontal-position error ε_x and intersection-angle error ε_φ are plotted in Figure 7 and Figure 8, respectively.

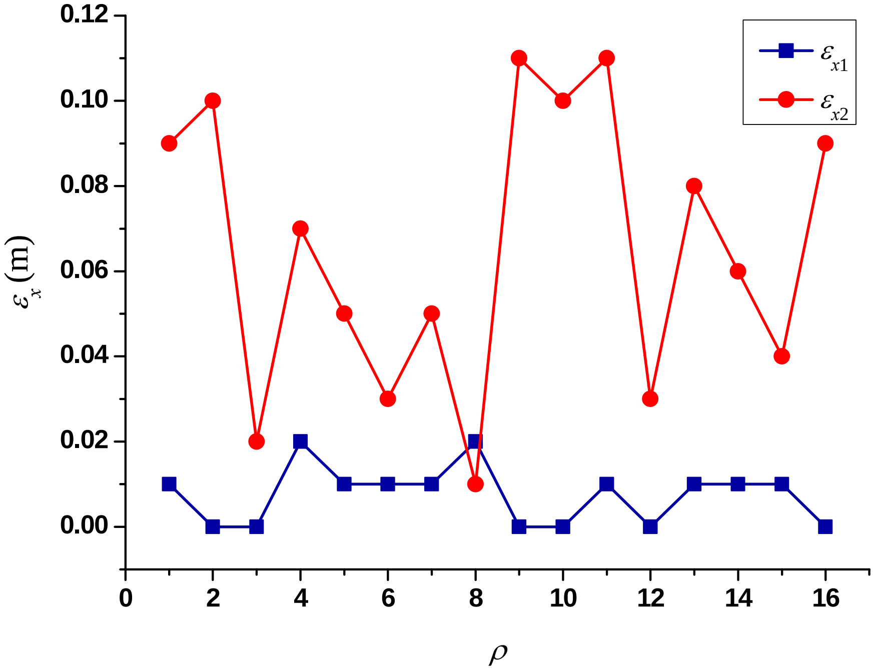

Figure 7. Contrast of horizontal-position error at the final position

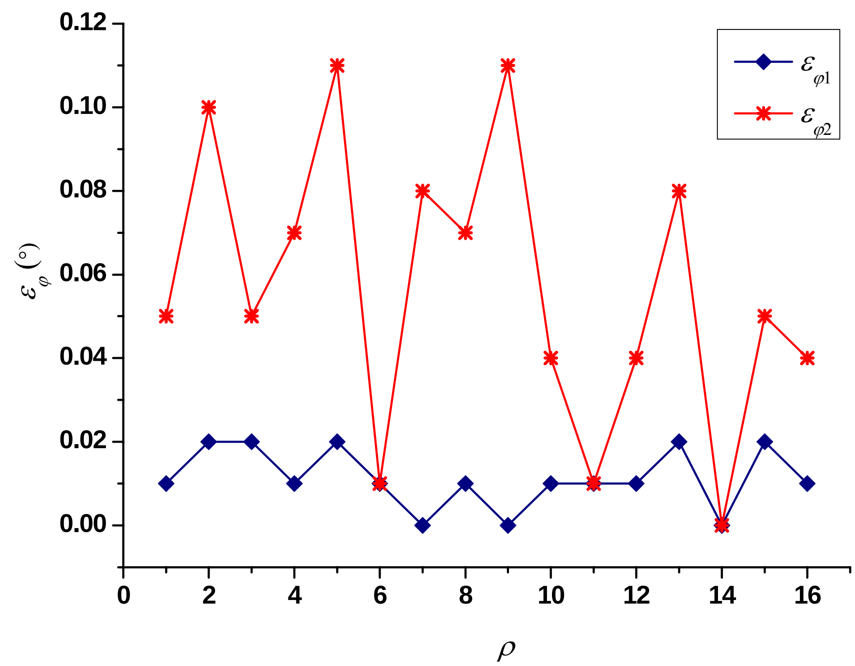

Figure 8. Contrast of two intersection-angle error at the final position

Figure 7 shows that the error ε_{x1} is not more than 0.02 (m). However, the error ε_{x2} almost approaches 0.11(m). Also, it can be seen that the error range of Figure 8 is the same with that of Figure 7. Obviously, the VUD fuzzy controller outperforms the general fuzzy controller in accuracy at the final position.

IV. CONCLUSIONS

VUD fuzzy controllers are capable of high control accuracy by the online operation of expansion–contraction factors. This paper has presented the formula of VUD fuzzy controllers and the method of designing VUD fuzzy rules using input–output data pairs. First, one input–output data pair leads to a fuzzy rule. Second, a value is assigned to the strength of any fuzzy rule. Third, the fuzzy rule that holds the maximum strength is selected to represent the set of conflicting fuzzy rules. Our method has been applied to a backing-up controller. By computer simulation experiment, a VUD backing-up fuzzy controller designed by the method has been proven to outperform the general backing-up fuzzy controller.

ACKNOWLEDGMENTS

The authors are grateful for the projects supported by the National Natural Science Foundation of China under Grant 61074069, by the construct program of the key discipline in Hunan province, and by the Planned Science and Technology Project of Hunan Province of China under Grant 2011FJ3097.

The authors are very grateful to Prof. S. C. Mukhopadhyay and anonymous reviewers for their constructive comments and suggestions that help in the quality improvement of this paper

REFERENCES

[1] Seok-Yong Oh, Dong-Jo Park, “Self-Tuning fuzzy controller with variable universe of discourse”, In Proceedings of the IEEE International Conference on Systems, Man and Cybernetics, pp.2628-2632, 1995.

[2] Hong-xing Li, “Adaptive fuzzy control based on variable universe”, Science in China (Series E), vol.42, no.1, 1999, pp.10-20.

[3] Hong-xing Li, Zhi-hong Miao, Jia-yin Wang, "Variable universe stable adaptive fuzzy control of nonlinear system", Science in China (Series E), vol.45, no.3, 2002, pp.225-240.

[4] Hong-xing Li, Zhi-hong Miao, Jia-yin Wang, "Variable universe adaptive fuzzy control on the quadruple inverted pendulum", Science in China (Series E), vol.45, no.2, 2002, pp.213-224.

[5] Hong-xing Li, Jia-yin Wang, Yan-bin Feng, Yundong Gu, "Hardware implementation of the quadruple inverted pendulum with single motor", Progress in Natural Science, vol.14, no.9, 2004, pp.822-827.

[6] Jiang Wang, Guo-dong Qiao, Bin Deng, "Observer-based robust adaptive variable universe fuzzy control for chaotic system", Chaos, Solitons & Fractals, vol.23, no.3, 2005, pp.1013-1032.

[7] Jiang Wang, Wen-jie Si, Hui-yan Li, "Robust ISS-satisficing variable universe indirect fuzzy control for chaotic systems", Chaos, Solitons and Fractals, vol.39, no.1, 2009, pp.28-38.

[8] Jing Wang, Wei-wei Zhang, "Chaos control via variable universe fuzzy theory in auto gauge control system", 2010 International Conference on Computer Application and System Modeling, pp.363-369, 2010.

[9] Wei-wei Shan, Yuan Ma, Newcomb, R.W., Dong-ming Jin, "Analog Circuit Implementation of a Variable Universe Adaptive Fuzzy Logic Controller", IEEE Transactions on Circuits and Systems-II: Express Briefs, vol.55, no.10, 2008, pp.976-980.

[10] Zu-qiang Long, Xi-ming Liang, Li-rong Yang, "Some approximation properties of adaptive fuzzy systems with variable universe of discourse", Information Sciences, vol.180, no.16, 2010, pp.2991-3005.

[11] Zu-Qiang Long, Yue-Bing Xu, Can Liu. Fuzzy Control Algorithm Based on Variable Universe of Discourse and Its Expansion-Contraction Factors. Journal of Convergence Information Technology, vol. 7, no.19, 2012, pp.570-577.

[12] A. M. Yazdani, A. Ahmadi, S. Buyamin, M. F. Rahmat, F. Davoudifar and H. A. Rahim, "Imperialist competitive algorithm-based fuzzy PID control methodology for speed tracking", International Journal on Smart Sensing and Intelligent Systems, vol.5, no.3, 2012, pp.717-741

[13] T. K. Dakhlallah, M. A. Zohdy and O.M. Salim, “Type-2 Fuzzy Kalman Hybrid Application for Dynamic Security Monitoring Systems based on Multiple Sensor Fusion”, International Journal on Smart Sensing and Intelligent Systems, vol.4, no.4, 2011, pp.607-629.

[14] Liu Chang, Cao Ming-zhi, Han Feng, Shi Gui-ming and Liu Ren-jie, “A Model for Fuzzy Data Correlation of AIS and Radar”, International Journal on Smart Sensing and Intelligent Systems, vol.5, no.4, 2012, pp.843-858.

[15]Yong-ping Pan, Meng Joo Er, Dao-ping Huang, Qin-ruo Wang, “Adaptive Fuzzy Control With Guaranteed Convergence of Optimal Approximation Error”, IEEE Transaction on Fuzzy Systems, vol.19, no.5, 2011, pp.807–818.

[16] Li-xin. Wang, “A Course in Fuzzy Systems and Control”, Prentice Hall PTR, USA, 1997.

[17]Yue-zhong Tang, Nai-yao Zhang,Yan-da Li, “Stable fuzzy adaptive control for a class of nonlinear systems”, Fuzzy Sets and Systems, vol.2, no.104, 1999, pp.279-288.