www.s2is.org
International Journal on Smart Sensing and Intelligent Systems
ISSN 1178-5608

RESEARCH ON LONGITUDINAL CONTROL ALGORITHM FOR FLYING WING UAV BASED ON LQR TECHNOLOGY

Yibo LI[1*], Chao Chen[1], Wei Chen[2]

[1]College of automation Shen Yang Aerospace University, Shenyang 110136 Liaoning, China

[2]AVIC Cheng Du aircraft industrial (group) Co., Ltd. Chengdu 610000, China

Emails: lyb20040612@aliyun.com, chenchao2368989@163.com, 8952939@qq.com

*Correspondent author: Yibo LI, lyb20040612@aliyun.com

Submitted: June.30, 2013 ***Accepted: Nov.23, 2013*** ***Published: Dec.16, 2013***

Abstract- Linear Quadratic Regulator (LQR) is widely used in many practical engineering fields due to good stability margin and strong robustness. But there is little literature reports the technology that has been used to control the flying wing unmanned aerial vehicles (UAV). In this paper, aiming at the longitudinal static and dynamic characteristics of the flying wing UAV, LQR technology will be introduced to the flying wing UAV flight control. The longitudinal stability augmentation control law and longitudinal attitude control law are designed. The stability augmentation control law is designed by using output feedback linear quadratic method. It can not only increase the longitudinal static stability, but also improve the dynamic characteristics. The longitudinal attitude control law of the flying wing UAV is designed by using command tracking augmented LQR method. The controller can realize the control and maintain the flight attitude and velocity under the condition without breaking robustness of LQR. It solves the command tracking problems that conventional LQR beyond reach.

Considering that some state variables of the system are difficult to obtain directly, a control method that called quasi-command tracking augmented LQR is designed by combing with the reduced order observer, it retains all the features of command tracking augmented LQR and more suitable for the application of practice engineering. Finally, the control laws are simulated under the environment of Matlab/Simulink. The results show that the longitudinal control laws of the flying wing UAV which are designed based on LQR can make the flying wing UAV achieve satisfactory longitudinal flying quality.

Index terms*: Flying wing UAV, UAV modeling, augmented LQR method, longitudinal stability augmentation, longitudinal attitude control, dimension reduction observer.*

I. INTRODUCTION

Because flying wing UAV adopts the technology of wing-fuselage blending, horizontal tail and vertical tail of the conventional configuration are canceled, the plane looks like a lifting surface. It not only improves the lift-to-drag ratio, reduces the Radar Cross Section, but also enlarges the range of flight envelope and cuts down on energy consumption [1][2]. But this exclusive pneumatic layout has brought many problems to the design of flight control system. On the one hand, because of the aspect ratio of the flying wing UAV is large and the fuselage is short, there is no tail plane or horizontal tail, which results in the decrease of the longitudinal static stability and the control effectiveness. On the other hand, the vertical tail of the aircraft is canceled made the transverse lateral damping of aircraft declined, meanwhile we need to add new control mechanism to achieve the yaw of plane. Undoubtedly these will increase the difficulty in designing the flight control law [3][4].

In the method of multivariable feedback control system design, the LQR technology is widely used in many practical engineering fields [5-11], especially in the control of conventional layout fixed-wing UAV and unmanned helicopter [12-16], which has many advantages such as more than 60° phase margin, infinite amplitude margin and strong robustness. In literature [12][13], it has realized the stability augmentation control of conventional layout fixed-wing UAV by using conventional LQR method. It has effectively solved the problems that UAV is disturbed by air current easily and has poor flight stability. The shortage is that there are large numbers of feedback gain to seek, and it is not convenient for the application of practical engineering. The explicit and implicit model following technology, based on linear quadratic regulator theory, are respectively applied to design unmanned helicopter and fixed-wing UAV autopilot in literature [14][15], although the method has achieved the attitude control and hold of UAV, it needs the high-precision reference model, and the controller structure is also more complicated. Literature [16] presents a combined control method based on active modeling and traditional LQG control theory， which can be effectively adapted to model uncertainty and applied to flight control of unmanned helicopter, the result shows that the method can ensure the flight stability of the unmanned helicopter in uncertain wind environment, however the shortage is that LQG control increases the complexity of the control system owing to estimate of the whole state variables, but it needn't in practical engineering.

At present, the design of flight control law of flying wing UAV mainly adopts classic control theory such as the root locus method and frequency domain analysis method[17]. Although the method is reliable and intuitive, there will be heavy workload by using classic control methods to design feedback loop, and sometimes it's difficult to satisfy the design requirements for complex flight control system with multiple inputs multiple outputs and strong coupling. LQR technology with its strong robustness has been successfully and widely used in the fields of the fixed-wing UAV, unmanned helicopter and other engineering. But there are no reports on the flight control of the flying wing UAV. Therefore, the LQR technology with its good robustness will be introduced into the flying wing UAV flight control in this paper, and the longitudinal stability augmentation control law and longitudinal attitude control law of the flying wing UAV will be designed respectively. The design of stability augmentation control law is accomplished by using output feedback linear quadratic method. It can not only increase the longitudinal static stability, but also improve the dynamic characteristics. Meanwhile, the numbers of feedback gain have been reduced compared with the conventional LQR state regulator. It is convenient for the applications of practical engineering. A special control system augmented method and conventional LQR method are combined together to obtain a command tracking augmented LQR method that can used control the attitude of the flying wing UAV. The controller has strong robustness and simple structure, it realizes tracking control with zero static error of the flight velocity and pitch angle, thus it solves the command tracking problems that conventional LQR regulator beyond reach. Considering that some state variables of the system are difficult to obtain directly (e.g. angle of attack), a control method that called quasi-command tracking augmented LQR is designed by combing with the reduced order observer, it retains all the features of command tracking augmented LQR method and more suitable for the application of practical engineering.

Finally, the longitudinal model of the flying wing UAV will be established and the control method will be simulated under the environment of Matlab/Simulink. The results show that the longitudinal control laws for the flying wing UAV based on LQR can make the flying wing UAV achieve satisfactory longitudinal flying qualities.

II. LONGITUDINAL MODELING OF THE FLYING WING UAV

a. Control surfaces of the flying wing UAV

Flying wing UAV has no horizontal stabilizer and vertical tail, so control mechanism of the aircraft can only be installed on the trailing edge of the wing as shown in figure 1.

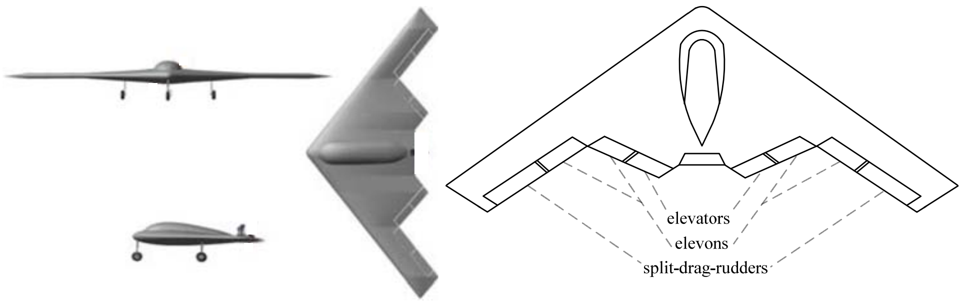


Figure 1. The high-altitude long-endurance flying wing UAV

There is a pair of elevators in the inner side of the trailing edge of the UAV, which is used to control pitch and lift of the plane. The laterals of elevator are provided with symmetrical elevons, which are used for lift enhancement and rolling motion; A new kind of control mechanism named split-drag-rudder is adopted to control the lateral motion of the UAV. Two split-drag-rudders are on the tail edge near the tip wing, which are far from the symmetry plane. When the split-drag-rudder open up a certain angle in one side, the drag-force will be increased on the same side and get an unbalanced yawing moment, which lead the UAV yawing to the same side. While when the two split-drag-rudders are opened up on both two sides, the drag force will be increased noticeably. So the split-drag-rudder can be used to the velocity control of the UAV, such as in approaching and landing, air refueling and etc. Due to the split-drag-rudders are mounted on the tail edge near the tip wing, the distance between the control mechanism and gravity of the UAV is far, the effects on longitudinal pitching motion of the UAV by drag rudder can't be ignored, so it should be taken into consideration while modeling.

b. Longitudinal motion equation of the flying wing UAV

Considering the UAV motion is a very complex dynamic process, it is a nonlinear time-varying system in the flight process, the whole movement is influenced by various factors. For example, earth curvature, atmospheric motion, elastic deformation of the plane, acceleration of gravity and so on. It will be very complex to take various factors into account, which makes the modeling hardly realized. Therefore, we need to make the following assumptions of the flying wing UAV motion system in the modeling process [18][19].

(1) The UAV is rigid and the quality is constant.

(2) The earth fixed axis is regarded as an inertial coordinate system

(3) The acceleration of gravity g is a constant;

(4) The plane ***XOZ*** of the body coordinate system of the flying wing UAV is symmetric, not only the geometry appearance of the UAV is symmetric, but also the internal quality distribution.

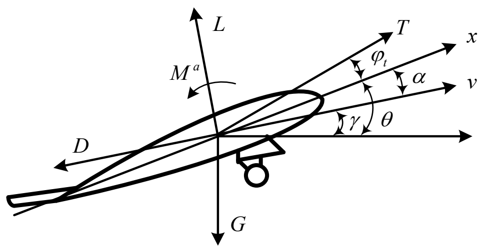


Figure 2. Force analysis of the flying wing UAV longitudinal motion

The longitudinal dynamics and the kinematics model of the flying wing UAV are built based on the above assumptions. Longitudinal stress analysis to the UAV (select a state to climbing process of the UAV) is shown as in Figure 2. Longitudinal motion equation of the flying wing UAV can be described as:

$$\begin{cases} m\dot{v} = T\cos(\alpha+\phi_t) - D + mg\sin\gamma \\ mv\dot{\gamma} = T\sin(\alpha+\phi_t) + L + mg\cos\gamma \\ I_z\ddot{\theta} = M_z \\ \gamma = \theta - \alpha \end{cases} \quad (1)$$

Where m is the quality of the UAV, life force $\boldsymbol{L}$ and drag force $\boldsymbol{D}$ are aerodynamic forces of the UAV, and pitching moment $\boldsymbol{M_z}$ is aerodynamic moment, $\boldsymbol{T}$ is the thrust of the UAV, $\boldsymbol{\varphi_t}$ is the intersection angle between the thrust $\boldsymbol{T}$ and the body axis $\boldsymbol{x}$, $\boldsymbol{I_z}$ is the yaw moment of the inertia, γ is the path angle of the UAV. Aerodynamic force and aerodynamic moments are nonlinear function about the speed v, the atmosphere density ρ of the height of flight, pitch rate q, angle of attack α and the rudder deflection angle. The nonlinear functions are expressed according to the aerodynamic principle as follows :

$$\left.\begin{array}{l} D = \frac{1}{2}\rho V^2 SC_d = \frac{1}{2}\rho V^2 S(C_{D0} + C_D^{\alpha}\alpha + C_D^{v}V + C_D^{\delta_e}\delta_e + C_D^{\delta_d}\delta_d) \\ L = \frac{1}{2}\rho V^2 SC_L = \frac{1}{2}\rho V^2 S(C_{L0} + C_L^{\alpha}\alpha + C_L^{v}V + C_L^{q}q + C_L^{\delta_e}\delta_e) \\ M_z = \frac{1}{2}\rho V^2 ScC_M = \frac{1}{2}\rho V^2 Sc(C_{M0} + C_M^{\alpha}\alpha + C_M^{v}V + \frac{c}{2V}C_M^{q}q + C_M^{\delta_e}\delta_e + C_M^{\delta_d}\delta_d) \end{array}\right\} \quad (2)$$

where S is the wing reference area, c is the wing mean geometric chord, $C_{D0}, C_D^{\alpha}, C_D^{v}$ etc are the aerodynamic derivatives of the UAV, $\boldsymbol{\delta_e}$ is the elevator angle, $\boldsymbol{\delta_d}$ is the split drag rudder angle. Thrust T of the UAV is a nonlinear function about the atmosphere density, the flight velocity and throttle opening. The expression is:

$$T = T(\rho, v, \delta_t) \quad (3)$$

where $\boldsymbol{\delta_t}$ is the throttle opening. Substitute above formula (2) (3) into formula (1), we can get complete longitudinal mathematical model of the flying wing UAV. Since the model consists of nonlinear differential equations, it is not convenient to analysis the system and design the control law. Therefore, we select a typical flight status to linear processing, which is based on the disturbance theory and the coefficient freezing method, the differential equations with constant coefficients of the system are:

$$\begin{bmatrix} \Delta\dot{v} \\ \Delta\dot{\alpha} \\ \Delta\dot{q} \\ \Delta\dot{\theta} \\ \Delta\dot{h} \end{bmatrix} = \begin{bmatrix} n_{1v} & n_{1\alpha} & 0 & n_{1\theta} & 0 \\ n_{2v} & n_{2\alpha} & 1 & n_{2\theta} & 0 \\ n_{3v} & n_{3\alpha} & n_{3q} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ n_{5v} & n_{5\alpha} & 0 & n_{5\theta} & 0 \end{bmatrix} \begin{bmatrix} \Delta v \\ \Delta\alpha \\ \Delta q \\ \Delta\theta \\ \Delta h \end{bmatrix} + \begin{bmatrix} n_{1\delta_e} & n_{1\delta_t} & n_{1\delta_d} \\ n_{2\delta_e} & n_{2\delta_t} & 0 \\ n_{3\delta_e} & 0 & n_{3\delta_d} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\delta_e \\ \Delta\delta_t \\ \Delta\delta_d \end{bmatrix} \tag{4}$$

Where $n_{1v} = \dfrac{T_v \cos(\alpha_0 + \varphi_t) - D_v}{m}$, $n_{1\alpha} = g\cos\gamma_0 - \dfrac{T_0 \cos(\alpha_0 + \varphi_t) + D_\alpha}{m}$, $n_{1\theta} = g\cos\gamma_0$, $n_{1\delta e} = -\dfrac{D_{\delta e}}{m}$,

$n_{1\delta d} = -\dfrac{D_{\delta d}}{m}$, $n_{1\delta t} = \dfrac{T_{\delta t}\cos(\alpha_0 + \varphi_t)}{m}$, $n_{2v} = -\dfrac{T_v \sin(\alpha_0 + \varphi_t) + L_v}{mv_0}$, $n_{2\delta e} = -\dfrac{L_{\delta e}}{mv_0}$

$n_{2\alpha} = \dfrac{g\sin\gamma_0}{\gamma_0} - \dfrac{T_0\sin(\alpha_0 + \varphi_t) + L_\alpha}{mv_0}$, $n_{2\theta} = \dfrac{g\sin\gamma_0}{v_0}$, $n_{3\delta e} = \dfrac{M_{\delta e}}{I_y}$

$n_{2\delta t} = -\dfrac{T_{\delta t}\sin(\alpha_0 + \varphi_t)}{mv_0}$, $n_{3v} = \dfrac{M_v}{I_y}$, $n_{3\alpha} = \dfrac{M_\alpha}{I_y}$, $n_{3q} = \dfrac{M_q}{I_y}$, $n_{3\delta d} = \dfrac{M_{\delta d}}{I_y}$

$n_{5v} = \sin\gamma_0$, $n_{5\alpha} = v_0\cos\gamma_0$, $n_{5\theta} = v_0\cos\gamma_0$.

The $v_0, \gamma_0, \alpha_0, T_0$ are known as parameters of typical flight status, longitudinal modeling of the UAV is completed.

III. THE FLYING WING UAV LONGITUDINAL STABILITY AUGMENTATION CONTRAL

As the fuselage of the flying wing UAV is short, it results in the control effectiveness of elevator and elevon on the tail edge of the UAV low. In order to improve the maneuverability, the static stability can be properly relaxed. The aerodynamic center will shift backward, which may weaken the UAV's longitudinal static stability when the UAV flying at transonic or at high angle of attack. As a compromise between the longitudinal stability and maneuverability, the aircraft's center of gravity can be configured at the position between the aerodynamic center at high angle of attack and high-speed and the aerodynamic center at low-speed and low angle of attack. Therefore, the static stability can be maintained at low speed and low angle of attack, and the instability at high speed and high angle of attack is still acceptable. As a result, only at high-speed and high angle of attack, longitudinal stability augmentation is needed.

a. Output feedback linear quadratic regulator

Although the traditional linear quadratic regulator can achieve stability augmentation of the UAV, it needs all the state variables information of the system and heavy workload, which is not conducive to engineering practice. Therefore, there is output feedback linear quadratic method to design the longitudinal stability augmentation control law of the flying wing UAV. Combined with section Ⅱ.b, consider the following three rudder loop models:

Elevator: $\delta_e = \frac{20}{S+20} u_e$, throttle thrust: $\delta_t = \frac{20}{S+20} u_t$, and split-drag-rudder: $\delta_d = \frac{40}{S+20} u_d$.

The longitudinal augmented state equation and output equation of the flying wing UAV are:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \tag{5}$$

Where $x = [\Delta v \;\; \Delta\alpha \;\; \Delta q \;\; \Delta\theta \;\; \Delta\delta_e \;\; \Delta\delta_t \;\; \Delta\delta_d]^T$ is the system state vector, $u = [\Delta u_e \;\; \Delta u_t \;\; \Delta u_d]^T$ is the control vector, and $y = [\Delta v \;\; \Delta\alpha \;\; \Delta q \;\; \Delta\theta]^T$ is the output of the UAV. Choose form for output feedback:

$$u = Ky \tag{6}$$

where ***K*** is the feedback gain matrix of the corresponding dimension. Substitute (6) into the above formula (5), we obtain the following state equation of closed loop system:

$$\dot{x} = (A + BKC)x = A^* x \tag{7}$$

The purpose of designing the stability augmentation control law is to adjust the UAV state, so that any errors of the initial conditions can be preserved to zero, which can ensure the flight stability. Thus we can minimize the following quadratic cost function by selecting the control input ***u***:

$$J = \frac{1}{2}\int_0^\infty (x^T Q x + u^T R u) dt \tag{8}$$

where, ***Q*** and ***R*** are the weighting matrices, ***Q*** is the semi-positive definite symmetric matrix and ***R*** is the positive definite symmetric matrix. The selection of ***Q*** and ***R*** can be compromised between the adjustment speed and control function of state variables. Greater control weighting matrix ***R*** can obtain smaller control ability, whereas greater state weighting matrix ***Q*** can speed up the adjustment of the state variable. The choice of ***Q*** and ***R*** also affects the pole position of the closed-loop system. The anticipant time domain characteristics of the closed-loop system can be achieved by reasonable weight matrix configuration. Substitute (6) and (5) into the above formula (8), we can easily obtain:

$$J = \frac{1}{2}\int_0^\infty x^T (Q + C^T K^T RKC) x dt \tag{9}$$

It can be concluded that simply selecting the appropriate feedback matrix ***K*** can obtain the aim of minimizing quadratic cost function from (9), which converts a dynamic optimization problem into a static problem which is easy to be solved.

Assume that a positive definite symmetric matrix ***P*** can be found to build a Lyapunov function of x. If the function satisfies the Lyapunov stability theorem, the closed-loop system (7) is asymptotically stable. The Lyapunov function can be defined as：

$$V(x) = x^T P x \tag{10}$$

Combining (7) we take the derivative of ***V(x)***:

$$\dot{V}(x) = x^T (A^{*T} P + PA^*) x \tag{11}$$

Then the following equation can be obtained by using the integrand of (9) and the property of the Lyapunov function ***V(x)***:

$$\frac{d(x^T P x)}{dt} = x^T (A^{*T} P + PA^*) x = -x^T (Q + C^T K^T RKC) x \tag{12}$$

Because we have assumed that the closed-loop system is asymptotically stable, the quadratic cost function can be written as:

$$J = \frac{1}{2} x^T(0) P x(0) - \lim_{t\to\infty} \frac{1}{2} x^T(t) P x(t) = \frac{1}{2} x^T(0) P x(0) \tag{13}$$

From (13), we can calculate the quadratic cost function of the closed-loop system as long as the initial condition ***x(0)*** are known, and this is irrelevant to other states under feedback control (6). As (12) must satisfy all the initial conditions, all the state trajectories ***x(0)*** satisfy the following Lyapunov equation:

$$f = Q + C^T K^T RKC + A^{*T} P + PA^* = 0 \tag{14}$$

From (14) we can find that if matrix ***Q*** and matrix ***K*** are given, auxiliary matrix ***P*** can be determined by Lyapunov function, and it is independent on the state of the system.

In conclusion, in terms of any feedback matrix ***K*** with fixed value, if there is a non-negative definite symmetric matrix ***P*** which satisfies the Lyapunov equation (14) and the closed-loop system is stable, the quadratic performance index is relevant to the initial condition ***x(0)***and the matrix P, which is independent of system states.

To simplify the solving of feedback gain ***K***, tr(AB)=tr(BA), which describes the relationship of matrix trace, is introduced. Thus (13) can be rewritten as:

$$J = \frac{1}{2} tr[Px(0)x^T(0)] \tag{15}$$

Visibly, under the constraint condition of state equations (7), the problem which obtains the feedback matrix ***K*** by minimizing the quadratic cost function (9) is converted to the problem which solves feedback matrix ***K*** by minimizing (15) under (14) with the auxiliary symmetrical matrix ***P***. But seen from (15), $x(0)\,x^T(0)$ is depend on initial conditions, which are not expected to get because initial states may not be pre-determined in many practical engineering. Therefore, we assume that the initial state is evenly distributed in the unit sphere, namely $x(0)\,x^T(0)$ is the unit matrix. So the problem of solving performance indicators (15) is converted to solving the expectation $E(J)$ for performance indicators, which evades the choice of the initial value.

Next, we use the solving method of the extreme value problem with constraint conditions to solve matrix ***K*** and matrix ***P*** [20]. First, we introduce the Lagrange matrix factor $\lambda \in R^{4\times4}$, and then construct Hamilton function as:

$$H = F + tr(f\lambda) \tag{16}$$

where $F = tr[Px(0)x^T(0)]$. We make the variation to (16) respectively for ***K***, ***P***, ***λ*** and make them zero, then obtain the following equations:

$$\begin{cases} \dfrac{\partial H}{\partial K} = RKC\lambda C^T - B^T P\lambda C^T = 0 \\ \dfrac{\partial H}{\partial P} = A^*\lambda + \lambda A^{*T} + x(0)x^T(0) = 0 \\ \dfrac{\partial H}{\partial \lambda} = Q + C^T K^T RKC + A^{*T}P + PA^* = 0 \end{cases} \tag{17}$$

The three equations above are necessary conditions for the solution of output feedback linear quadratic regulator. ***R*** is a positive definite matrix and non-singular, so the output feedback matrix K can be obtained as:

$$K = R^{-1}B^T P\lambda C^T (C\lambda C^T)^{-1} \tag{18}$$

Finally, we obtain the longitudinal stability augmentation control law of the flying wing UAV:

$$u = -Ky = -R^{-1}B^T P\lambda C^T (C\lambda C^T)^{-1} Cx$$

The longitudinal stability augmentation control system structure of the flying wing UAV designed by output feedback linear quadratic regulator is illustrated in Figure 3.

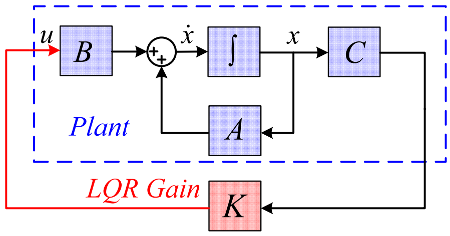


Figure 3. Block diagram of the control system designed by output feedback linear quadratic regulator method

Simplex algorithm or iterative method can be used when we solve the feedback gain matrix K with computers [21][22]. The specific steps of iterative method are shown as followings:

Step 1: Parameter initialization

Set $i = 0$, then make matrix $\boldsymbol{A_i = A - BK_iC}$ asymptotically stable by selecting the initial gain $\boldsymbol{K_i}$ with the eigenvalue configuration method.

Step 2: Iterative process

Make the i-th iteration, and solve $\boldsymbol{P_i}$, $\boldsymbol{\lambda_i}$ and cost function $J_i = 1/2\, tr[P_i x(0) x^T(0)]$ with following Lyapunov equations:

$$\begin{cases} Q + C^T K_i^T R K_i C + A_i^{*T} P + P A_i^* = 0 \\ A_i^* \lambda_i + \lambda_i A_i^{*T} + x(0)x^T(0) = 0 \end{cases}$$

The correction value of the feedback matrix K is calculated through, $\Delta K = R^{-1}B^T P_i \lambda_i C^T (C\lambda_i C^T)^{-1} - K_i$ and the amended feedback matrix is $K_{i+1} = K_i + \varepsilon \Delta K$. The ε is chosen to make matrix A_{i+1} asymptotically stable, in the meantime, make $J_{i+1} \le J_i$. When J_{i+1} is close enough to J_i, go to the step 3, otherwise set i=i+1 and go to the step 2 to continue the calculation.

Step 3: Valuation

Set $K = K_i$ and $J = J_i$, then the iteration ended.

IV. THE FLYING WING UAV LONGITUDINAL ATTITUDE CONTROL

In previous section, we have designed the longitudinal stability augmentation control law of the flying wing UAV with the use of output feedback linear quadratic technology, and completed the longitudinal stability augmentation control of the UAV. On this basis, we will design the flying wing UAV longitudinal attitude control law with an command tracking augmented LQR method, which is the combination of a special control system augmented method and the conventional LQR. Given the angle of attack in the engineering practice is hard to be directly detected, a quasi-command tracking augmented LQR method will be designed with the combination of reduced-dimension observer to solve this problem.

a. Command tracking augmented LQR control method

Although conventional LQR and output feedback linear quadratic regulator can realize the stability augmentation of the system in a certain equilibrium state, it is difficult for them to achieve the tracking of the input instructions. Therefore, in this paper, the following augmented LQR has been considered to design the longitudinal attitude control law of the flying wing UAV. All the state variables of the system are assumed to be detected.

Longitudinal state equation and output equation of the flying wing UAV after stability augmentation are known as:

$$\begin{cases} \dot{x} = A^* x + B^* u \\ y = C^* x \end{cases} \tag{19}$$

Set the control input $u = \begin{bmatrix} \Delta u_t & \Delta u_e \end{bmatrix}^T$, and the output $y = \begin{bmatrix} \Delta v & \Delta \theta \end{bmatrix}^T$. The control law $\boldsymbol{u}$ should be designed to make the system's output y can be tracked on a given input signal: *r(t)=C×1(t)*, where $\boldsymbol{C}$ is a constant matrix of corresponding dimensions. Set the output error of the system is *e(t)=r(t)-y(t)* . The differentiation of (19) is:

$$\begin{cases} \ddot{x} = A^* \dot{x} + B^* \dot{u} \\ \dot{e} = \dfrac{d(r-y)}{dt} = -\dot{y} = -C^* \dot{x} \end{cases} \tag{20}$$

Set the augmented state vector $\tilde{x} = \begin{bmatrix} \dot{x}^T & e^T \end{bmatrix}^T$, and get the following augmented system:

$$\dot{\tilde{x}} = \tilde{A}\tilde{x} + \tilde{B}\tilde{u} \tag{21}$$

Where $\tilde{u} = \dot{u}$, $\tilde{A} = \begin{bmatrix} A^* & 0 \\ -C^* & 0 \end{bmatrix}$, $\tilde{B} = \begin{bmatrix} B^* \\ 0 \end{bmatrix}$. To keep all the state variables of the augmented system being zero, that is $\tilde{x} = \begin{bmatrix} \dot{x}^T & e^T \end{bmatrix}^T = 0$, linear quadratic regulator is designed for (21). It will ensure the system's output zero static error, so as to achieve the purpose of tracking given input signal. Set the quadratic cost function of formula (21):

$$\tilde{J} = \frac{1}{2}\int_0^{\infty} [\tilde{x}^T Q \tilde{x} + \tilde{u}^T R \tilde{u}] dt \quad (22)$$

Where $\boldsymbol{Q}$ is a positive semi-definite symmetric weighting matrix of corresponding dimension, and $\boldsymbol{R}$ is a positive definite symmetric weighting matrix. The control law obtained with LQR method is:

$$\tilde{u} = -R^{-1}\tilde{B}^T P x = \tilde{K} x \quad (23)$$

where $\tilde{K}$ is constant state feedback gain, and $\boldsymbol{P}$ is positive definite symmetric matrix. It is obtained by the following Riccati equation:

$$P\tilde{A} + \tilde{A}^T P - P\tilde{B}R^{-1}\tilde{B}^T P + Q = 0$$

Expressing $\tilde{K}$ as partitioned matrix according to $\dot{x}$ and e, (23) can be rewritten as follows:

$$\dot{u} = \tilde{u} = \tilde{K}\tilde{x} = \begin{bmatrix} K_{\dot{x}} & K_e \end{bmatrix} \times \begin{bmatrix} \dot{x} \\ e \end{bmatrix} = K_{\dot{x}}\dot{x} + K_e e \quad (24)$$

By integrating both sides of (24), we can obtain that longitudinal attitude control law of the flying wing UAV:

$$u(t) = K_{\dot{x}} x(t) + K_e \int_0^t e(t) dt \quad (25)$$

The flying wing UAV longitudinal attitude control system structure designed by the method of command trace augmented LQR is illustrated in Figure 4:

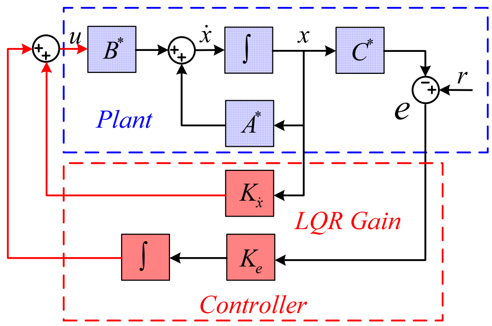


Figure 4. Block diagram of the control system designed by command tracking augmented LQR method

It can be seen from Figure 4 that the output y of the model is feedback into the input of the controller, and then get the error signal *e(t)* after being subtracted with a given input command. Because the *e(t)* passes through the integrator, it can be judged that this control method can eliminate the steady-state error of the system. Furthermore, this controller not only uses all the state variables of the system, but also the output information. Therefore, it will predictably have a good tracking effect.

b. Quasi-command tracking augmented LQR control method

When designing controller with the command tracking augmented LQR method, all the state variables of the system have to be detected. However, some state variables are difficult or even impossible to be directly detected in practical engineering applications, such as the angle of attack. To this end, there is a quasi-command tracking augmented LQR method designed with reduced-order observer. The specific process is discussed as follows.

Because the system (19) is completely observable, there must exist a linear transformation $x = T\bar{x}$ which divides the state variables into two parts: one cannot be detected, the other can. Here the transformation matrix $T = \begin{bmatrix} Z_0 \\ C^* \end{bmatrix}^{-1}$ is selected, in which Z_0 should ensure that $\boldsymbol{T}$ is non-singular. The formula (19) will be transformed and divided as follows:

$$\begin{cases} \dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}u \\ \bar{y} = \bar{C}\bar{x} \end{cases} \tag{26}$$

where $\bar{x} = \begin{bmatrix} \bar{x}_1 & \bar{x}_2 \end{bmatrix}^T$, $\bar{x}_1 = \Delta\bar{\alpha}$ is the state vector that cannot be detected, and $\bar{x}_2 = [\Delta\bar{v} \quad \Delta\bar{q} \quad \Delta\bar{\theta}]$ is the state vector that can be detected. $\bar{C}=C^*T=[0 \quad I]$, $\bar{A}=T^{-1}A^*T=\begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix}$, $\bar{B}=T^{-1}B^*=\begin{bmatrix} \bar{B}_1 \\ \bar{B}_2 \end{bmatrix}$, $\bar{y}=\bar{x}_2$. formula (26) can be written as:

$$\begin{cases} \dot{\bar{x}}_1 = \bar{A}_{11}\bar{x}_1 + \bar{A}_{12}\bar{x}_2 + \bar{B}_1 u \\ \bar{A}_{21}\bar{x}_1 = \dot{\bar{x}}_2 - \bar{A}_{22}\bar{x}_2 - \bar{B}_2 u \end{cases} \tag{27}$$

Set $U = \bar{A}_{12}\bar{x}_2 + \bar{B}_1 u$, $Y = \dot{\bar{x}}_2 - \bar{A}_{22}\bar{x} - \bar{B}_2 u$, (3-9)can be rewritten as:

$$\begin{cases} \dot{\bar{x}}_1 = \bar{A}_{11}\bar{x}_1 + U \\ Y = \bar{A}_{21}\bar{x}_1 \end{cases} \tag{28}$$

U and Y are rectifiable based on known ***u*** and directly obtained $\bar{x}_2$ via $\bar{y}$. Thus the state reconstruction of the subsystem to be observed can be realized simply by designing a full-order observer for (28). The full-dimensional observer designed by (28) is as follows:

$$\dot{\hat{\bar{x}}}_1 = (\bar{A}_{11} - \bar{K}_e \bar{A}_{21})\hat{\bar{x}}_1 + \bar{K}_e Y + U \tag{29}$$

Where $\hat{\bar{x}}_1$ is the estimated value of $\bar{x}_1$, $\bar{K}_e$ is the output error feedback matrix of state observer. By substituting U and Y into (29), we obtain:

$$\dot{\hat{\bar{x}}}_1 = (\bar{A}_{11} - \bar{K}_e \bar{A}_{21})\hat{\bar{x}}_1 + (\bar{A}_{12} - \bar{K}_e \bar{A}_{22})\bar{y} + (\bar{B}_1 - \bar{K}_e \bar{B}_2)u + \bar{K}_e \dot{\bar{y}} \tag{30}$$

To eliminate $\dot{\bar{y}}$, the variable $\hat{\bar{\zeta}} = \hat{\bar{x}}_1 - \bar{K}_e \bar{y}$ is introduced and substituted into (30), we obtain:

$$\dot{\hat{\bar{\zeta}}} = (\bar{A}_{11} - \bar{K}_e \bar{A}_{21})\hat{\bar{\zeta}} + [(\bar{A}_{12} - \bar{K}_e \bar{A}_{22}) + (\bar{A}_{11} - \bar{K}_e \bar{A}_{21})\bar{K}_e]\bar{y} + (\bar{B}_1 - \bar{K}_e \bar{B}_2)u \tag{31}$$

Finally, combing with $\hat{\bar{\zeta}} = \hat{\bar{x}}_1 - \bar{K}_e \bar{y}$, we obtain the estimated value of the entire state vector $\bar{x}$ as follows:

$$\hat{\bar{x}} = \begin{bmatrix} \hat{\bar{x}}_1 \\ \bar{x}_2 \end{bmatrix} = \begin{bmatrix} \hat{\bar{\zeta}} + \bar{K}_e \bar{y} \\ \bar{y} \end{bmatrix} = \begin{bmatrix} I \\ 0 \end{bmatrix} \hat{\bar{\zeta}} + \begin{bmatrix} \bar{K}_e \\ I \end{bmatrix} \bar{y} \tag{32}$$

Transforming (32) back to the original system, the state estimated value of the system (19) is $\hat{x} = T\hat{\bar{x}}$.

As can be seen from (31) and (34), as long as the output error feedback matrix of the state observer $\bar{K}_e$ is obtained, it will be able to complete the design of reduced order observer. The selection of the matrix $\bar{K}_e$ will directly affect the convergence rate of the error e_1, where $e_1 = \bar{x}_1 - \hat{\bar{x}}_1$. For single-input systems, $\bar{K}_e$ is generally obtained via the dual relationship between state feedback and state observer. But for complex multi-input system, it will be more complex to solve the state feedback matrix by pole assignment method. To this end, we choose conventional LQR method to obtain $\bar{K}_e$. Firstly, the dual system of formula (28) is written as:

$$\begin{cases} \dot{\bar{x}}_1^* = \bar{A}_{11}^T \bar{x}_1^* + \bar{A}_{21}^T U^* \\ Y^* = \bar{x}_1^* \end{cases} \tag{33}$$

Then we design linear quadratic regulator for (33), and solve the following Riccati equation:

$$P\bar{A}_{11}^T + \bar{A}_{11}P - P\bar{A}_{21}^T R^{-1} \bar{A}_{21} P + Q = 0 \tag{34}$$

The feedback gain matrix can be obtained as $K^* = -R^{-1}\overline{A}_{21}P$. Finally, we transpose matrix K^* and obtain $\overline{K}_e = K^{*^T} = -P\overline{A}_{21}^T(R^{-1})^T$.

Through the design of the reduced order observer, the state variable $\Delta\alpha$ of the system that cannot be detected can be estimated. So command tracking augmented LQR control method can be adopted to design the longitudinal attitude control law of the UAV. We still set $\tilde{K}$ as the augmented LQR feedback matrix of the system (21) against the corresponding quadratic cost function (22). Express $\tilde{K}$ into block matrix $\tilde{K} = \begin{bmatrix} K_{\hat{x}_1} & K_{x_2} & K_e \end{bmatrix}$ according to $\hat{x}_1$, x_2 and e. Then the longitudinal attitude control law of the flying wing UAV can be obtained as:

$$u(t) = K_{\hat{x}_1}\hat{x}_1(t) + K_{x_2}x_2(t) + K_e\int_0^t e(t)dt \tag{35}$$

The structure of the flying wing UAV longitudinal attitude control system that designed by the quasi-command tracking augmented LQR control method is illustrated in figure 5.

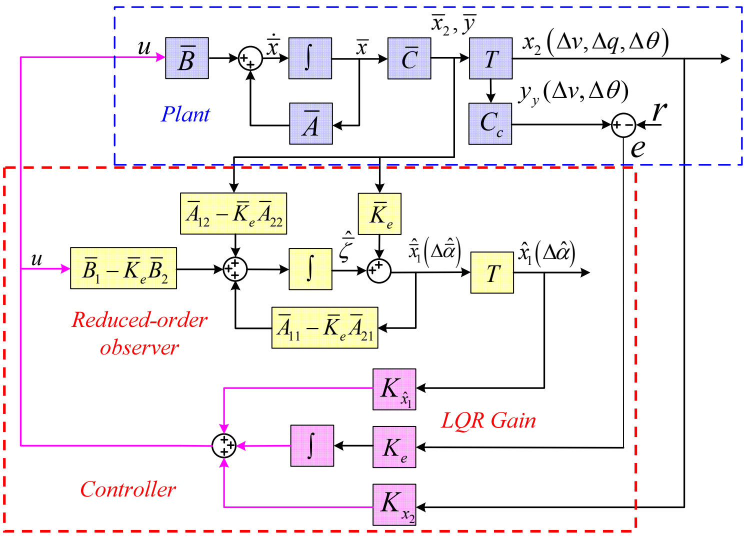


Figure 5. Block diagram of the control system designed by quasi-command tracking augmented LQR method

V. DESIGN EXAMPLE

In this section, the longitudinal stability augmentation control law and the longitudinal attitude control law of the flying wing UAV will be simulated. Now we research on the longitudinal of the high-altitude long-endurance flying wing UAV, and select an altitude at 2000m, Mach 0.805

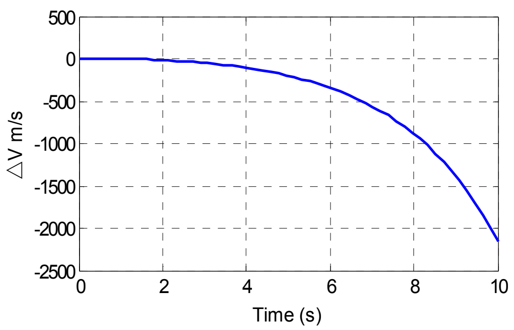
as the typical state for analysis. The flying wing UAV is linearized in the flight state; the augmented state equation and output equation are obtained as follows:

$$\begin{cases} \dot{x}_m = A_m x_m + B_m u \\ y = C_m x_m + D_m u \end{cases} \tag{36}$$

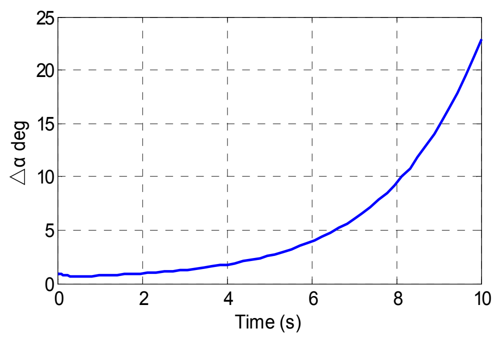
Where $x_m = \begin{bmatrix} \Delta v & \Delta\alpha & \Delta q & \Delta\theta & \Delta\delta_e & \Delta\delta_t & \Delta\delta_d \end{bmatrix}^T$, $u_m = \begin{bmatrix} \Delta u_e & \Delta u_t & \Delta u_d \end{bmatrix}^T$

$$A_m = \begin{bmatrix} -0.0127 & 6.2136 & 0 & -9.3718 & 0.0058 & 0.111 & 0.7384 \\ -0.004 & -1.9889 & 1 & -0.0411 & -0.0032 & -0.0002 & 0 \\ -0.0024 & 6.3838 & -2.4646 & 0 & -0.2437 & 0 & -0.3132 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -20 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -20 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -20 \end{bmatrix} B_m = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 20 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 40 \end{bmatrix} C_m = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 57.3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 57.3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 57.3 & 0 & 0 & 0 \end{bmatrix} D_m = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

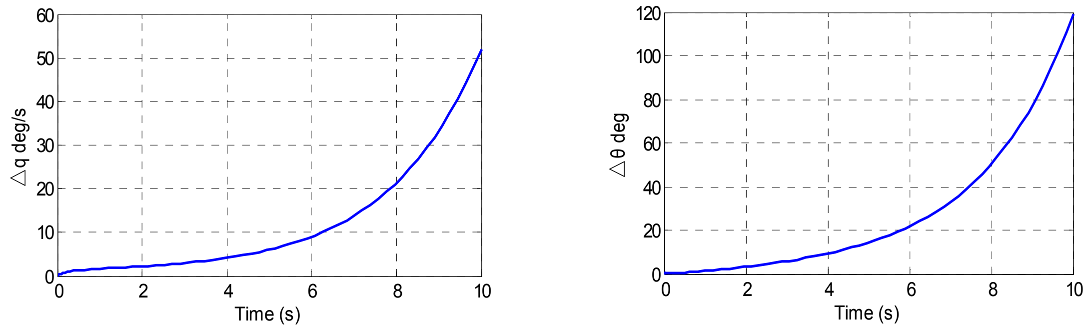
The eigenvalues of the open-loop system according to formula (36) are : λ_1=-4.5775, λ_2=0.4419, $\lambda_{3,4}$=-0.0665±0.359i, $\lambda_{5,6,7}$=-20, where $\lambda_{1,2}$ are the corresponding characteristic roots of longitudinal short period mode of the flying wing UAV, and $\lambda_{3,4}$ are the corresponding characteristic roots of long period mode. These four eigenvalues play a decisive role in the longitudinal movement of the UAV. When the UAV is subject to external disturbance or given input, the changing rule of longitudinal various parameters over time is the superposition of the two motion modes. $\lambda_{5,6,7}$ are the poles of three rudder loops. When the open-loop system is disturbed by 1° angle of attack, the response curves of the UAV longitudinal parameters are shown in Figure 6.



(a) The flight velocity response curve



(b) The angle of attack response curve



(c) The pitch rate response curve (d) The pitch angle response curve

Figure 6. The disturbance response curve of the flying wing UAV before stability

Figure 6 shows that the response curves of the parameters are diverging after the UAV is disturbed, because there exists the positive root in the UAV short period mode which is caused by the flying wing UAV longitudinal static instability. Therefore, it is essential to conduct longitudinal stability augmentation control for the flying wing UAV.

The longitudinal stability augmentation control law of the UAV, can be designed as ***u=Ky*** by use the output feedback linear quadratic regulator, where $K \in R^{3\times4}$. In order to get a satisfying stability augmentation effect, the appropriate weighting matrices ***Q*** and ***R*** augmentation should be firstly selected before the output feedback by LQR application. Considering that the UAV longitudinal static instability will lead to the short period mode of the UAV diffuse, in the quadratic cost function, the state $\Delta\alpha^2$ andΔq^2 which are closely relevant to short period mode should be weighted by element q_α of the weighting matrix ***Q*** if we want to obtain the short period mode with a satisfying stability. In the long period mode, under damping can be seen from the characteristic values $\lambda_{3,4}$, it is necessary for the state Δv^2 and $\Delta\theta^2$ which are closely relevant to the long period mode, to be weighted by the element q_b of the weighting matrix ***Q***. As the extended state variables are not discussed, there is no need to weight them. As a result, the weighting matrix ***Q*** can be rewritten as $Q=\mathrm{diag}\{q_b,q_a,q_a,q_b,0,0,0\}$. In terms of R, the form of $R=\rho \times I$ is used to prevent oversize control input. Where ρ is the design parameter and I is a unit matrix of corresponding dimension.

After selecting and checking repeatedly, it can be concluded that the longitudinal stability augmentation of the UAV will achieve the best when $Q=\mathrm{diag}\{50,10,10,50,0,0,0\}$ and $\rho=1$. The optimal feedback matrix $\boldsymbol{K}$ is obtained as follows by the use of the iterative solution method mentioned in section III:

$$K=\begin{bmatrix} -1.6073 & -22.8329 & -23.3958 & -26.5004 \\ 7.2136 & 10.1877 & 0.9967 & -14.1970 \\ 3.2225 & 2.4844 & -2.4463 & -7.6770 \end{bmatrix}$$

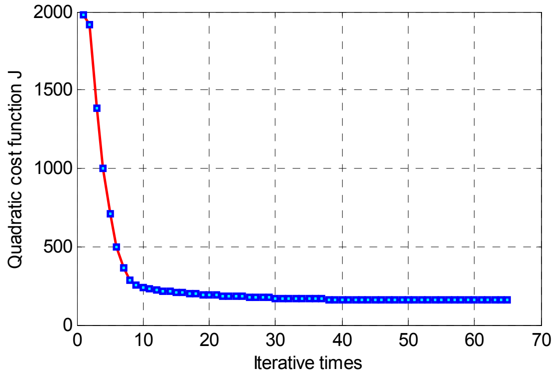
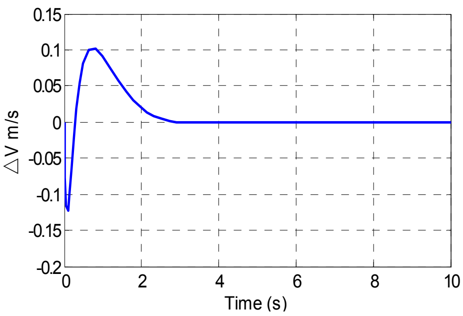
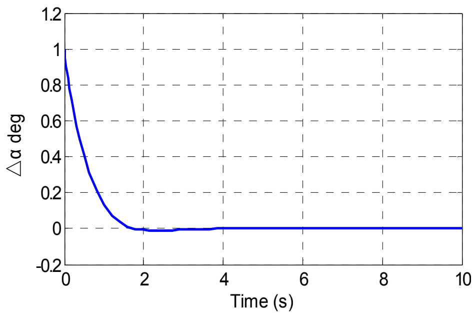


Figure 7.   The changing curve of the quadratic cost function J

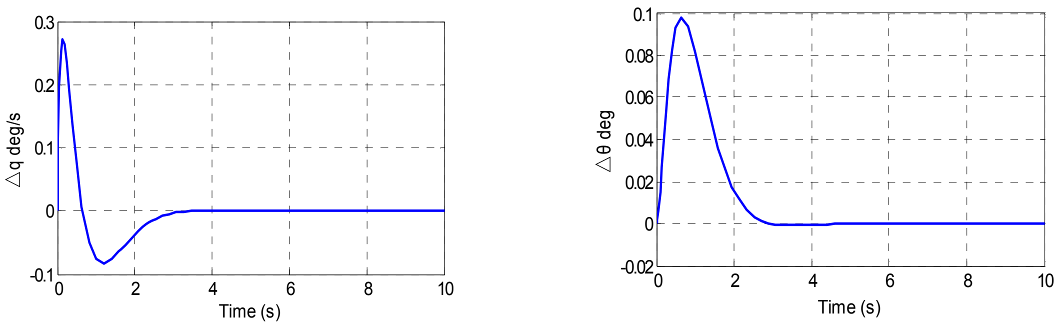
The changing curve of quadratic cost function during iterative process is illustrated in Figure 7. The eigenvalues of the closed-loop system after stability augmentation are $\lambda_{1,2}$=-1.526±0.764i, $\lambda_{3,4}$=-10.437±9.01i, λ_5=-5.884, λ_6=-14.657, λ_7=-20. When the closed-loop system is disturbed by 1° angle of attack, the response curves of the UAV longitudinal parameters are shown in Figure 8.It can be seen from Figure 8 and the closed-loop system characteristic roots, the longitudinal dynamic quality of the flying wing UAV after stability augmentation has improved significantly.



(a).The flight velocity response curve



(b).The angle of attack response curve



(c).The pitch rate response curve (d).The pitch angle response curve

Figure 8. The disturbance response curve of the flying wing UAV after stability augmentation

We have achieved the stability augmentation control, the longitudinal attitude control will be completed based on it. In order to verify the two kinds of attitude control methods mentioned in the third section, here we only control the flight velocity and the pitch angle. The stability augmented Δv and Δθ are chosen as the system output and u_t, u_e are chosen as the system control input. The two augmented LQR control methods mentioned in the third section are applied to design the attitude control law u_t and u_e, so that the command tracking of the flight velocity and pitch angle can be achieved. It is assumed that all the states of the system are measurable when we use command tracing augmented LQR method to design the control laws. While the angle of attack which is assumed immeasurable can be estimated by the reduced order observer when the command tracking augmented LQR method is adopted to design the control laws.

The state equation and output equation of the flying wing UAV after stability augmentation are:

$$\begin{cases} \dot{x}(t) = A^* x(t) + B^* u(t) \\ y(t) = C^* x(t) \end{cases} \tag{37}$$

Where $x=[\Delta v \quad \Delta\alpha \quad \Delta q \quad \Delta\theta \quad \Delta\delta_e \quad \Delta\delta_t]^T$, $u=[\Delta u_t \quad \Delta u_e]^T$, $y=[\Delta v \quad \Delta\theta]^T$. According to the methods in Section Ⅳ.a, equation (37) can achieve a new augmented state equation:

$$\dot{\tilde{x}} = \tilde{A}\tilde{x} + \tilde{B}\tilde{u} \tag{38}$$

Select quadratic cost function (22) and design the linear quadratic regulator for (38). After selecting and checking repeatedly, it can be concluded that the command trace will achieve the best when $\boldsymbol{R}$=diag{1,1}, $\boldsymbol{Q}$=diag{20,20,20,20,1,1,500,2000}. The optimal feedback matrix $\boldsymbol{K}$ is obtained as follows:

$$K = \begin{bmatrix} k_{\Delta\dot{v}}^{u_t} & k_{\Delta\dot{a}}^{u_t} & k_{\Delta\dot{q}}^{u_t} & k_{\Delta\dot{\theta}}^{u_t} & k_{\Delta\dot{\delta}_e}^{u_t} & k_{\Delta\dot{\delta}_t}^{u_t} & k_{\Delta e_v}^{u_t} & k_{\Delta e_\theta}^{u_t} \\ k_{\Delta\dot{v}}^{u_e} & k_{\Delta\dot{a}}^{u_e} & k_{\Delta\dot{q}}^{u_e} & k_{\Delta\dot{\theta}}^{u_e} & k_{\Delta\dot{\delta}_e}^{u_e} & k_{\Delta\dot{\delta}_t}^{u_e} & k_{\Delta e_v}^{u_e} & k_{\Delta e_\theta}^{u_e} \end{bmatrix}$$
$$= \begin{bmatrix} -14.59 & 90.15 & 90.17 & 380.32 & -4.77 & -0.012 & 7.89 & -185.56 \\ -27.22 & -62.95 & 8.57 & 116.62 & -0.018 & -3.62 & 11.75 & 124.75 \end{bmatrix}$$

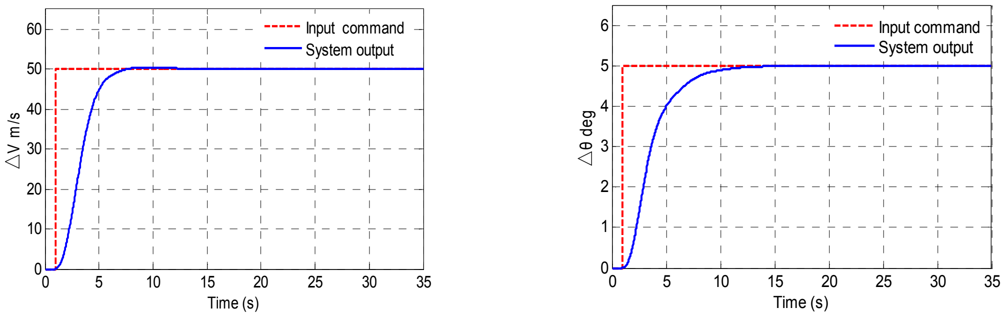
Finally, the flying wing UAV flight velocity and pitch angle control laws are designed by the command tracking augmented LQR method：

$$\left.\begin{aligned} u_v = u_t &= k_{\Delta\dot{v}}^{u_t}\Delta v + k_{\Delta\dot{a}}^{u_t}\Delta a + k_{\Delta\dot{q}}^{u_t}\Delta q + k_{\Delta\dot{\theta}}^{u_t}\Delta\theta \\ &\quad + k_{\Delta\dot{\delta}_e}^{u_t}\Delta\delta_e + k_{\Delta\dot{\delta}_t}^{u_t}\Delta\delta_t + k_{\Delta e_v}^{u_t}\int\Delta e_v dt + k_{\Delta e_\theta}^{u_t}\int\Delta e_\theta dt \\ u_\theta = u_e &= k_{\Delta\dot{v}}^{u_e}\Delta v + k_{\Delta\dot{a}}^{u_e}\Delta a + k_{\Delta\dot{q}}^{u_e}\Delta q + k_{\Delta\dot{\theta}}^{u_e}\Delta\theta \\ &\quad + k_{\Delta\dot{\delta}_e}^{u_e}\Delta\delta_e + k_{\Delta\dot{\delta}_t}^{u_e}\Delta\delta_t + k_{\Delta e_v}^{u_e}\int\Delta e_v dt + k_{\Delta e_\theta}^{u_e}\int\Delta e_\theta dt \end{aligned}\right\} \tag{39}$$

When quasi-command tracking augmented LQR method is used to design attitude control laws, in addition to design the reduced order observer for the state variable $\Delta\alpha$ that cannot be detected, the rest of the design process is exactly the same with the above. To this end, there is no longer detailed description. Equation (37) is designed according to the reduced order observer in section Ⅳ.b, the output error feedback matrix of state observer is $\bar{K}_e = \begin{bmatrix} 0.3615 & 0.3714 & 0 \end{bmatrix}$. Finally, the flying wing UAV flight velocity and pitch angle control laws are designed by the quasi-command tracking augmented LQR method：

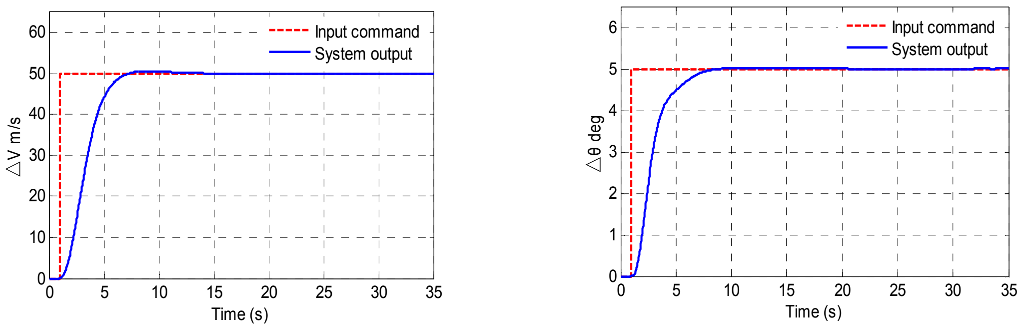
$$\left.\begin{aligned} u_v = u_t &= k_{\Delta\dot{v}}^{u_t}\Delta v + k_{\Delta a}^{u_t}\Delta\hat{a} + k_{\Delta\dot{q}}^{u_t}\Delta q + k_{\Delta\dot{\theta}}^{u_t}\Delta\theta \\ &\quad + k_{\Delta\dot{\delta}_e}^{u_t}\Delta\delta_e + k_{\Delta\dot{\delta}_t}^{u_t}\Delta\delta_t + k_{\Delta e_v}^{u_t}\int\Delta e_v dt + k_{\Delta e_\theta}^{u_t}\int\Delta e_\theta dt \\ u_\theta = u_e &= k_{\Delta\dot{v}}^{u_e}\Delta v + k_{\Delta\dot{a}}^{u_e}\Delta\hat{a} + k_{\Delta\dot{q}}^{u_e}\Delta q + k_{\Delta\dot{\theta}}^{u_e}\Delta\theta \\ &\quad + k_{\Delta\dot{\delta}_e}^{u_e}\Delta\delta_e + k_{\Delta\dot{\delta}_t}^{u_e}\Delta\delta_t + k_{\Delta e_v}^{u_e}\int\Delta e_v dt + k_{\Delta e_\theta}^{u_e}\int\Delta e_\theta dt \end{aligned}\right\} \tag{40}$$

We will use the two groups of attitude control laws to conduct simulation to the UAV longitudinal linear model. In order to simplify the following expressions, set the formula (39) as the controller ① and the formula (40) as the controller ②. Now the flying wing UAV linear model is selected when the flight status is h=2000m，Ma=0.805. Assume that the initial state of the linear model is an equilibrium state. when the simulation time t = 1s, the flight velocity will be given a step signal of 50m/s and the pitch angle will be given a step signal of 5°. The simulation results are shown in Figure 9 and Figure 10.



(a) The flight velocity step response curve (b) The pitch angle step response curve

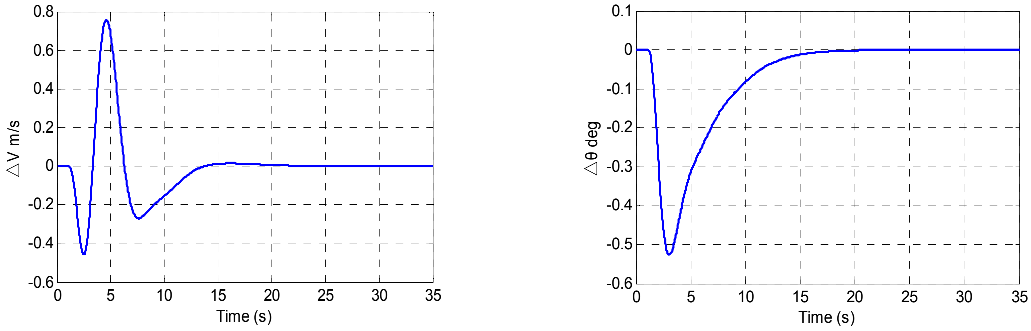
Figure 9. Simulation results with controller ①



(a) The flight velocity step response curve (b) The pitch angle step response curve

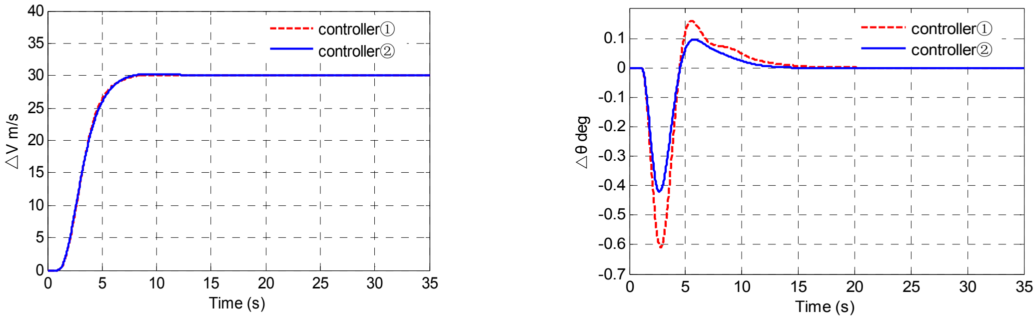
Figure 10. Simulation results with controller ②

Figure 9, 10 show that the two controllers are able to make the input signals be tracked without steady-state error. Meanwhile, there is high adjusting precision, short transition time and small overshoot in the response process. As reduced order observer is introduced in controller②, in order to analyze the effect on the command tracking augmented LQR control by the reduced order observer, we make the difference between figures 9 and 10, and then obtain system output difference curves of the two control methods, which are shown in Figure 11：



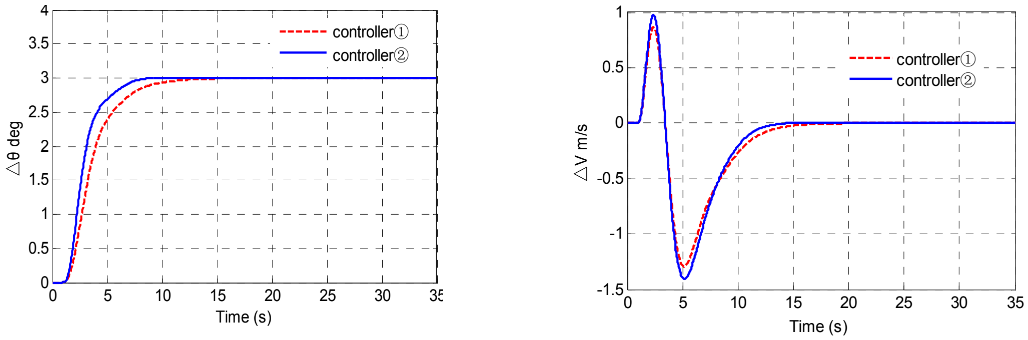
(a) The flight velocity difference curve (b) The pitch angle difference curve

Figure 11. The system outputs difference curve of two control methods



(a) The flight velocity response curve (b) The pitch angle disturbance curve

Figure 12. The velocity loop step response curve



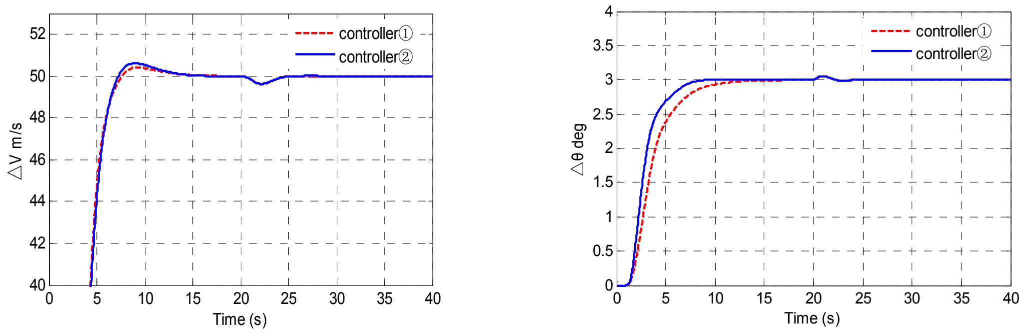
(a) The pitch angle response curve (b) The flight velocity disturbance curve

Figure 13. The pitch angle loop step response curve

Figure 11 shows that the fluctuating range of the system output difference curves are small in the whole response process. So it can be inferred that the introduction of the reduced order observer make little effect on command tracking augmented LQR, and the controller ② almost keep the full performance of the controller ①.

In order to verify the decoupling performance of the two controllers，the same controlled model as above will be selected, the flying speed will be given a step signal of 30m / s and the pitch angle will be given a step signal of 3° . The simulation results are shown in figure 12 and figure 13.

Figure.12 shows that when speed is controlled only, the maximum disturbance of the pitch angle is less than 0.7, and the disturbance is 0 after 15s. Figure.13 shows that when pitch angle is controlled only, the maximum disturbance of the speed is less than 1.5m/s, the disturbance becomes 0 after 15s. Therefore, both controllers have strong decoupling performance.



(a) The flight velocity response curve (b) The pitch angle response curve

Figure 14. The robustness simulation curve of the two controllers

In order to verify the robustness of the two controllers, we presume that when the UAV is in flight with a given speed of 50m/s and a given pitch angle of 5° , it will get a constant value of gust disturbance torque of 10N.m at t = 20s. Now use of the linear model mentioned above, the response curve obtained is shown in figure 14. The figure 14 shows that the impact of the disturbance torques on the flight speed and the pitch angle are little. Therefore, both controllers have good disturbance rejection ability and robustness

VI. CONCLUSIONS

In this paper, we take high-altitude long-endurance flying wing UAV as a platform. First of all, longitudinal mathematical model is established based on its special aerodynamic layout and unique control surfaces. Then we combine LQR technology to design the longitudinal stability augmentation control law and the attitude control law of the UAV respectively. The stability augmentation control is achieved by using output feedback linear quadratic method that not only improve the longitudinal static stability and the dynamic characteristics, but also reduce the numbers of feedback compared with conventional LQR state regulator. The attitude control law of the flying wing UAV uses a special control system augmented method and conventional LQR method to obtain a command tracking augmented LQR method. The simulation results show that the control law effectively realizes the command tracking of the angle of pitch and the flight velocity. Besides, the controller has strong robustness and decoupling performance. It can be seen from the time domain performances of the control system that the overshoot, the accommodation time and the steady accuracy are very ideal. Finally, when the system state variables can not be detected all, we also designed quasi-command tracking augmented LQR control method. It retains all the features of command tracking augmented LQR control method and more suitable for the application of practice engineering. Above all, the simulation results show that the longitudinal control laws of the flying wing UAV based on LQR enable the UAV to achieve satisfactory longitudinal flying quality.

REFERENCES

[1] R.M. Wood and X.S. Bauer, “Flying wings/flying fuselages”, AIAA paper 2001-0311, January 2001, pp. 1-7.

[2] W.R. Sear, “Flying Wing airplanes-The XB-35/YB-49 program”, AIAA paper 80-3036 in Evolution of Aircraft Wing Design Sytnposium, pp. 57-59, March 1980.

[3] S. Esteban, “Static and dynamic analysis of analysis of an unconventional plane flying wing”, AIAA Atmospheric Flight Mechanics Conference and Exhibit, Montreal, Canada. August 2001, pp. 6-9.

[4] A.R. Weyl, “Tailless Aircraft and Flying Wings A Study of Their Evolution and Their Problems”, Aircraft Engineering, vol. 16, No. 5, pp. 8-13, January 1945.

[5] G. Yang, J.G. Sun and Q.H. Li, "Augmented LQR Method for Aeroengine Control Systems", Journal of Aerospace Power, vol. 19, No. 1, pp. 153-158, February 2004.

[6] W.K. Lai, M. F. Rahmat and N.A. Wahab, "Modeling and controller design of pneumatic actuator system with control valve", International Journal on Smart Sensing and Intelligent Systems, vol. 5, No. 3, pp. 624-644, September 2012.

[7] R. Ghazali, M. Rahmat, Z. Zulfatman et al., "Perfect tracking control with discrete-time lqr for a non-minimum phase electro-hydraulic actuator system". International Journal on Smart Sensing and Intelligent Systems, vol. 4, No. 3, pp.424-439, September 2011.

[8] L. Gargouri, A. Zaafouri and A. Kochbati et al., "LQG / LTR Control of a Direct Current Motor", Systems, Man and Cybernetics, 2002 IEEE International Conference on. IEEE, vol. 5, pp. 5-10, May 2002.

[9] O. Rehman, B. Fidan and I.R. Petersen, "Minimax LQR control design for a hypersonic flight vehicle", 16th AIAA/DLR/DGLR International Space Planes and Hypersonic Systems and Technologies Conference, October 2009.

[10] R.W. Huang and Y.D Zhao, "Soft landing control of electromagnetic Valve actuation for engines by using LQR", Tsinghua Science and Technology, vol. 47, No. 8, 2007, pp. 1338-1342.

[11] S.B. McCamish, M. Romano and S. Nolet et al., "Flight testing of multiple-spacecraft control on SPHERES during close-proximity operations", Journal of Spacecraft and Rockets, vol. 46, No. 6, pp. 1202-1213, November–December 2009.

[12] X.J. Xing, J.G. Yan and D.L. Yuan, "Augmented-stability controller design and its simulation or a UAV based on LQR theory", Flight Dynamics, vol. 29, No. 5, pp. 54-56, October 2011.

[13] J.W. Choi, G. Daniel and L.R. Mariano, "Control System Modeling and Design for a Mars Flyer, MACH-1 Competition", AIAA Guidance, Navigation and Control Conference and Exhibit, Honolulu, Hawaii, August 2008.

[14] Y. Tao, "Application of Explicit-Model Following Control technology in the Attitude Control System for the Unmanned Shipboard Helicopter", Journal of Naval Aeronautical Engineering Institute, vol. 24, No. 5, pp. 543-546. September 2009.

[15] J. Andersson, P. Krus and K. Nilsson, "Optimization as a support for selection and design of aircraft actuation systems", Proceedings of Seventh AIAA/USAF/NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimization, St. Louis, USA, September 1998.

[16] Y.B. Li, W.Z. Liu and Q. Song, "Improved LQG control for unmanned helicopter based on active model in wind environment", Flight Dynamics, vol. 30, No. 4, pp. 318-322, August 2012.

[17] L. Zhang and Z. Zhou, "Study on Longitudinal Control Laws for High Altitude Long Endurance Tailless Flying-wing Unmanned Aerial Vehicles", Science Technology and Engineering, vol. 7, No. 16, pp. 1671-1819, August 2007.

[18] M.L. Zhang, "Flight Control System", National defense of Industry Press, 2nd edn, pp. 41-42, 1994.

[19] E.Q. Yang, "Research on the key technology of Unmanned Combat Aerial Vehicle flight control system", Ph.D. Dissertation, Automation Science and Electrical Engineering Dept. Beijing Univ. of Aeronautics and Astronautics, Beijing, China, May 2006.

[20] G. Garcia, J. Daafouz and J. Bernussou, "Output Feedback Disk Pole Assignment for Systems with Positive Real Uncertainty", Automatic Control, IEEE Transactions on, vol. 41, No. 9, 1996, pp. 1385-1391.

[21] D.E. Miller and M. Rossi, "Simultaneous Stabilization With Near Optimal LQR Performance", Automatic Control, IEEE Transactions on, vol. 46, No. 10, 2001, pp. 1543-1555.

[22] D.D. Moerder and A. Calise, "Convergence of a Numerical Algorithm for Calculating Optimal Output Feedback Gains", Automatic Control, IEEE Transactions on, vol. 30, No. 9, 1985, pp. 900-903.