



*vehicle body, suspension deformation, and tire displacement are obtained through the invariant equation associated with Laplace transform. Secondly, two optimization methods for the system parameters of passive suspension are presented, and the design objectives are to designate the invariant points as the local maximum points of the amplitude-frequency curves so as to improve the control performance of vibration isolation. At last, the control performances of the two improved passive suspension systems are compared with those of the original passive suspension, the active suspension, and the semi-active suspension by some performance indexes. The results show that the two optimization methods could greatly improve the control performance of passive suspension systems.*

**Index terms:** Vehicle suspension, invariant points, parameters optimization, passive control

## I. INTRODUCTION

With the development of the modern vehicle engineering, the riding comfort has attracted more and more attention from the scientists and engineers [1-8]. As a key device, the suspension system is important to affect not only the driving security but also the riding comfort to passengers. Because the vibration isolation parameters of passive suspension are fixed, it is difficult to obtain the satisfactory isolation results in whole-frequency range. Due to the limitation of passive suspension, the semi-active or active suspension is presented and studied to improve the riding comfort. A lot of works have concentrated on this subject, where different control strategies and controllable dampers are adopted in vehicle suspension. In recent years, many scholars have conducted a lot of studies on active and semi-active vibration isolation algorithm and experiments, and have made some important conclusions [9-18]. There are some shortcomings for active suspension and semi-active suspension, such as large energy consumption, bad reliability and so on, so that these two kinds of controllable suspensions have not been widely applied in vehicle engineering.



force respectively. If  $u$  equals to 0, the differential equations will be for passive suspension system. The state variables  $x_s$ ,  $x_t$  and  $x_r$  stand for suspension displacement, tire displacement and road input respectively.

Adding Eq. (1) and Eq. (2) together, the invariant equation of suspension is given as

$$m_s \ddot{x}_s + m_t \ddot{x}_t - k_t(x_r - x_t) = u. \quad (3)$$

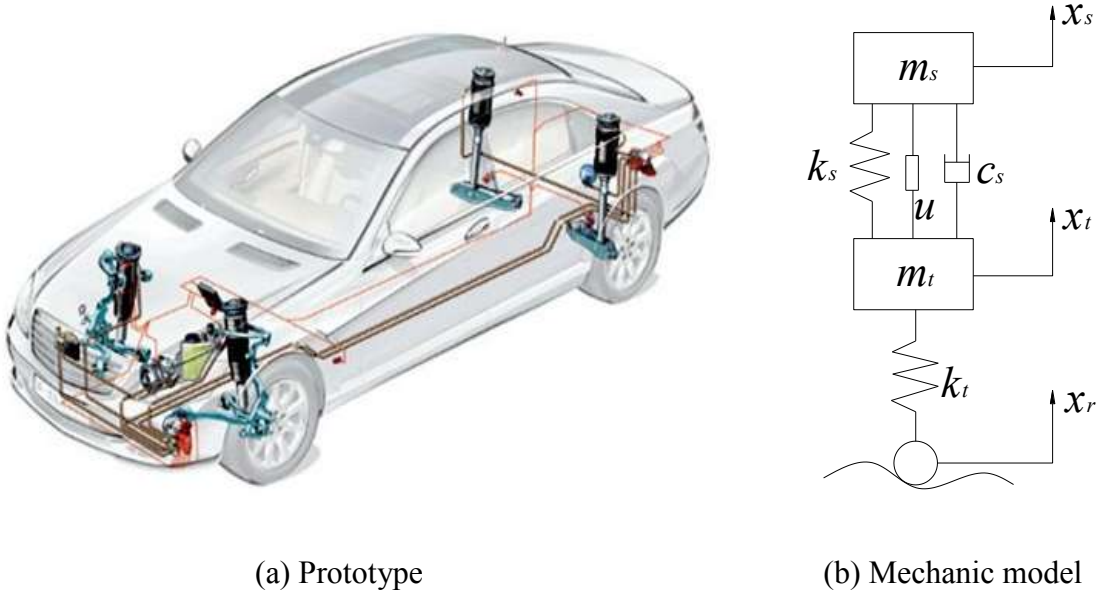


Figure 1. Prototype and mechanic model for a quarter vehicle

It could be found that there are no suspension parameters in this equation, which means that the active, semi-active and passive suspensions are all under the control of the invariant equation. Supposing the initial condition is zero, the Laplace transform of Eq. (3) could be obtained

$$m_s \ddot{X}_s(s) + m_t \ddot{X}_t(s) - k_t[X_r(s) - X_t(s)] = u. \quad (4)$$

If the road roughness is used as input, three transfer functions that describe control performance could be defined. The transfer function for riding comfort is

$$H_{\ddot{x}_s-r}(s) = \frac{\ddot{X}_s(s)}{X_r(s)}. \quad (5)$$

The transfer function for suspension deformation is



$$\omega = \sqrt{\frac{k_t}{m_s + n_t}}. \quad (14)$$

In Eq. (9), there is not any other invariant point for tire deformation except for the invariant point  $H_T(0) = 0$ . That is to say, the wheels will vary with road tightly around  $\omega = 0$ .

Furthermore, it is found that if other types of transfer functions are adopted, there is still an invariant point at  $\omega = 0$  for the transfer function of the riding comfort, with the only different form in the invariant point. Meanwhile, the situations of the invariant points for other two transfer functions, i.e. the transfer functions for the suspension deformation and tire deformation, are similar to the transfer function for the riding comfort.

For example, three other types of transfer functions could be defined to describe the control performance if the road speed is used as input. The transfer function for riding comfort is

$$H_{\ddot{x}_s - r}(s) = \frac{\ddot{X}_s(s)}{\dot{X}_r(s)}. \quad (15)$$

The transfer function for suspension deformation is

$$H_D(s) = \frac{X_s(s) - X_r(s)}{\dot{X}_r(s)}. \quad (16)$$

The transfer function for tires deformation is

$$H_T(s) = \frac{X_t(s) - X_r(s)}{\dot{X}_r(s)}. \quad (17)$$

If Eq.(15) to Eq.(17) are plugged into Eq.(4), one could obtain the relationships between the three transfer functions after simplification and classification

$$m_s H_{\ddot{x}_s - r}(j\omega) + (k_t - n_t \omega^2) H_T(j\omega) = -m_t \omega, \quad (18)$$

$$-n_s \omega^2 H_D(j\omega) + [k_t - (m_s + n_t) \omega^2] H_T(j\omega) = -(m_s + n_t) \omega, \quad (19)$$

$$[k_t - m_t + n_s] \omega [H_{\ddot{x}_s - r}(j\omega) - ik_t \omega - \gamma] - (k_t - n_t \omega^2) H_D(j\omega) = 0. \quad (20)$$

There are still invariant points at  $\omega = 0$  and  $\omega = \omega_0$  obtained from Eq. (18) and Eq. (19) for the



could be considered as the local maximum points of the amplitude-frequency equations to restrain the shape of the amplitude-frequency curves. This will make the control performance of vibration isolation for passive suspension improved. Two design methods for optimizing the system parameters of passive suspension, based on the abovementioned thought, are given as follows.

#### a. Method 1

The first method is based on the principle directly, that is to say, the two invariant points are respectively designed as local maximum points of the amplitude-frequency equations for vehicle body acceleration and suspension deformation. Letting the derivatives of  $|H_{\ddot{x}_s-r}(\omega)|$  at  $\omega = \sqrt{k_t/m_t}$  and  $|H_D(\omega)|$  at  $\omega = \sqrt{k_t/(m_s + m_t)}$  equal to zero respectively, one can obtain

$$c_s^2 k_t + k_s (-k_t m_s + k_s m_t) = 0, \quad (27)$$

$$-k_t m_s m_t + k_s (m_s + m_t)^2 = 0. \quad (28)$$

Then the optimal suspension parameters by method 1 are

$$k_s = \frac{k_t m_s m_t}{(m_s + m_t)^2}, \quad (29)$$

$$c_s = \sqrt{\frac{k_t m_s^3 m_t (m_s + m_t)}{(m_s + m_t)^4}}. \quad (30)$$

The obtained  $k_s$  and  $c_s$  could make the two invariant points as the local maximum points of the amplitude-frequency curves, which will improve the control performance.

#### b. Method 2

In method 2, the amplitude-frequency curve for vehicle body acceleration is designed as local maximum at  $\omega_1$  and  $\omega_2$  simultaneously, and the amplitude-frequency curve of suspension





$$k'_c=8437 \text{ N/m}, \quad c'_c=1833 \text{ N s/m}. \quad (36)$$

Based on method 2 and selecting  $a_i = \quad (i = ,2,3,4)$ , the optimal parameters are shown as

$$k'_c = 6999.4 \text{ N/m} \text{ and } c'_c = 1265 \text{ N s/m}. \quad (37)$$

Three ways are adopted to compare the vibration isolation performances of the improved passive suspensions with those of original passive suspension, active suspension and semi-active suspension. The first way is to compare the theoretical transfer function curves for two improved passive suspensions with those of the active and original passive suspension, and the results are shown in Fig. 2 to Fig. 4. Due to the strong nonlinearity, the theoretical transfer function curves for the semi-active suspension could not be presented here.

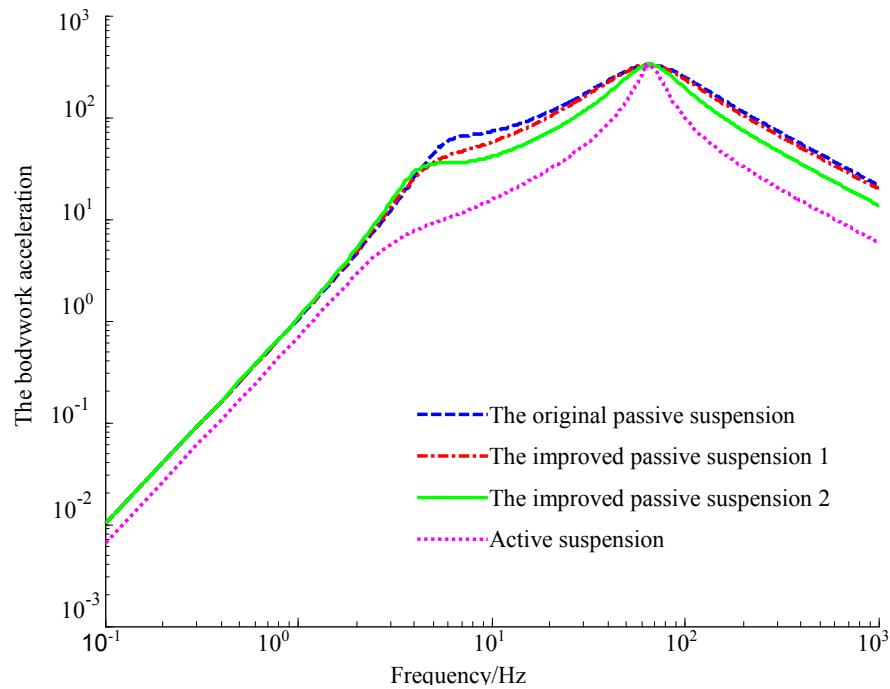


Figure 2. Comparison of the acceleration of vehicle body



two improved passive suspensions, the original passive suspension, the active and the semi-active suspension, where the deterministic sine road excitation with amplitude as 0.01m and different frequencies is adopted. The square value for vehicle body acceleration, suspension deformation and tire deformation are used as evaluation indicators after they become stable. The results are illustrated in Fig.5 to Fig 7.

The third way is to study the statistical responses of the abovementioned five kinds of suspension to the stochastic excitation. Based on the model for road surface suggested in references [7-8], stochastic excitation of road roughness with different levels ( Grade A, B, C, D, E) are constructed for numerical integration and then the statistical properties of system response to stochastic excitation of five kinds of road roughness are summarized in Table 1.

The conclusions can be drawn based on Fig. 2 to Fig. 7 and Table 1.

- (1) The vehicle body accelerations of the two improved passive suspensions have been significantly reduced compared with those of original suspension, so that the riding comfort for passive suspension is enhanced remarkably. This means the main design objectives of the two presented methods has been achieved and the two presented methods could improve the control performance of the passive suspension.
- (2) From the observation of the suspension deformation, it could be found that the suspension deformations of two improved suspensions are slightly bigger than those of original suspension, but it is still in the design range for suspension. In addition, it could be found that the suspension deformations of two improved suspensions are still smaller than those of active and semi-active suspension. This could guarantee the improved passive suspensions have much feasibility in vehicle engineering.
- (3) From the observation of tire deformation, the two improved passive suspensions are similar to the original passive suspension. They all are superior to the semi-active and active suspension, especially at high-frequency band. This means it is much easier to keep the tire life of the passive suspensions than the semi-active and active suspension.

Although the experimental verification could not be fulfilled to certify the effectiveness of the

presented optimization methods, the results by numerical simulation present much more reliable information. Accordingly, these optimization methods may be applied in vehicle engineering or other similar engineering fields.

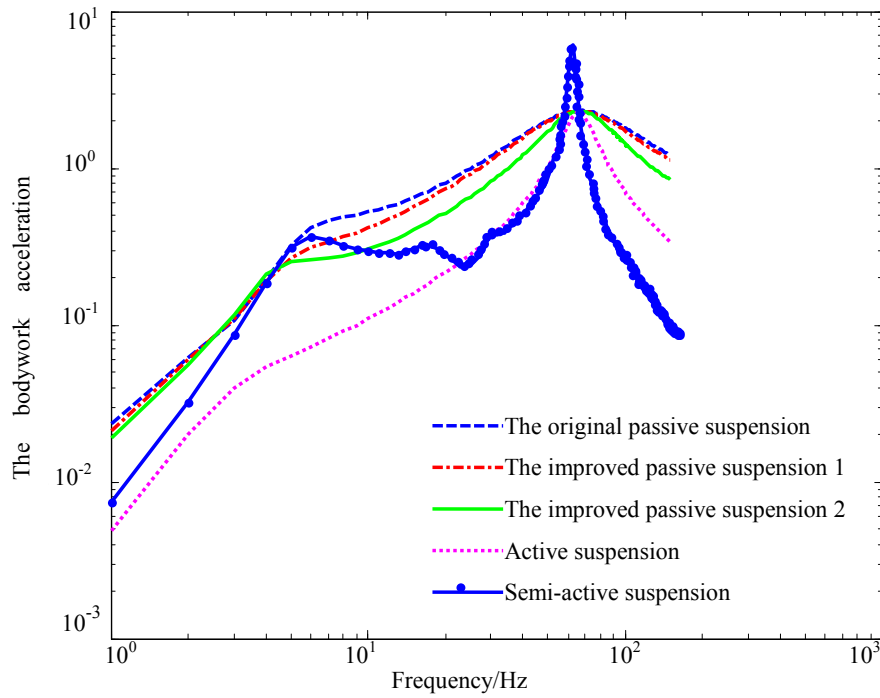


Figure 5. Comparison of the acceleration of vehicle body

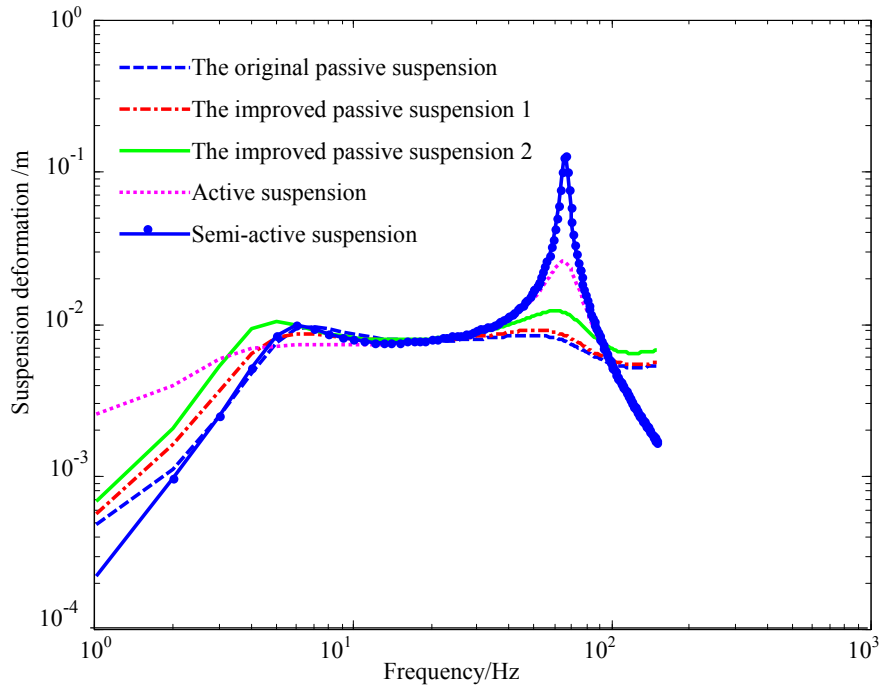


Figure 6. Comparison of the suspension deformation

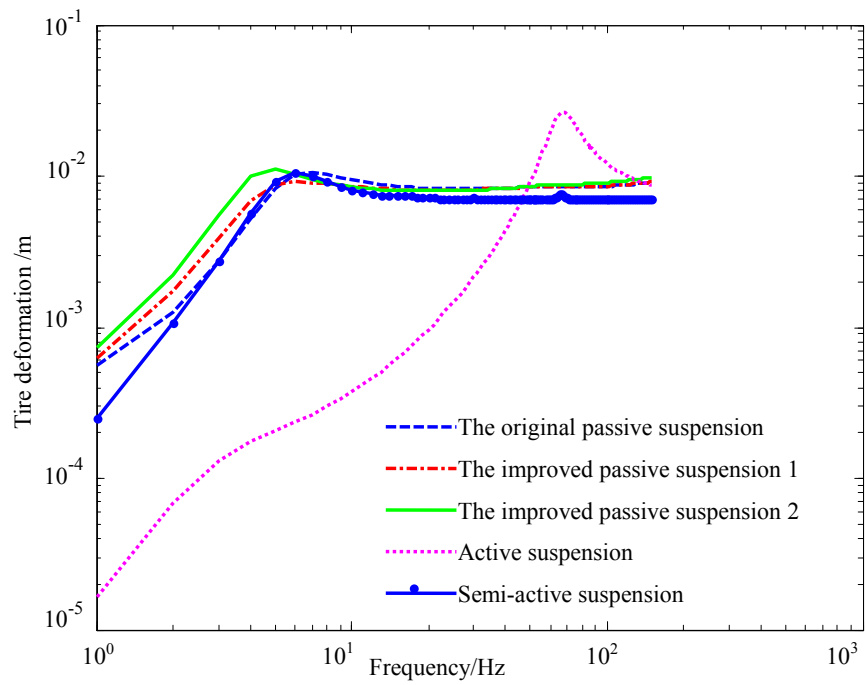


Figure 7. Comparison of tire deformation

Table 1 . The mean square value of response to the road levels

Road levels	Types of suspension	Vehicle body acceleration $/(m^2 \cdot s^{-2})$	Suspension deformation $/10^{-3} \cdot m^2$	Tire deformation $/10^{-3} \cdot m^2$
A-class road	Original passive suspension	0.2684	0.0027	0.0030
	Improved passive suspension 1	0.2501	0.0028	0.0029
	Improved passive suspension 2	0.2103	0.0033	0.0034
	Semi-active suspension	0.1909	0.0042	0.0036
	Active suspension	0.1754	0.0035	0.0015
B-class road	Original passive suspension	0.5408	0.0054	0.0060
	Improved passive suspension 1	0.5044	0.0056	0.0059
	Improved passive suspension 2	0.4235	0.0066	0.0068
	Semi-active suspension	0.4013	0.0086	0.0070
	Active suspension	0.3534	0.0070	0.0031
C-class road	Original passive suspension	1.0743	0.0108	0.0119
	Improved passive suspension 1	1.0018	0.0111	0.0118
	Improved passive suspension 2	0.8434	0.0131	0.0135
	Semi-active suspension	0.7948	0.0173	0.0148
	Active suspension	0.7015	0.0139	0.0061
D-class road	Original passive suspension	2.1474	0.0216	0.0239
	Improved passive suspension 1	1.9984	0.0221	0.0236
	Improved passive suspension 2	1.6806	0.0260	0.0269
	Semi-active suspension	1.5769	0.0349	0.0299
	Active suspension	1.4079	0.0278	0.0124
E-class road	Original passive suspension	4.3302	0.0434	0.0480
	Improved passive suspension 1	4.0361	0.0444	0.0472

	Improved passive suspension 2	3.3945	0.0523	0.0538
	Semi-active suspension	3.2505	0.0692	0.0564
	Active suspension	2.8165	0.0555	0.0248

## V. CONCLUSIONS

Based on the invariant equation, the invariant points of amplitude-frequency equations for vehicle body acceleration, suspension deformation and tire deformation of passive suspension are obtained. According to the invariant point theory, two design methods for optimizing the system parameters of passive suspension are presented. Then the performance indicators of improved passive suspensions are compared with those of original passive suspension, active suspension and semi-active suspension analytically and numerically. The results show that the control performance of the two improved suspensions can be greatly improved. These methods can also be applied to solve other problems about vibration isolation in engineering, and they provide a simple way to design the system parameters for the appropriate engineering structure of vibration isolation.

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