

Figure 8 The third mode shape

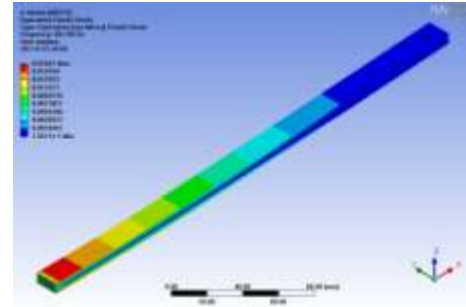


Figure 9 The first strain mode shape

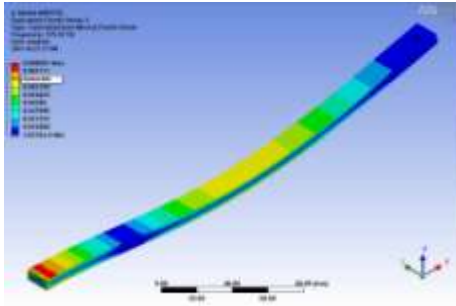


Figure 10 The second strain mode shape

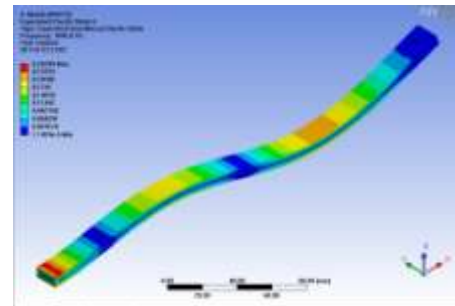


Figure 11 The third strain mode shape

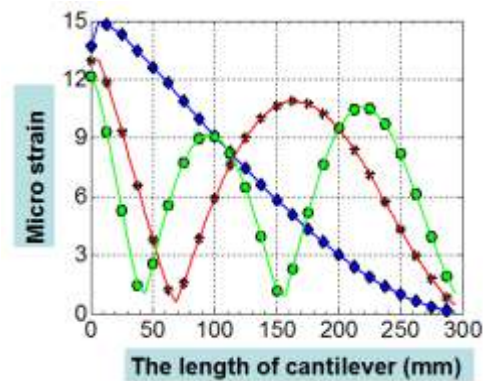




Figure.12 The strain on cantilever ( the first strain mode shape,

 The second strain mode shape,  the third strain mode shape)

B. Experiment

1. Experimental evaluation of FBG sensors and piezoelectric accelerator for monitoring vibration

This section presents the experimental program for evaluation the performance of FBG-based strain sensors and piezoelectric accelerator for beam's vibration measurement. The experiments were conducted in two and three modes in order to verify the proposed measurement methodology.

2. Experimental-1 set-up and result

The laboratory tests were performed on a simple support cantilever. The FBG-based strain gauges, which were manufactured by the Wuhan University of Technology, China, had a gauge length of 5mm. The acceleration vibration was measured by a piezoelectric accelerometer (4508-B, Bruel & Kjaer) in the frequency range 10~139 kHz. There are 3 measuring points of FBG and 1 acceleration sensors on the beam which were employed to record the beam's surface strain and the displacement vibration simultaneously. The sensors' position is shown in Figure 14. High strength epoxy adhesive RS 159-3957 was used for bonding the strain measuring FBG sensor for beam. Figure 13 show a close view of various sensors bonded on beam, Figure 14 shows the location of FBGs on beam.

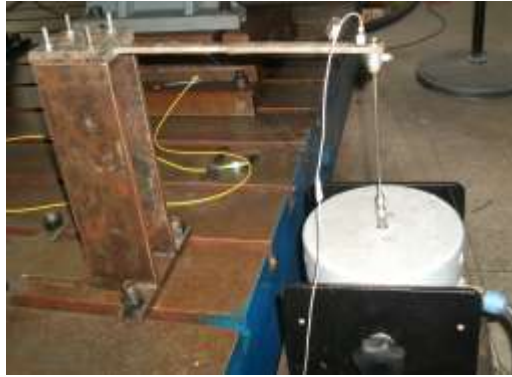


Figure. 13 An experimental rig for measuring strain

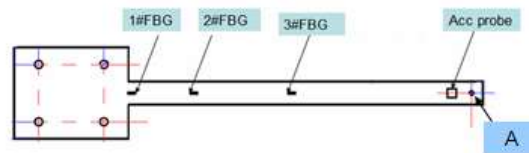


Figure.14 The beam and the FBGs' positions

Figure 15 shows the experimental setup for measuring the response of the beam. The beam was clamped rigidly at one end. An electric-driven shaker, used to generate external excitation to the beam, is attached to the bottom of the beam at a position 5mm from the left end as indicated in the drawing. The strain measured by FBG sensors and the displacement measured by the piezoelectric accelerometer were recorded simultaneously.

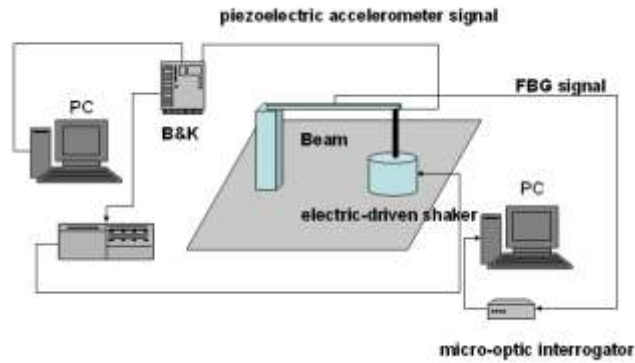


Figure 15. The whole experimental set-up

The FBG sensors were wired to an interrogator which was controlled using a personal computer. In order to sense the first to the third vibration modes, four sets of FBG were mounted at locations $x=8mm$, $x=90.4mm$, $x=159.3mm$ and $x=215.3mm$, respectively, to detect the strain signals. The positions of sensors were determined by the method presented in section III. A. Note that the first set was mounted at $x=8mm$ instead of $x=0mm$ due to the dimensions of gauges.

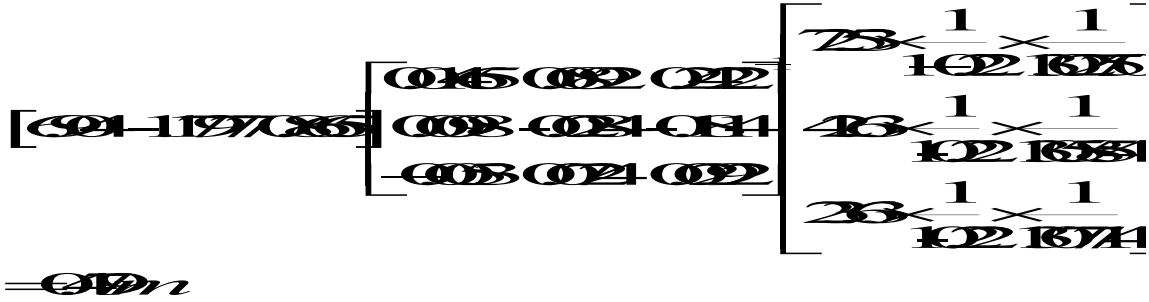
Assume that strain and deflection signals are composed of first-three modes, neglecting the

higher modes. The coefficient, listed in Table 2, is $\frac{\partial^2 Y_i(x_i)}{\partial x^2}$, come from mode shape analysis, by substituting these coefficient into Equation (22), we obtain the estimated displacement for the three modes.

Table 2 The physical and mechanical properties of the beam

	1# POINT	2#POSITION	3#POSITION
First mode	0.01465	0.0098	-0.0053
Second mode	0.0892	-0.0284	0.0724
Third mode	0.2422	-0.1814	0.0292

For example, when the cantilevered beam was excited by a harmonic force of 30 Hz, the FBG's strain response was $72.53\mu\epsilon$, $42.63\mu\epsilon$, $23.63\mu\epsilon$, respectively. The displacement of A point on beam (Figure.14) is:



The measurement error of beam tip can be obtained by an accelerator:

$$\frac{0.0011}{0.11} \times 10^{-8}$$

3. Experimental-2 set-up

Figure 16 shows an experimental rig for rotating beam, the beam, with four FBG mounted, is as same as section III.B. A motor drive a gearbox and the gearbox drive a shaft, the motor power is 45KW, the motor speed could change from 100 to 3000rpm from frequency control, the gearbox ratio is 1:5 and the maximum speed is 15000rpm. The beam is fixed the end of rotating shaft; the Figure 16 shows a close-up view of a rotary optical coupler using an optical circulator.

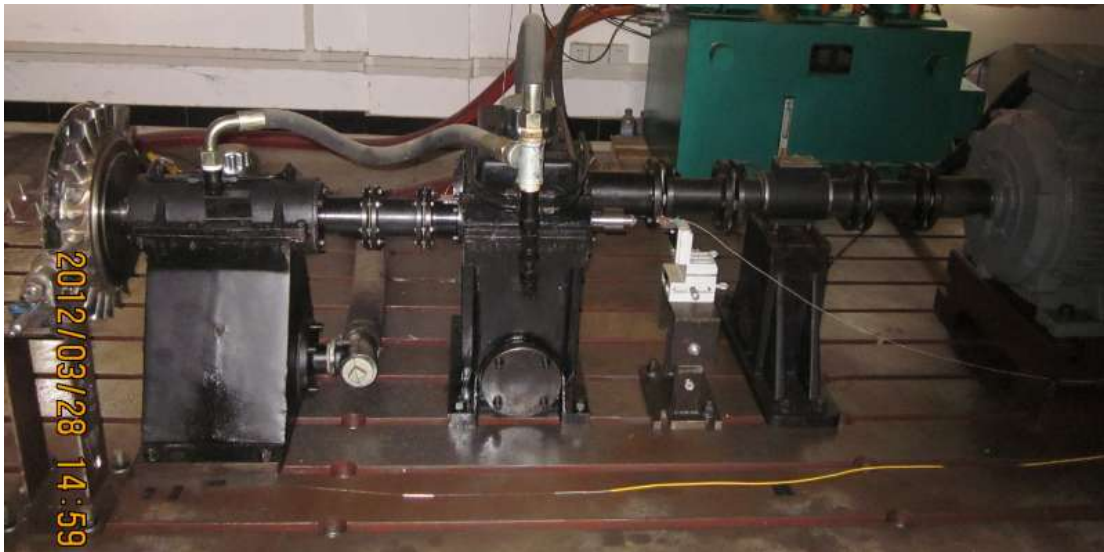


Figure 16. Experiment-2 set-up

IV. RESULTS AND DISCUSSION

A. Results

This section presents the results of strain measurements and an evaluation of displacement vibration of rotating a beam, by gradually increasing the rotational speed to 1200rpm and the 3-FBG's wavelengths were simultaneously recorded at a sampling rate of 2kHz.

Figure 17 shows micro strain of the FBG sensor mounted at 8mm of beam in range of 10-1200 rpm. Rotational speed was controlled by a motor controller. Rotational speed started at 10 rpm, at 10~15 second intervals, to 1200rpm at the end. From Figure 17, the dynamic strain shows oscillating vibration around a mean value. The mean strain decreases with rotational speed, which means the beam was stretched under centrifugal force. The oscillating vibration means that experiment was carried out under an elastic strain condition.

Figure 18 shows the measured dynamic strain against time at constant rotational speeds (approximately 325 rpm, with the same FBG sensor in Figure 17). The results of spectrum analysis show that 7.26 Hz, 14.53Hz, 55Hz cycles existed (Figure. 19). These means are doubled in frequency relating to rotating speed (7.26 Hz, 14.53Hz) and a frequency related to natural frequency of a cantilevered beam (55Hz, the simulation results is 60.199 Hz in section III, the difference came from the boundary condition).

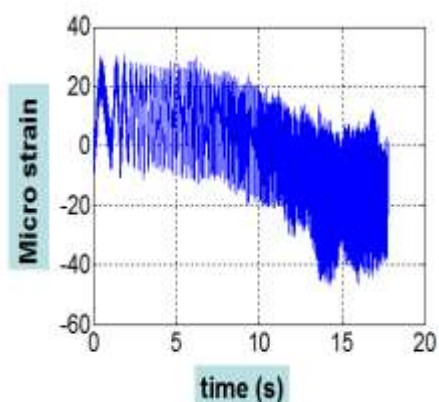


Figure 17 Strain on 1# FBG in range 0~900rpm

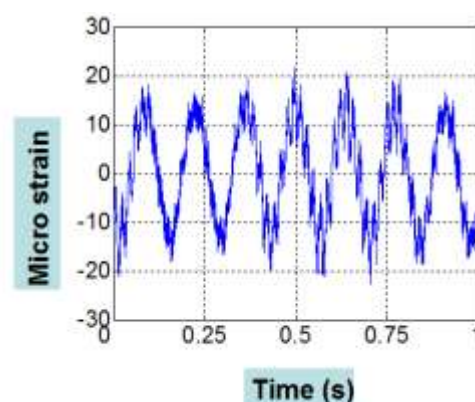


Figure 18 Strain on 1# FBG at 325rpm

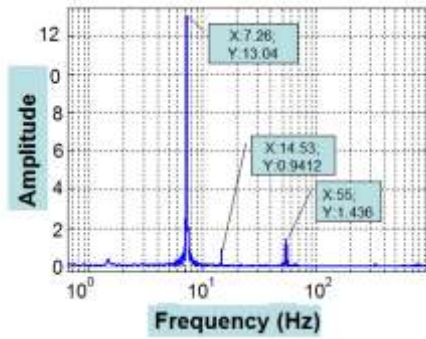


Figure. 19 Spectrum analysis of Figure 17

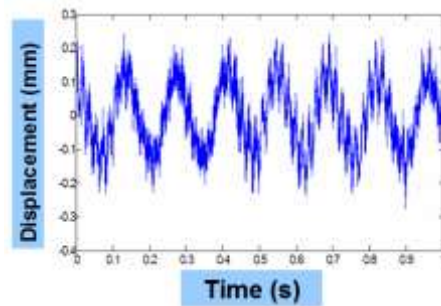


Figure. 20 The vibration estimation of the beam's tip

Figure 20 shows the vibration estimation of A point of beam at constant rotational speeds 325 rpm, blue line represent the case considering only the first three modes. From experimental-1, we know that the measurement error approximately 8.86%

B. Discussion

The errors came from the following causes.

- 1) The clamping condition of rotating cantilever beam does not yield a perfect fixed boundary condition.
- 2) The errors came from the FBG. Ideally, the sensors are moved to a more advantageous position. However, due to size of the FBG, the signals from the FBG are average values of a surface range. On the other hand, the sensitivity of the FBG is the source of the errors.
- 3) The relative error comes from the piezoelectric accelerometer which was used to calibrate the displacement vibration. To achieve higher displacement measurement, a sensor with higher sensitivity could be used.
- 4) 8.86% error is for consider only three. Due to measuring three modes there was an error of 8.86%. By taking more modes into account, errors will be reduced.
- 5) Since the location of the FBG is calculated based on analytical modes, it induced some errors.

V. CONCLUSIONS

The method of using an FBG to measure dynamic strain and estimate the displacement vibration of a rotating, cantilevered beam has been validated. This method is based on the fact that the vibration displacement can be expressed in terms of an infinite number of vibration modes and be related to the dynamic strains through the strain-displacement relationship. From the results, it can be concluded that this new monitoring method can be applied to a rotor blade.

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