



## Active Modeling Based Yaw Control of Unmanned Rotorcraft

Yan Peng, Wenqing Guo, Mei Liu and Shaorong Xie  
School of Mechatronics Engineering and Automation  
Shanghai University  
Shanghai, China  
Emails: [pengyan@shu.edu.cn](mailto:pengyan@shu.edu.cn)

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*Submitted: Oct. 10, 2013    Accepted: Feb. 2, 2014    Published: Mar. 10, 2014*

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*Abstract- With the characteristics of input nonlinearity, time-varying parameters and the couplings between main and tail rotor, it is difficult for the yaw dynamics of Rotorcraft to realize good tracking performance while maintaining stability and robustness simultaneously. In this paper, a new kind of robust controller design strategy based on active modeling technique is proposed to attenuate the uncertainties pre-described in the yaw control of unmanned systems. Firstly, by detailed analysis, the uncertainties are introduced into the new-designed yaw dynamics model by using the concept of modeling errors. Then, Kalman filter is used to estimate the modeling errors simultaneously, which is used subsequently to design the robust controller. Finally, the new strategy is tested with respect to the unmanned Rotorcraft system to show the feasibility and validity of it.*

**Index terms:** Unmanned Rotorcraft, Active modeling technique, Model error, Kalman filter (KF).

## I. INTRODUCTION

With the advantages of low cost, small volume, convenience for transportation, small land for taking-off and landfall, unmanned Rotorcraft is widely used in both military and civilian areas. Designing a suitable yaw control system becomes an important objective of unmanned Rotorcraft. When traveling, the Rotorcraft will suffer from many kinds of uncertainties, which can be classified as model uncertainties (unknown parameters) and environment disturbances which will greatly deteriorate the autonomous ability. It is clear that a controller which can give accurate estimations of these uncertainties will improve the steering control result. To be sure, many researchers have been aware of the model dependence issue, and various techniques, such as robust control and adaptive control, have been suggested to make the control system more tolerant of the unknowns in physical systems.

Many control strategies have been applied on the controller design of Rotorcraft, such as PID, LQR/LQG and so on. The complicated dynamics of rotorcraft leads to both parametric and dynamic uncertainty, so the controller should be robust to those effects and advanced control strategies need to be used in order for a RUAV to fly autonomously.

Many robust controllers have achieved some robust performances, such as  $H_\infty$ ,  $H_2$  disturbance attenuation, and guaranteed cost control method. Castillo [1] proposed a proportional-integral-derivative (PID) controller combined with a fuzzy logic controller, while Shin [2] and Kumar [3] put forward a linear quadratic controller, Kumar [4] and Suresh [5] raised a neural controller. Cai[6] suggested a robust and nonlinear control method for a small electric helicopter using quaternion feedback, and Nejjari[7] proposed a scheme to control the heading using the PID feedback/feedforward method. Nonaka and Sugizaki [8] came up with an attitude control scheme using the integral sliding mode to overcome the ground effect. Besides, Joelianto[9] suggested a model predictive control method to handle the transition between the various modes of autonomous unmanned helicopters. Shin [10] developed a position tracking control system for a rotorcraft-based unmanned aerial vehicle (RUAV) using robust integral of the signum of the error (RISE) feedback and neural network (NN) feed forward terms. In addition, Cai[11] applied a so-called robust and perfect tracking (RPT) control technique to the design and implementation of the flight control system of a miniature unmanned rotorcraft.

The  $H_\infty$  control strategy can provide an advanced method and perspective for designing control systems [12]; so many investigators are working to develop robust  $H_\infty$  controllers for unmanned small-scale helicopters with their own specific missions. Gadewadikar[13] suggested a static output feedback  $H_\infty$  controller with static gains only to control inner and outer loops. They obtained a simple static output feedback controller using the  $H_\infty$  control scheme and demonstrated that the controller could overcome wind disturbances. Zhao [14] presented an adaptive robust  $H_\infty$  control scheme for yaw control with fixed and variable gains to compensate for the effect of uncertainties. Dharmayanda[15] presented state space model identification of a small-scale helicopter, and applied the  $H_\infty$  control scheme to obtain a longitudinal and lateral motion controller for the Raptor 620 helicopter. Jeong<sup>[16]</sup> presented an H-infinity attitude control system design for a small-Scale autonomous helicopter.

These traditional robust and adaptive controllers always aim at model uncertainties, and these methods have strong restriction on the description form and system structure, so these methods have limitation in applicability and validity and hard to have good performance in yaw control. We'll show in this paper with active modeling, we don't need to know as much as we are told. In fact, the unknown dynamics and disturbance can be actively estimated with joint estimation and compensated in real time and this makes the controller more robust and less dependent on the detailed mathematical model of the physical process. Simulations conducted on the home-developed Unmanned Rotorcraft demonstrate the performance of the controller.

## II. YAW DYNAMICS

Rotorcraft platforms mainly compose of five channels, the main rotor, tail rotor, fuselage, horizontal tale and vertical fin. While hovering and low-speed flying, the forces and torques created by the main rotor and tail rotor play the dominant role.

The rotorcraft as a test case is constructed by Shanghai University (Fig.1).



Figure 1. Unmanned rotorcraft

Without regard to the effects of the fuselage, horizontal tale and vertical fin, with the method of model identification, a simplified equation for the yaw dynamics extracted from all states dynamic equation is described as follows[17]:

$$\begin{aligned} \dot{\varphi} &= q \sin \phi \sec \theta + r \cos \phi \sec \theta \\ I_{zz} \dot{r} &= (I_{xx} - I_{yy}) / pq + (N_{mr} + N_{fus} - Y_{mr} l_{mr} - Y_{tr} l_{tr} - Y_{fus} l_{fus} - Y_{vf} l_{vf}) \end{aligned} \quad (1)$$

where  $\phi$ ,  $\theta$  and  $\varphi$  are roll, pitch and yaw angle respectively;  $q$  and  $r$  are pitch, yaw angular velocity respectively;  $I_{xx}$ ,  $I_{yy}$  and  $I_{zz}$  are Rotorcraft inertia about  $x$ ,  $y$  and  $z$  axis;  $Y$  is the resultant force of  $y$  axis in body-fixed frame;  $N$  is resultant moment of  $z$  axis in body-fixed frame; the subscript  $mr$  (main rotor) denote main rotor;  $tr$  (tail rotor) denote scull;  $vf$  (vertical fin) denote vertical fin;  $ht$  (horizontal tale) denote horizontal tail;  $fus$  (fuselage) denote the influence of body and aerodynamic;  $l_{mr}$ ,  $l_{tr}$ ,  $l_{fus}$  and  $l_{vf}$  are distances from acting force to Rotorcraft center of gravity. For yaw course control of independent channel, the other states are all zero, so Eq.(1) can be simplified as

$$\begin{aligned} \dot{\varphi} &= r \\ I_{zz} \dot{r} &= N_{mr} + N_{fus} - Y_{mr} l_{mr} - Y_{tr} l_{tr} - Y_{fus} l_{fus} - Y_{vf} l_{vf} \end{aligned} \quad (2)$$

In low speed flight state, the force and moment produced by the main rotor and tail rotor play a leading role, so the yaw course control dynamic equation can be rewritten as

$$\begin{cases} \dot{\varphi} = r \\ I_{zz} \dot{r} = -Q_{mr} + T_{tr} l_{tr} + b_1 r + b_2 \varphi \end{cases} \quad (3)$$

where  $Q_{mr}$  is the main rotor moment;  $T_{tr}$  is the scull force;  $b_1$  and  $b_2$  are constant damping coefficient.  $Q_{mr}$  and  $T_{tr}$  are coupled, but by analysis of the relation curve, we can find that the relation between  $Q_{mr}$  and  $\theta_{mr}$  can be described as the following second degree curve

$$Q_{mr} = k_{Q_2} \theta_{mr}^2 + k_{Q_1} \theta_{mr} + k_{Q_0} \quad (4)$$

where  $k_{Q_2}$ ,  $k_{Q_1}$  and  $k_{Q_0}$  are time varying parameters depending on the geometrical shape of the paddle and revolving speed of main rotor.

The relationship between balance force of tail rotor and its elongation can be described as

$$T_r = k_{T2}\theta_r^2 + k_{T1}\theta_r + k_{T0} \quad (5)$$

$K_{T0}$ ,  $K_{T1}$  and  $K_{T2}$  are time varying parameters based on blade geometry and rotor speed.

By taking equations (4) and (5) into (3), the yaw course model of unmanned rotorcraft is described as

$$\begin{cases} \dot{\varphi} = r \\ I_{zz}\dot{r} = -(k_{Q_2}\theta_{mr}^2 + k_{Q_1}\theta_{mr} + k_{Q_0}) + (k_{T2}\theta_r^2 + k_{T1}\theta_r + k_{T0})l_{tr} + b_1r + b_2\varphi \end{cases} \quad (6)$$

Define  $\mathbf{x} = [x_1, x_2]^T = [\varphi, r]^T$  as system states, and  $\mathbf{y} = \varphi$  as system output, then equation (6) can be written as

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = b_1x_1(t) + b_2x_2(t) + \psi(u) \end{cases} \quad (7)$$

, which can also be described as

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}\psi(u) \quad (8)$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ b_1 & b_2 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$u(t) = (k_{T2}\theta_r^2 + k_{T1}\theta_r + k_{T0})l_{tr} - (k_{Q_2}\theta_{mr}^2 + k_{Q_1}\theta_{mr} + k_{Q_0}) \quad (9)$$

$u(t)$  is control input.

During the simplification process, many influence factors are neglected, which will result in model uncertainty and environment disturbances. In order to get better control effect, we introduce  $f(\varphi, \dot{\varphi}, \omega, t) = a(\varphi, \dot{\varphi}, t) + \omega(t)$  as system model error, in which  $a(\varphi, \dot{\varphi}, t)$  denote model uncertainty and  $\omega(t)$  denote environment disturbance. The system can be changed as the following form with uncertainties

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t) + \mathbf{E}f(\mathbf{x}, \dot{\mathbf{x}}, \omega, t) \\ \mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) \end{cases} \quad (10)$$

where,  $\mathbf{x}(t) \in R^n$  is the system state vector;  $u^t \in R^m$  is the system control input vector;  $\mathbf{y}(t) \in R^p$  is the measurement output vector.

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ 0 & -1/T \end{bmatrix}; \quad \mathbf{B} = \begin{bmatrix} 0 \\ b \end{bmatrix}; \quad b = k/T; \quad \mathbf{E} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

### III. CONTROL SCHEME BASED ON ACTIVE ESTIMATION

We have introduced the yaw control dynamics. The success of the controller is tied closely to the timely and accurate estimation of the disturbance, so in this section we'll introduce the KF based joint estimation to estimate the AUV's states and model error, and give the controller online model to compensated uncertainties in real time.

Joint estimation means using the same estimation approach to simultaneous estimate system states and parameters. It increases the estimation's degree of accuracy. Using KF to resolve the problem of joint estimation is by means of combining the system states and model error into augmented state variables, and then constituting augmented dynamic model.

Considering the course control dynamics with model error as in (10), and define

$$h(x, \dot{x}, \omega, t) = \dot{f}(x, \dot{x}, \omega, t)$$

(10) can be rewritten as

$$\begin{cases} \dot{x} = Ax + Bu + Ef(x, \dot{x}, \omega, t) \\ \dot{f}(x, \dot{x}, \omega, t) = h \\ y = Cx \end{cases} \quad (11)$$

In KF based joint estimation, the model error which includes all modeling uncertainties and environmental disturbances is appended onto the true state vector. The augmented state vector is  $x^a = [x \ f]$ . With respect to the course dynamics of AUV, the success of the controller is tied closely to the timely and accurate estimation of the disturbance  $f(x, \dot{x}, \omega, t)$ . If we can get an approximate analytical expression of  $f(\psi, \dot{\psi}, w, t)$ , which is sufficiently close to its corresponding part in physical reality, we can get better performance results. The augmented state space form of the system is:

$$\begin{cases} \dot{x}^a = \bar{A}x^a + \bar{B}u + \bar{E}h \\ y = \bar{C}x^a \end{cases} \quad (12)$$

with

$$\bar{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -1/T & 1/T \\ 0 & 0 & 0 \end{bmatrix}$$

$$\bar{B} = [0 \quad b \quad 0]^T, \bar{C} = [1 \quad 0 \quad 0], \bar{E} = [0 \quad 0 \quad 1]^T$$

Construct the whole states kalman estimator

$$\begin{cases} \dot{z} = \bar{A}z + \bar{B}u + K_g[y - z] \\ K_g = P\bar{C}^T R^{-1} \\ \dot{P} = \bar{A}P + P + P\bar{A}^T - P\bar{C}^T R^{-1} \bar{C}P + DQD^T \end{cases} \quad (13)$$

where  $z = [z_1, z_2, z_3]^T$  is the estimator state vector,  $z_i \approx x_i, i=1,2,3$ , the third state of the estimator  $z_3$  approximates  $f$ .  $K_g$  is the gain of kalman estimator,  $P$  is the estimation error covariance,  $Q$  is process noise covariance matrix,  $R$  is the measurement noise covariance matrix. Take the estimated model error into the system as compensatory item:

$$u = (-z_3 + u_0)T_0 / K_0 \quad (14)$$

In order to illustrate the universal applicability of model error based controller, we use the well-known pole-placement method to design linear controller

$$u_0 = k_1(\psi_d - z_1) - k_2 z_2 \quad (15)$$

where  $\psi_d$  is the desired trajectory,  $k_1, k_2$  are control gain. The control structure is proposed in Fig.2.

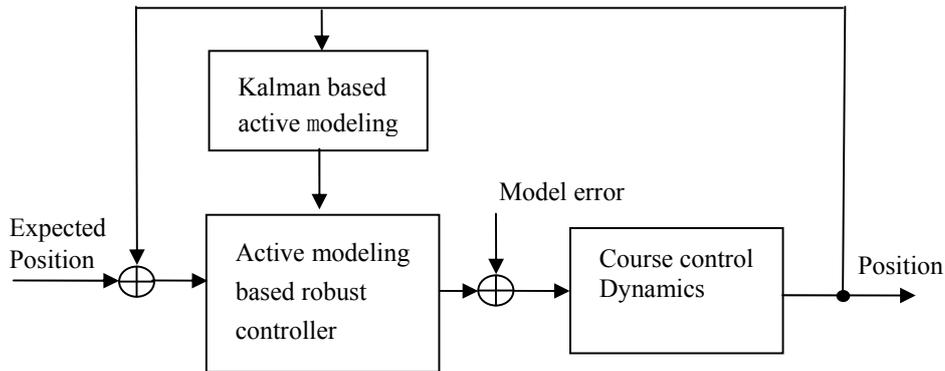


Figure 2. Active estimation enhanced control scheme

#### IV. STABILITY ANALYSIS

The key problem of the control design of the system with disturbances is its infection to system stability. That is, if  $f(x, \dot{x}, w, t)$  is completely unknown, can we guarantee the

stability of the system in any sense? So in this section we will discuss the stability problem of system (12) under the co-effect of estimator (13) and controller (14) (15).

**Theorem 1.** Considering the following system

$$\begin{cases} \dot{x} = Ax + Bu + f_d(x) + D\Delta \\ y = Cx \end{cases} \quad (16)$$

where  $x$  is the system state vector,  $u$  is the control input vector,  $f_d(x)$  is the unknown nonlinear state, and  $\|f_d(x)\| \leq \sigma\|x\|$  ( $\sigma$  is positive constant),  $\Delta$  is environmental disturbance. If there exist a feedback control law  $u = Kx$  and a positive matrix  $P$  which can meet the need of the following inequality

$$P(A + BK) + (A + BK)^T P + (1 + \sigma^2)P + \frac{2}{\gamma^2} PFF^T P + \frac{1}{2} C^T C \leq 0 \quad (17)$$

, the whole closed-loop system has  $L_2$ -gain less than or equal to  $\gamma$  from  $\Delta$  to  $y$ .

**Proof:** Take  $V = x^T P x$  as a candidate Lyapunov function of system (16), and its first time derivative is:

$$\begin{aligned} \dot{V}(x) &= V_x[Ax + BK + f_d(x)] + V_x D\Delta = 2x^T P[Ax + BK + f_d(x)] + 2x^T P D\Delta \\ &= -\frac{\gamma^2}{2} \left\| \Delta - \frac{2}{\gamma^2} D^T P x \right\|_2^2 + 2x^T P[Ax + BK + f_d(x)] + \frac{2}{\gamma^2} x^T P D D^T P x + \frac{\gamma^2}{2} \|\Delta\|_2^2 \end{aligned} \quad (18)$$

According to the nonlinear system input output  $L_2$ -gain stability lemma, if there is a positive  $P$  which can satisfy the following inequality

$$x^T P[Ax + BKx + f_d(x)] + [x^T A^T + x^T K^T B^T + f_d^T(x)] P x + \frac{2}{\gamma^2} x^T P D D^T P x + \frac{1}{2} x^T C^T C x \leq 0, \quad \text{then it}$$

can guarantee the  $L_2$ -gain stability of the closed loop system, and the gain is less or equal to  $\gamma$ .

Because  $\|f_d(x)\| \leq \sigma\|x\|$ , so

$$\begin{aligned} x^T P f_d(x) + f_d^T(x) P x &= -[x^T - f_d^T(x)] P [x - f_d(x)] + x^T P x + f_d^T(x) P f_d(x) \\ &\leq x^T P x + f_d^T(x) P f_d(x) \leq (1 + \sigma^2) x^T P x \end{aligned} \quad (19)$$

then

$$\begin{aligned}
 & x^T P[Ax + BKx + f_d(x)] + [x^T A^T + BKx + f_d^T(x)]Px + \frac{2}{\gamma^2} x^T PDD^T Px + \frac{1}{2} x^T C^T Cx \\
 & \leq x^T P(Ax + BKx) + x^T A^T Px + (1 + \sigma^2)x^T Px + \frac{2}{\gamma^2} x^T PDD^T Px + \frac{1}{2} x^T C^T Cx
 \end{aligned} \tag{20}$$

Obviously, if we can find a positive  $P$  that satisfy (17), it can guarantee the whole closed-loop system's  $L_2$ -gain less than or equal to  $\gamma$  from  $\Delta$  to  $y$ .

The following lemma exists about the stability of Kalman estimator.

**Lemma 1.** To the following continuous system

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \tag{21}$$

$$E[w(t)] = 0, E[w(t)w^T(t)] = q(t)d(t-t)$$

$$E[v(t)] = 0, E[v(t)v^T(t)] = r(t)d(t-t)$$

$$E[w(t)v^T(t)] = 0.$$

, if the system is completely controllable and observable,  $q(t)$  and  $r(t)$  are positive, the Kalman estimator is uniformly asymptotic stable.

Let  $e = [e_1 \ e_2 \ e_3]^T = x - z$  be the estimation error, and we can get

$$\dot{e} = (\bar{A} - K_g^\infty \bar{C})e + \bar{E}h \tag{22}$$

where  $K_g^\infty$  is the steady Kalman gain. Take the controller (14), (15) into system (12), take  $\bar{y} = [y, \dot{y}]^T$ , and then we can get

$$\begin{aligned}
 \dot{\bar{y}} &= \begin{bmatrix} \bar{y}_2 \\ -\frac{1}{T_0} \bar{y}_2 + f + [-z_3 + k_1(\psi_d - z_1) - k_2 z_2] \end{bmatrix} \\
 \begin{cases} \dot{\bar{y}} = \begin{bmatrix} 0 & 1 \\ -k_1 & -k_2 \end{bmatrix} \bar{y} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ k_1 & k_1 & k_2 & 1 \end{bmatrix} \begin{bmatrix} \psi_d \\ e_1 \\ e_2 \\ e_3 \end{bmatrix} \\ y = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \bar{y} \end{cases} \tag{23}
 \end{aligned}$$

The following theorem exists about the closed-loop stability.

**Theorem 2.** To the system (22) (23) whose controller is  $u = -Kx$ , when the system model error satisfies the following condition

$$\begin{aligned}\|a(y, \dot{y}, t)\| &\leq Mx^2 + N\|x\| \\ \|w\| &\leq G\|x\|\end{aligned}$$

, it can guarantee the  $L_2$ -gain stability of the closed loop system from  $h$  to  $\bar{y} = [y, \dot{y}]$ .

**Proof:** To system (22),

$$\begin{aligned}\text{rank}[\bar{A}^{n-1}\bar{B} \ \bar{A}^{n-2}\bar{B} \ \dots \ \bar{B}] &= 3 \\ \text{rank}[\bar{C} \ \bar{C}\bar{A} \ \dots \ \bar{C}\bar{A}^{n-1}]^T &= 3\end{aligned}$$

, so the system is uniform completely controllable and uniform asymptotic observable. The Kalman estimator is uniform asymptotic stable according to lemma 1 and  $(A - K_g^\infty C)$  is Hurwitz matrix.  $h = \hat{f}(x, \dot{x}, w, t)$  can be written as  $\|h\| \leq 2M\|x\| + G + N$

Using Theorem 1 we can get a positive symmetric matrix which makes the following inequality comes into existence

$$P(A - KgC) + (A^T - C^T Kg^T)P + (1 + (\rho K)^2) + \left(\sum_{i=1}^p k_i\right)^2 P + \frac{2}{\gamma_1^2} PFF^T P + \frac{1}{2} C^T C \leq 0 \quad (24)$$

And it can guarantee the whole system's  $L_2$ -gain less than or equal to  $\gamma$  from  $h$  to  $e$ .

To (23), the desired course  $\psi_d$  is bounded, so  $\Delta = [\psi_d \ e_1 \ e_2 \ e_3]$  are bounded. According to theorem 1, the system is  $L_2$ -gain stable from  $\Delta$  to  $\bar{y} = [y, \dot{y}]$ , and the closed loop is  $L_2$ -gain stable from  $h1$  to  $\bar{y} = [y, \dot{y}]$

## V. SIMULATIONS

The concrete parameters of self-made rotorcraft are illustrated in Table 1.

Table 1. Concrete parameters of unmanned rotorcraft

Parameter symbol	Physical significance	parameter Initial value
$m$	Mass of the rotorcraft and load	7.7kg
$\rho$	Air density	1.2
$I_{xx}$	Inertia moment about X axis	0.1634kgm <sup>2</sup>
$I_{yy}$	Inertia moment about Y axis	0.5782kgm <sup>2</sup>
$I_{zz}$	Inertia moment about Z axis	0.6306kgm <sup>2</sup>
$l_{mr}$	Distances from main rotor acting force to Rotorcraft center of gravity	0.01mm
$l_{fus}$	Distance to rotorcraft acting force	-0.1m
$l_{ht}$	Distance to center of mass	0m
$a_{mr}$	Main rotor paddle lifting line slope	5.4
$b_{mr}$	Main rotor paddle number	2
$c_{mr}$	Main rotor width	0.058m
$R_{mr}$	Main rotor radius	0.782m
$R_{omr}$	Main rotor inner diameter	0.196m
$\Omega_{mr}$	Main rotor speed	8792.64rpm
$a_{tr}$	Tail rotor paddle lifting line slope	5.4
$b_{tr}$	Tail rotor paddle number	2
$c_{tr}$	Tail rotor width	0.028m
$R_{tr}$	Tail rotor radius	0.1325m
$R_{otr}$	Tail rotor inner diameter	0.042m

### 5.1. System structure

The aircraft flight control system contains two parts, onboard flight control system and ground monitoring system. The structure chart is as follows,

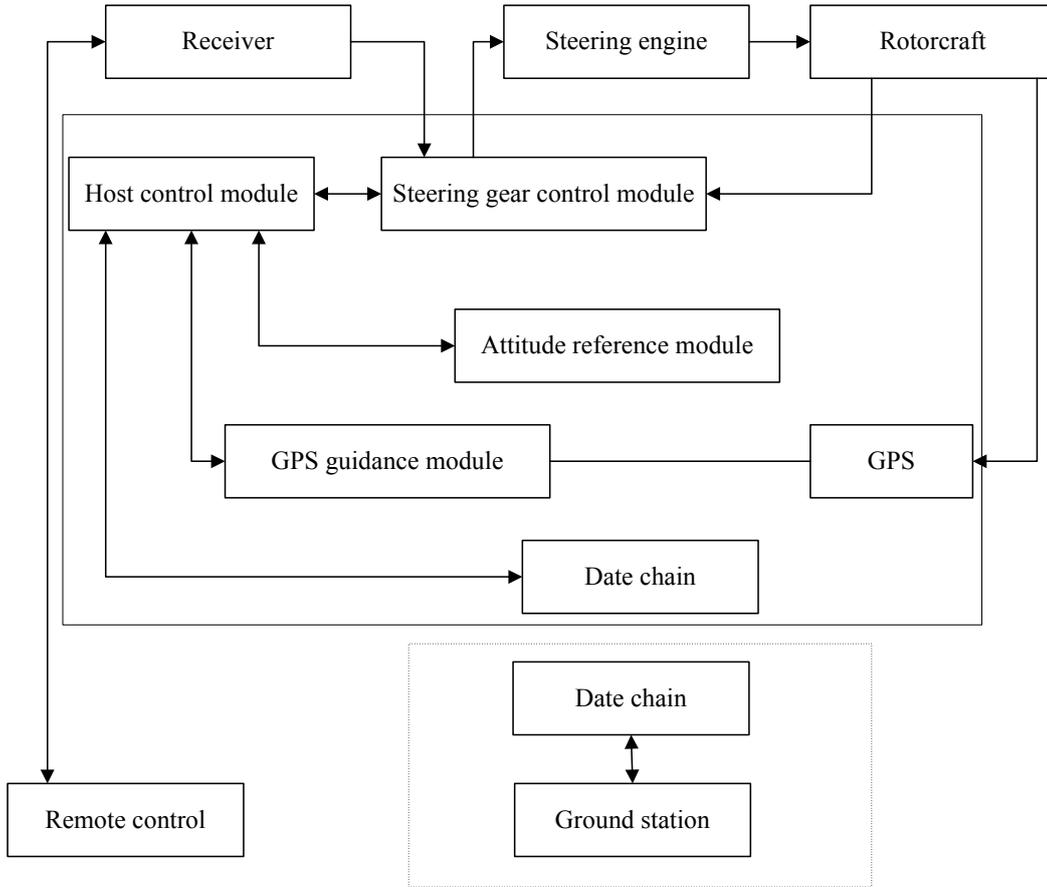


Figure 3. The structure chart of rotorcraft

### 5.2. Model identification

First, identify parameters of yaw course dynamic model by exponential forgetting least squares using flight test data,

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \lambda_1 x_1 + \lambda_2 x_2 + \omega_o + \omega_w + b_0 u \\ y = x_1 \end{cases}$$

where,  $\lambda_1 = -3.81$ ,  $\lambda_2 = -1.46$ ,  $b_0 = -65.8241$ .

### 5.3. Simulations

The desired heading angle is  $10^\circ$ . Give the model disturbances at 10s. The simulation parameters are set as follows: sampling time  $T = 0.1s$ ,  $k_p = 19.5$ ,  $k_d = 8$ .  $x_3(t) = \omega_0(t)$  describes the model

uncertainty and environment disturbance. The simulations are illustrated under different disturbances, which are as follows,

- 1) Sinusoidal disturbance at  $t = 10s$

$$x_3(t) = \begin{cases} 0, & t < 10s \\ 20\cos(0.5\pi t) + \text{ProcessNoise}(1,t), & t \geq 10s \end{cases}$$

The tracking control results are illustrated by Figure 4-Figure 8 (desired yaw angel is red line and the actual heading is black line).

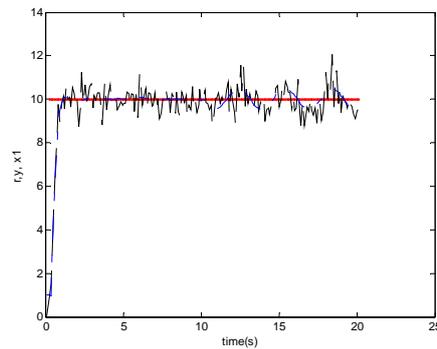


Figure 4. Yaw tracking result using active modeling based disturbance rejection control

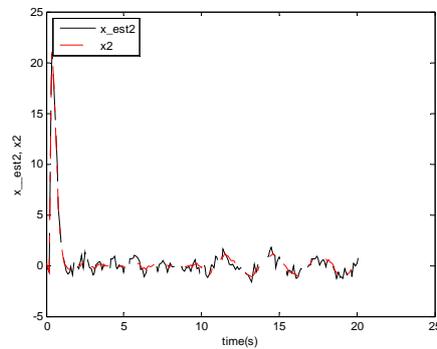


Figure 5. Yaw angle velocity

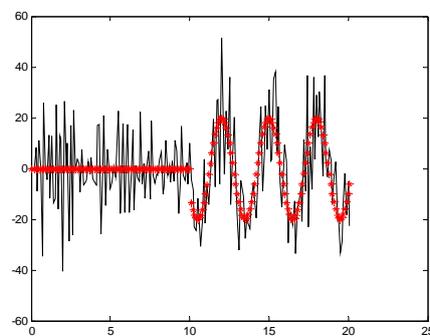


Figure 6. Model uncertainty and environment disturbance

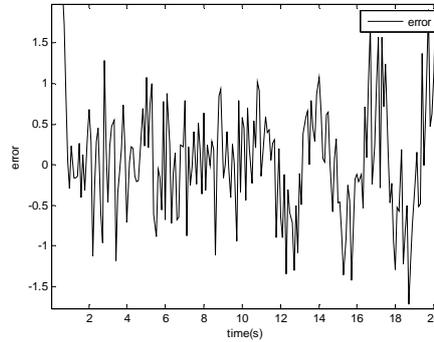


Figure 7. Yaw error

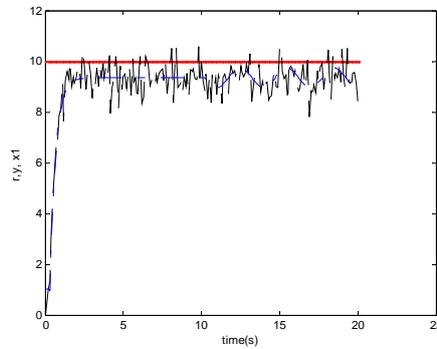


Figure 8. Yaw tracking result without active modeling

We can see that active modeling based yaw controller felt the variation of model error variations, made the KF react quickly and track the change successfully after a short period of adaptation, and regulate the controller adaptively based on the actual model variation. The states estimated by KF are not influenced by the noise covariance at  $t=10s$ . The controller without active modeling has certain adaptive ability to disturbances, but it can't reject the effect of the disturbances, and the system's real trajectory keeps away from the desired trajectory.

2) Step disturbance at  $t=10s$

$$x_3(t) = \begin{cases} 0 & t < 10 \\ 10 & t \geq 10 \end{cases}$$

The tracking control results are illustrated by Figure 9-Figure13 (desired yaw angle is red line and the actual heading is black line).

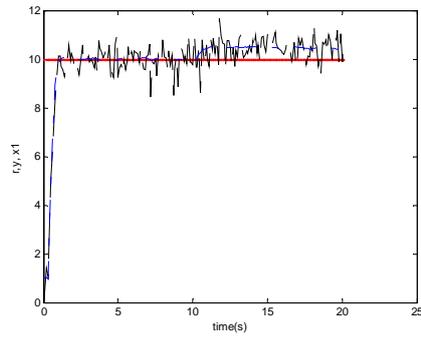


Figure 9. Yaw tracking result using active modeling based disturbance rejection control

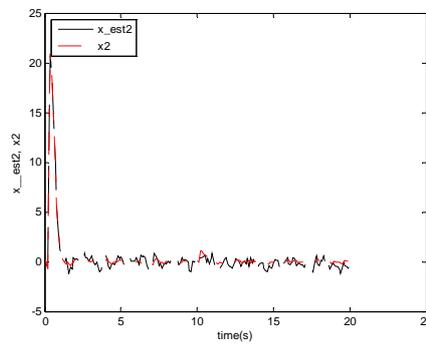


Figure 10. Yaw angle velocity

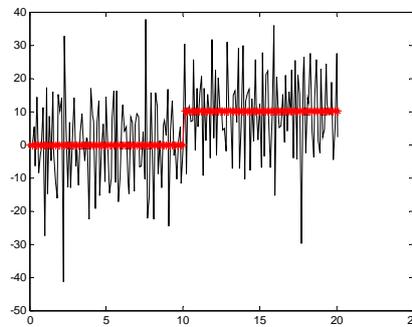


Figure 11. Model uncertainty and environment disturbance

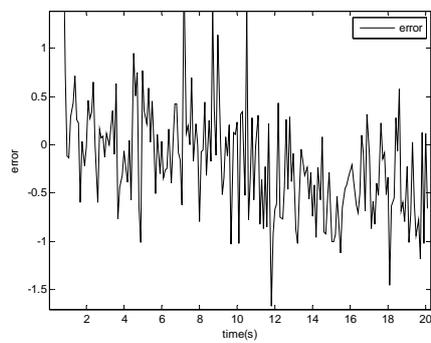


Figure 12. Yaw error

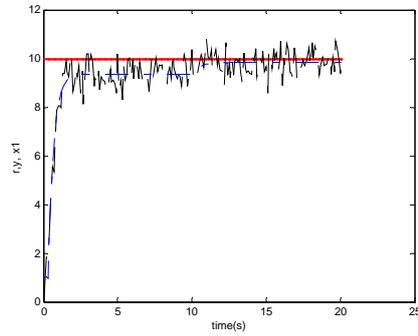


Figure 13. Yaw tracking result without active modeling

The control results have fast response speed without overshoot in step disturbance, which show good dynamic property.

3) Pulse disturbance at  $t=10s$

$$x_3(t) = \begin{cases} 0 & t < 10 \text{ \& } t > 10 \\ 100 & t = 10 \end{cases}$$

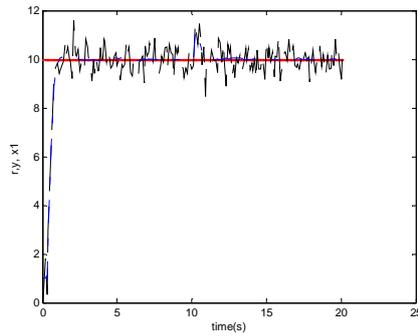


Figure 14. Yaw tracking result using active modeling based disturbance rejection control

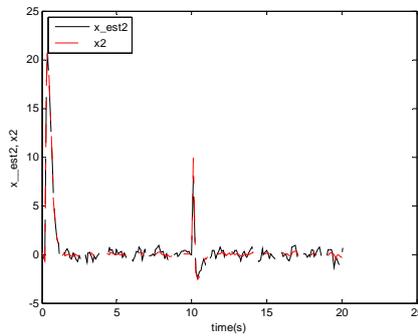


Figure 15. Yaw angle velocity

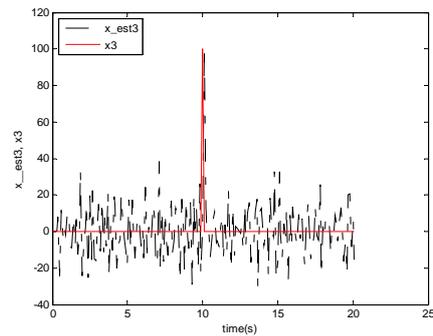


Figure 16. Model uncertainty and environment disturbance

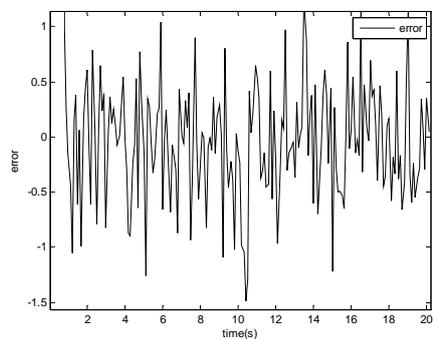


Figure 17. Yaw error

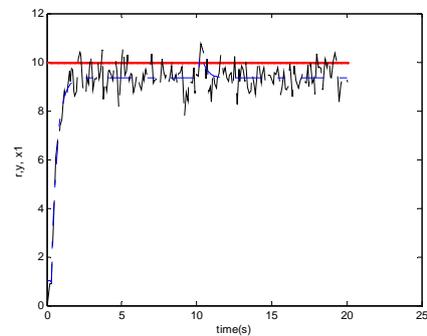


Figure 18. Yaw tracking result without active modeling

By analysis of the response curves, we can see that active based tracking controller has good disturbance rejection ability, which makes the aircraft snap back to desired heading.

#### 5.4. Experiment results

The control objective of the experiment is to track a desired heading by using active modeling based course controller. During the experiment, the desired heading angle is set as  $270^\circ$ , the disturbance are given manually, and sent to the aircraft by wireless-LAN; besides, the system

itself has the modeling uncertainty and environmental disturbance, so the controller should regulate adaptively on the sum of all these disturbances.

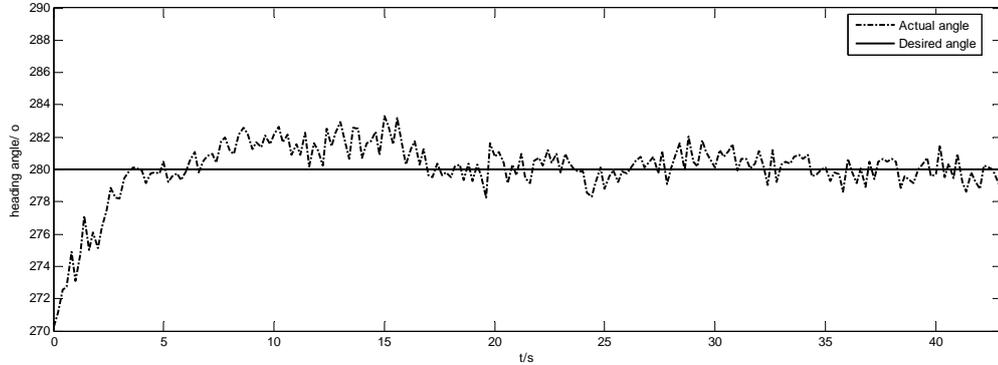


Figure 19. Yaw tracking experiment result

The result is illustrated in Figure 19, in which the designed trajectory is given by solid line, and the actual trajectory is given by dashed line. The experiment result clearly indicates that our control system design using active modeling technique is successful.

## VI. CONCLUSIONS

This paper proposes a new course control dynamic model and an active modeling based disturbance rejection controller considering the external disturbances and other uncertain factors. The controller induces all uncertainties into the system as model error, appends it onto the true state vector as augmented state and gives it joint estimation. The estimated model error is taken into the system as compensatory item. Besides, the simulation and experiment results show that this algorithm has a good estimation and prediction ability.

## VII. ACKNOWLEDGEMENTS

This work is supported by National Natural Science Funds (60705028), “Chen Guang” project (No.10CG43) and Innovation Program (No.12YZ009) supported by Shanghai Municipal Education Commission and Shanghai Education Development Foundation, Shanghai Municipal Science and Technology Commission (No.12140500400, No.10170500400). The authors also gratefully acknowledge the helpful comments and suggestions of the reviewers, who have improved the presentation.

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