www.s2is.org
International Journal on Smart Sensing and Intelligent Systems
ISSN 1178-5608

Chaos Synchronization via Linear Matrix Inequalities: A Comparative Analysis

Hanéne Mkaouar [1] and Olfa Boubaker [2]

[1,2] National Institute of Applied Sciences and Technology

INSAT, Centre Urbain Nord BP. 676 – 1080 Tunis Cedex, Tunisia

Emails: [1] Hanene.mkaouar@gmail.com, [2] olfa.boubaker@insat.rnu.tn

Submitted: Mar. 27, 2004 ***Accepted: May 3, 2014*** ***Published: June 1, 2014***

Abstract- In this paper, three chaos synchronization approaches using Linear Matrix Inequality (LMI) tools are evaluated and compared. The comparative analysis is supported by four examples of Piecewise affine (PWA) chaotic systems: The Chua's original circuit, the Chua's modified system, the Lur'e like circuit and the five-scroll attractor system. To evaluate the performances of each synchronization approach, we examine first, the practical implementation of the LMIs. We analyze then, by simulation results, the feasibility of each approach for each PWA chaotic system. The elapsed time for solving the predefined LMIs and the influence of their tuning parameters' domain belonging on the feasibility and the performances of each approach are finally the considered comparative criteria.

Index terms: Chaos synchronization, Linear Matrix Inequalities, Piecewise Affine systems, Lyapunov stability, comparative analysis.

I. INTRODUCTION

Synchronization is a universal concept in nonlinear theory [1, 2] extensively investigated in many engineering applications. Chaos synchronization is, particularly, considered as the most important research field in this area. Synchronization of chaos is a phenomenon that may occur when two, or more, chaotic oscillators are coupled, or when a chaotic oscillator drives another chaotic oscillator. Pecora and Carroll [3] are first to introduce a control method to synchronize two identical chaotic systems with different initial conditions. Many approaches have been then developed for the synchronization of chaotic systems. They include active control [4], adaptive control [5, 6], back-stepping control [7], impulsive control [8, 9], sliding mode control [10], active sliding mode control [11] back-stepping sliding mode control [12], predictive control [13], linear state feedback control [14] and linear robust state feedback control [15].

Recently, chaos synchronization approaches based on linear state feedback control laws are the most applied techniques due to their simple implementation. Generally, a set of algebraic synchronization conditions are derived using Lyapunov approach and solved using suitable Linear Matrix Inequalities [16], [17], [18], [19], [20], [21], [22], [23], [24], [25]. Most of these approaches investigated in a time delay approach which imposes some numerical constraints on the delay domain beyond which the LMIs to be solved are not feasible [22], [23], [24], [25].

On the other hand, in the engineering field, various processes, which dynamics exhibit switching between linear dynamics and whose behavior need to be synchronized, can be described by a Piecewise affine (PWA) models. For such systems, the transitions between the linear dynamics are either governed by the laws resulting from approximation of nonlinear dynamics [26], [27] or from the intrinsic characteristics of certain components [28], [29], [30]. Very few works are reported on chaos synchronization of PWA systems [21], [31], [32], [33], [34], [35]. The lack of results can be justified by the complexities introduced by the switching nature of the vector-field of piecewise linear systems [36], [37], [38].

In this paper, three chaos synchronization approaches based on linear state feedback control laws and LMI tools are compared using four examples of PWA chaotic systems. The comparative analysis is based on the practical implementation of the LMIs, the feasibility of each approach for each PWA chaotic system, the elapsed time for solving the predefined LMIs, the number of tuning parameters and the influence of their domain's belonging on the feasibility and the performances.

This paper is organized as following: First, synchronization schemes of the different approaches via LMIs will be presented in section 2. The PWA master slave systems on which the comparative study is based are exposed in section 3. Chaos synchronization synthesis and comparative analysis are finally presented on section 4 and section 5, respectively.

II. SYNCHRONIZATION VIA LMIS

1. Jiang and Zheng approach [16]

Consider the chaotic master-slave system described by:

$$\begin{cases} \dot{x} = Ax + g(x) + h(t) \\ \dot{z} = Az + g(z) + h(t) + Bu \\ u = K(x - z) \end{cases} \tag{1}$$

where $A \in \Re^{n\times n}$ is a constant matrix, $h(t) \in \Re^{n}$ is an external input signal and $g(x)$ and $g(z)$ are continuous nonlinear function satisfying:

$$\|g(x) - g(z)\| \le \rho\, \|x - z\| \tag{2}$$

where ρ is a Lipchitz constant.

The error dynamics for the master slave system (1) are then given by:

$$\dot{e} = \dot{x} - \dot{z} = (A - BK)e + g(x) - g(z) \tag{3}$$

Theorem:

If a suitable matrix $B \in \Re^{n\times m}$ is chosen such that the pair (A, B) is controllable, if there exist matrices $Q \in \Re^{n\times n}$ and $Y \in \Re^{n\times m}$ such that the following LMI are satisfied:

$$Q = Q^T > 0 \tag{4}$$

$$\begin{bmatrix} AQ + QA^T + 2(\delta - \rho)Q - YB^T - BY^T & \rho I + Q \\ \rho I + Q & -I \end{bmatrix} < 0 \tag{5}$$

then the coupled system (1) is globally exponentially synchronized using the linear state feedback matrix gain:

$$K = Y^T Q^{-1} \tag{6}$$

2. Zhang, He and Wu approach [25]

Consider the chaotic master-slave system described by:

$$\begin{cases} \dot{x} = Ax + H\sigma(Dx) \\ \dot{z} = Az + H\sigma(Dz) + u \\ p = Cx \\ q = Cz \\ u = K(p(t_k) - q(t_k)) \end{cases} \tag{7}$$

where $A \in \Re^{n\times n}$, $H \in \Re^{n\times n_h}$, $D \in \Re^{n_h\times n}$ and $C \in \Re^{\ell\times n}$ are constant matrices. $p, q \in \Re^{\ell}$ are the output vectors of the master system and the slave system, respectively. $\sigma(.) : \Re^{n_h} \to \Re^{n_h}$ with $\sigma_i(.)$, $i = 1,2,...,n_h$, are nonlinear functions belonging to sectors $[0, \omega_i]$ and satisfying the sector condition:

$$\sigma_i(\xi)(\sigma_i(\xi) - \omega_i \xi) \le 0 \quad \forall \xi \tag{8}$$

$p(t_k)$ and $q(t_k)$ are discrete measurements of $p(t)$ and $q(t)$ at simplest instant t_k, respectively, such that t_k satisfies:

$$\Delta k = t_{k+1} - t_k \le h \qquad \forall k \ge 0 \tag{9}$$

For the system (7), the error dynamics can be described by:

$$\dot{e} = \dot{x} - \dot{z} = Ae + H\eta(De, z) - KCe(t_k) \tag{10}$$

where:

$$\eta(De, z) = \sigma(De + Dz) - \sigma(Dz) \tag{11}$$

Theorem :

Given constant scalars h > 0, ε and γ and a constant matrix $W = diag(\omega_1 \quad \cdots \quad \omega_{n_h})$ satisfying the conditions (8) and (9), then the master-slave system (7) is globally asymptotically synchronized if there exist any appropriately dimensioned matrices G, V, $M = \left[M_1^T\, M_2^T\, M_3^T\, M_4^T\, M_5^T\, M_6^T\, M_7^T\right]^T$, $N = \left[N_1^T\, N_2^T\, N_3^T\, N_4^T\, N_5^T\, N_6^T\, N_7^T\right]^T$, any positive diagonal matrices:

$$\Lambda = diag(\lambda_1, \lambda_2, ..., \lambda_{n_h}) \ge 0 \tag{12}$$

$$T = diag(t_1, t_2, ..., t_{n_h}) \ge 0 \tag{13}$$

$$F = diag(f_1, f_2, \ldots, f_{n_h}) \geq 0 \tag{14}$$

and any semi positive-definite matrices:

$$P = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ * & P_{22} & P_{23} \\ * & * & P_{33} \end{bmatrix} \geq 0 \tag{15}$$

where:

$$P_{11} > 0 \tag{16}$$

and

$$R = \begin{bmatrix} R_{11} & R_{12} \\ * & R_{22} \end{bmatrix} \geq 0 \tag{17}$$

$$Z = \begin{bmatrix} Z_{11} & Z_{12} \\ * & Z_{22} \end{bmatrix} \geq 0 \tag{18}$$

$$X = \begin{bmatrix} X_{11} & X_{12} & X_{13} & X_{14} & X_{15} & X_{16} & X_{17} \\ * & X_{22} & X_{23} & X_{24} & X_{25} & X_{26} & X_{27} \\ * & * & X_{33} & X_{34} & X_{35} & X_{36} & X_{37} \\ * & * & * & X_{44} & X_{45} & X_{46} & X_{47} \\ * & * & * & * & X_{55} & X_{56} & X_{57} \\ * & * & * & * & * & X_{66} & X_{67} \\ * & * & * & * & * & * & X_{77} \end{bmatrix} \geq 0 \tag{19}$$

such that the following LMIs hold:

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} & \Phi_{15} & \Phi_{16} & \Phi_{17} \\ * & \Phi_{22} & \Phi_{23} & \Phi_{24} & \Phi_{25} & \Phi_{26} & \Phi_{27} \\ * & * & \Phi_{33} & \Phi_{34} & \Phi_{35} & \Phi_{36} & \Phi_{37} \\ * & * & * & \Phi_{44} & \Phi_{45} & \Phi_{46} & \Phi_{47} \\ * & * & * & * & \Phi_{55} & \Phi_{56} & \Phi_{57} \\ * & * & * & * & * & \Phi_{66} & \Phi_{67} \\ * & * & * & * & * & * & \Phi_{77} \end{bmatrix} < 0 \tag{20}$$

$$\Psi_1 = \begin{bmatrix} X & \overline{N} \\ * & Z \end{bmatrix} \geq 0 \tag{21}$$

$$\Psi_2 = \begin{bmatrix} X & \overline{M} \\ * & Z \end{bmatrix} \geq 0 \tag{22}$$

where:

$\Phi_{11} = P_{13} + P_{13}^T + R_{11} + hZ_{11} + N_1 + N_1^T - \varepsilon GA - \varepsilon A^T G^T + hX_{11}$

$\Phi_{12} = P_{11} + R_{12} + hZ_{12} + N_2^T + \varepsilon G + A^T G^T + hX_{12}$

$\Phi_{13} = N_3^T + D^T WT + hX_{13}$

$\Phi_{14} = N_4^T - \varepsilon GH + D^T WF + hX_{14}$

$\Phi_{15} = P_{23}^T - N_1 + N_5^T + M_1 + \varepsilon VC - \gamma A^T G^T + hX_{15}$

$\Phi_{16} = -P_{13} + N_6^T - M_1 + hX_{16}$

$\Phi_{17} = N_7^T + hX_{17}$

$\Phi_{22} = R_{22} + hZ_{22} + G + G^T + hX_{22}$

$\Phi_{23} = D^T \Lambda + hX_{23}$

$\Phi_{24} = -GH + hX_{24}$

$\Phi_{25} = P_{12} - N_2 + M_2 + VC + \gamma G^T + hX_{25}$

$\Phi_{26} = -M_2 + hX_{26}$

$\Phi_{27} = hX_{27}$

$\Phi_{33} = -2T + hX_{33}$

$\Phi_{34} = hX_{34}$

$\Phi_{35} = -N_3 + M_3 + hX_{35}$

$\Phi_{36} = -M_3 + hX_{36}$

$\Phi_{37} = hX_{37}$

$\Phi_{44} = -2F + hX_{44}$

$\Phi_{45} = -N_4 + M_4 - \gamma H^T G^T + hX_{45}$

$\Phi_{46} = -M_4 + hX_{46}$

$\Phi_{47} = hX_{47}$

$\Phi_{55} = -N_5 - N_5^T + M_5 + M_5^T + \gamma VC + \gamma C^T V^T + hX_{55}$

$\Phi_{56} = -P_{23} - N_6^T + M_6^T - M_5 + hX_{56}$

$\Phi_{57} = -N_7^T + M_7^T + hX_{57}$

$$\Phi_{66} = -R_{11} - M_6 + M_6^T + hX_{66}$$

$$\Phi_{67} = -R_{12} - M_7^T + hX_{67}$$

$$\Phi_{77} = -R_{22} + hX_{77}$$

$$\overline{N} = \begin{bmatrix} -P_{33} & -P_{13}^T & 0 & 0 & 0 & P_{33} & 0 \\ N_1^T & N_2^T & N_3^T & N_4^T & N_5^T & N_6^T & N_7^T \end{bmatrix}^T$$

$$\overline{M} = \begin{bmatrix} -P_{33} & -P_{13}^T & 0 & 0 & 0 & P_{33} & 0 \\ M_1^T & M_2^T & M_3^T & M_4^T & M_5^T & M_6^T & M_7^T \end{bmatrix}^T$$

The linear state feedback matrix gain is given by:

$$K = G^{-1}V \tag{23}$$

3. Boubaker and Mkaouar approach [21]

Consider the master slave system described by:

$$\begin{cases} \dot{x} = A_j x + b_j, & x \in \Lambda_j,\ j \in \{1, \dots, N\} \\ \dot{z} = A_i z + b_i + Bu, & z \in \Lambda_i,\ i \in \{1, \dots, N\} \\ u = K(z - x) \end{cases} \tag{24}$$

where $A_i \in \Re^{n \times n}$, $A_j \in \Re^{n \times n}$, $b_i \in \Re^n$ $b_j \in \Re^n$ are two constant matrices and two constant vectors, respectively. $B \in \Re^{n \times m}$ is the control matrix.

Λ_j and Λ_i are partition of the state-space into polyhedral cells defined respectively by the following polytopic description:

$$\Lambda_j = \{x \mid H_j^T x + h_j < 0\} \tag{25}$$

$$\Lambda_i = \{z \mid H_i^T x + h_i < 0\} \tag{26}$$

where $H_j \in \Re^{n \times r_j}$, $h_j \in \Re^{r_j \times 1}$, $H_i \in \Re^{n \times r_i}$ and $h_j \in \Re^{r_i \times 1}$.

The error dynamics derived from (24) can be written as:

$$\dot{e} = (A_i + BK)e + A_{ij}x + b_{ij} \tag{27}$$

where $A_{ij} = A_i - A_j$, $b_{ij} = b_i - b_j$

Theorem :

If a suitable matrix $B \in \Re^{n\times m}$ is chosen such that the pairs (A_i, B) are controllable, for a given decay $\alpha_1 > 0$ and for all $i, j \in \{1, \ldots, N\}$, if there exist constant matrices $S \in \Re^{n\times n}$, $R \in \Re^{m\times n}$, $E_{ij} \in \Re^{r_i \times r_i}$ and $F_{ij} \in \Re^{r_j \times r_j}$ and constants β_{ij} and ξ_{ij}, such that the following LMIs are satisfied:

$$S = S^T > 0 \tag{28}$$

$$E_{ij} = diag\,(e_1, e_2, ..., e_{r_i}) < 0 \tag{29}$$

$$F_{ij} = diag\,(f_1, f_2, ..., f_{r_j}) < 0 \tag{30}$$

$$diag(\beta_{ij}, \xi_{ij}) < 0 \tag{31}$$

$$\begin{bmatrix} \xi_{ij} & \xi_{ij}|h_i|^T & \xi_{ij}|h_j|^T \\ * & \frac{1}{2}E_{ij} & 0 \\ * & * & \frac{1}{2}F_{ij} \end{bmatrix} < 0 \tag{32}$$

$$\begin{bmatrix} \Delta_i & A_{ij} & SH_i & 0 & \xi_{ij}b_{ij}|h_i|^T - \frac{1}{2}SH_iM_i & \xi_{ij}b_{ij}|h_j|^T \\ * & \beta_{ij}I & H_i & H_j & -\frac{1}{2}H_iM_i & -\frac{1}{2}H_jM_j \\ * & * & 2E_{ij} & 0 & 0 & 0 \\ * & * & * & 2F_{ij} & 0 & 0 \\ * & * & * & * & \frac{1}{2}E_{ij} - \xi_{ij}|h_i||h_i|^T & -\xi_{ij}|h_i||h_j|^T \\ * & * & * & * & * & \frac{1}{2}F_{ij} - \xi_{ij}|h_j||h_j|^T \end{bmatrix} < 0 \tag{33}$$

$$\Delta_i = A_i S + S A_i^T + BR + R^T B^T + \alpha_1 S - \xi_{ij} b_{ij} b_{ij}^T$$

then the master slave system (24) under the condition (25) and (26) is globally asymptotically synchronized and the linear state matrix gain is given by:

$$K = RS^{-1} \tag{34}$$

III. PIECEWISE CHAOTIC SYSTEMS

In this section, we have collected, from the literature, four piecewise affine chaotic systems. We recall their mathematical models and simulate their chaotic behavior in order to derive, in the following section, the comparative analysis.

1. The Original Chua circuit

The original Chua's oscillator can be described by the following dynamical model [39, 40]:

$$\begin{cases} \dot{x}_1 = -\frac{1}{R_1C_1}x_1 + \frac{1}{R_1C_1}x_2 - \frac{1}{C_1}g_1(x) \\ \dot{x}_2 = \frac{1}{R_1C_2}x_1 - \frac{1}{R_1C_2}x_2 + \frac{1}{C_2}x_3 \\ \dot{x}_3 = -\frac{1}{L}x_2 - \frac{R_0}{L}x_3 \end{cases} \tag{35}$$

with the nonlinear characteristic of the Chua's diode:

$$g_1(x) = G_b x_1 + 0.5(G_a - G_b)\left(|x_1 + E| - |x_1 - E|\right) \tag{36}$$

For the following parameters [41]: $C_1 = 5.75\ 10^{-9}F$ $C_2 = 21.32\ \ 10^{-9}F$, $L = 12\ \ 10^{-3}H$, $R_0 = 30.86\Omega$, $R_1 = 1.45\,k\Omega$, $G_a = -0.879\ \ 10^{-3}$, $G_b = -0.4124\ \ 10^{-3}$, $E = 1$ and the initial condition $x(0) = [0.5 \quad 0.5 \quad 0]^T$, a double scroll attractor is obtained as shown by Fig.1.

2. The modified Chua circuit

The modified Chua's circuit is described by [40, 42]:

$$\begin{cases} \dot{x}_1 = \alpha\left(x_2 - g_2(x)\right) \\ \dot{x}_2 = x_1 - x_2 + x_3 \\ \dot{x}_3 = -\beta x_2 \end{cases} \tag{37}$$

with the nonlinear characteristic of Chua's diode

$$g_2(x) = bx_1 + 0.5(a-b)\left(|x_1 + c| - |x_1 - c|\right) \tag{38}$$

For the following parameters [42]: $\alpha = 9$, $\beta = \frac{100}{7}$, $a = -\frac{1}{7}$, $b = \frac{2}{7}$ and $c = 1$ and the initial condition $x(0) = [0.1 \quad 0.1 \quad 0.1]^T$, a double scroll attractor is obtained as shown by Fig.2.

3. The lure like circuit

The Lur'e like system is described by [43]:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = -6.8x_1 - 3.9x_2 - x_3 + 12g_3(x) \end{cases} \tag{39}$$

with the nonlinear characteristic:

$$g_3(x) = \begin{cases} \kappa\, x_1 & if\ |x_1| < \dfrac{1}{\kappa} \\ sign\ (x_1) & otherwise \end{cases} \tag{40}$$

For $\kappa = 1.5$ and the initial condition $x(0) = [0.5 \quad 0.5 \quad 0]^T$, a chaotic behavior is obtained as shown by Fig.3.

4. The five scroll circuit

A more complete family of n-scroll has been obtained from a generalized Chua's circuit proposed in [42]. The n-scroll circuit is given by:

$$\begin{cases} \dot{x}_1 = \alpha(x_2 - g_4(x)) \\ \dot{x}_2 = x_1 - x_2 + x_3 \\ \dot{x}_3 = -\beta x_2 \end{cases} \tag{41}$$

with the piecewise linear characteristic:

$$g_4(x) = m_{2q-1}x_1 + \frac{1}{2}\sum_{i=1}^{2q-1}(m_{i-1} - m_i)(|x_1 + c_i| - |x_1 - c_i|) \tag{42}$$

For the following parameters: $\alpha = 9$, $\beta = \dfrac{100}{7}$, $m_0 = +\dfrac{0.9}{7}$, $m_1 = -\dfrac{3}{7}$, $m_2 = +\dfrac{3.5}{7}$, $m_3 = -\dfrac{2.7}{7}$

$m_4 = +\dfrac{4}{7}$, $m_5 = -\dfrac{2.4}{7}$, $c_1 = 1$, $c_2 = 2.15$, $c_3 = 3.6$, $c_4 = 6.2$ and $c_5 = 9$ and the initial condition $x(0) = [-1 \quad -1 \quad -1]^T$, a 5-scroll attractors are obtained as shown by Fig.4.

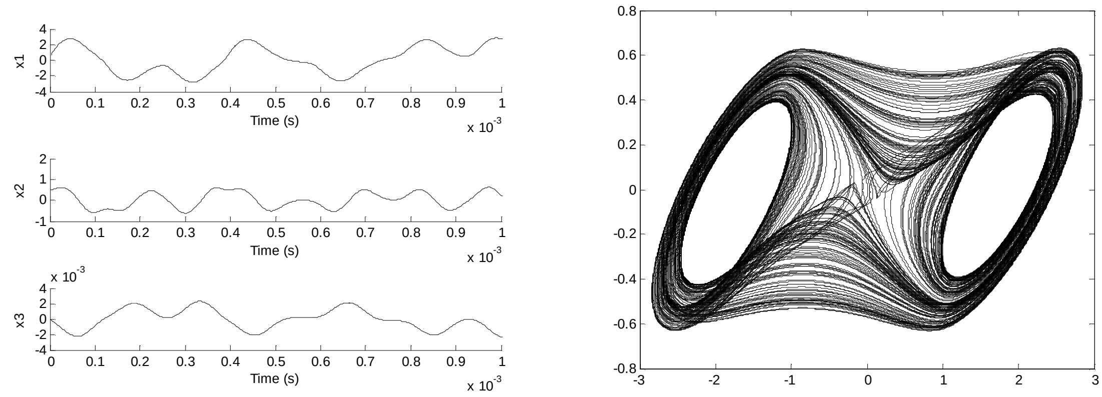


Figure 1. Chaotic behavior of the original Chua's circuit

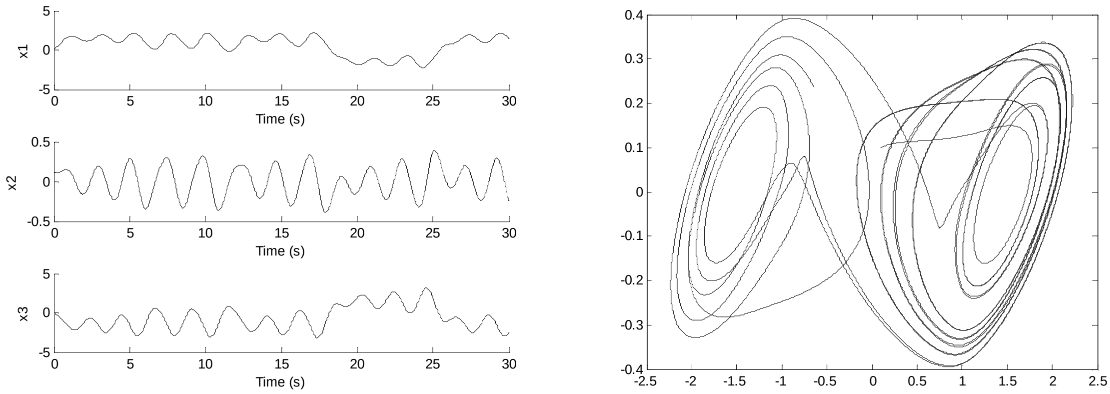


Figure 2. Chaotic behavior of the modified Chua's circuit

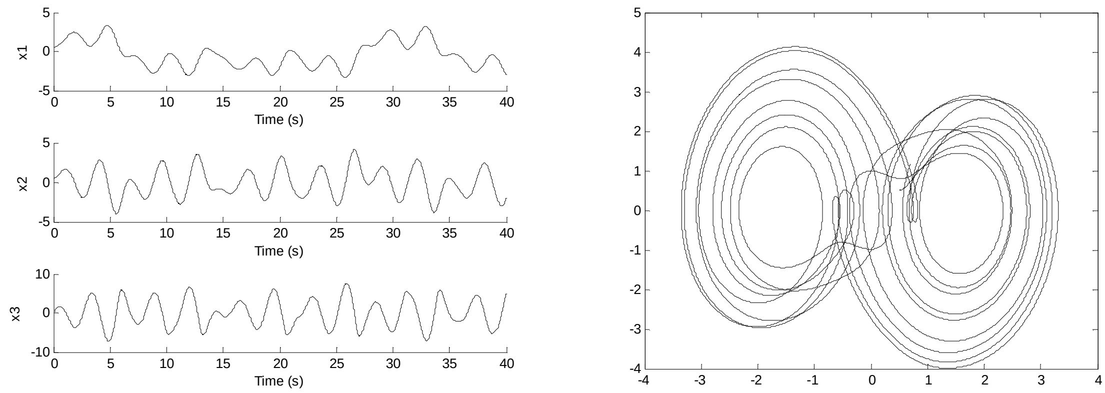


Figure 3. Chaotic behavior of the Lur'e like circuit

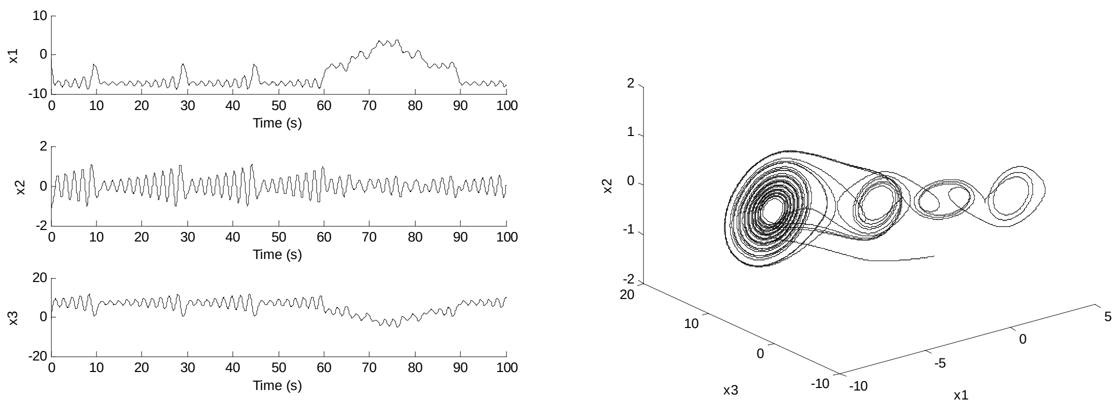


Figure 4. Chaotic behavior of the Five-scroll circuit

IV. CHAOS SYNCHRONIZATION SYNTHESIS

The objective of this section is to establish a comparative analysis between the three chaos synchronization approaches based on the four PWA systems via some defined criteria.

1. Original Chua's Circuits case study

Assume for the original Chua's circuit (35) that: $\mu_1 = 1/R_1C_1$, $\mu_2 = 1/C_1$ $\mu_3 = 1/R_1C_2$, $\mu_4 = 1/C_2$ and $\mu_5 = 1/L$.

Jiang and Zheng approach:

The matrices related to the master slave system (1) for the original Chua's circuit described by (35)-(36) are given then by:

$$A = \begin{pmatrix} -\mu_1 & \mu_1 & 0 \\ \mu_3 & -\mu_3 & \mu_4 \\ 0 & -\mu_5 & -R_0\mu_5 \end{pmatrix} \quad g(x) = \begin{pmatrix} -\mu_2 g_1(x) \\ 0 \\ 0 \end{pmatrix} \quad g(z) = \begin{pmatrix} -\mu_2 g_1(z) \\ 0 \\ 0 \end{pmatrix}$$

The Lipchitz parameter imposed by the condition (2) is computed as [44]:

$$\rho = \left| -\mu_2 \frac{G_a - G_b}{2} \right|$$

The tuning parameters δ is fixed at 0.5 where the computed parameter is $\rho = 4.0574\ 10^4$. After 16 iterations, the LMIs constraints (4)-(5) were found infeasible.

Zhang, He and Wu approach:

The matrices related to the master slave system (7) for the original Chua's circuit described by (35)-(36) are given by:

$$A=\begin{pmatrix} -\mu_1-\mu_2 G_b & \mu_1 & 0 \\ \mu_3 & -\mu_3 & \mu_4 \\ 0 & -\mu_5 & -R_0\mu_5 \end{pmatrix} \quad H=\begin{pmatrix} -\mu_2(G_a-G_b) \\ 0 \\ 0 \end{pmatrix}$$

$$C=D=\begin{pmatrix}1 & 0 & 0\end{pmatrix}$$

$$\sigma(D\,x)=0.5\left(\left|x_1+E\right|-\left|x_1-E\right|\right)$$

$$\sigma(D\,z)=0.5\left(\left|z_1+E\right|-\left|z_1-E\right|\right)$$

The tuning parameters ε and γ are fixed at 10 and 2, respectively. Under the conditions (8) and (9) we have $h=10^{-6}$ and $W=0.001$. After 52 iterations and for the initial conditions $x(0)=\left[0.5 \quad 0.5 \quad 0\right]^T$ and $z(0)=\left[-0.5 \quad -0.5 \quad 0\right]^T$, the LMIs constraints (12)-(22) were found feasible. The state matrix gain (23) is computed as $K=10^6\left[2.1119 \quad 0.4552 \quad 0.0011\right]^T$. Simulation results given by Fig.5 prove that chaos synchronization is well achieved with realistic control laws.

Boubaker and Mkaouar approach:

The matrices related to the master slave system (24) for the original Chua's circuit described by (35)-(36) are given by:

$$A_1=A_3=\begin{pmatrix} -(\mu_1+\mu_2 G_b) & \mu_1 & 0 \\ \mu_3 & -\mu_3 & \mu_4 \\ 0 & -\mu_5 & -R_0\mu_5 \end{pmatrix} \quad A_2=\begin{pmatrix} -(\mu_1+\mu_2 G_a) & \mu_1 & 0 \\ \mu_3 & -\mu_3 & \mu_4 \\ 0 & -\mu_5 & -R_0\mu_5 \end{pmatrix}$$

$$b_1=\begin{pmatrix} -\mu_2(G_a-G_b)E \\ 0 \\ 0 \end{pmatrix} \quad b_2=\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad b_3=\begin{pmatrix} \mu_2(G_a-G_b)E \\ 0 \\ 0 \end{pmatrix}$$

and the associate polytopic description (25)-(26) of the polyhedral cells are given by:

$$H_1=H_2=H_3=\begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}^T \quad h_1=\begin{bmatrix} -d \\ E \end{bmatrix} \quad h_2=\begin{bmatrix} -E \\ -E \end{bmatrix} \quad h_3=\begin{bmatrix} E \\ -d \end{bmatrix}$$

Where d and $-d$ are respectively an upper and lower bounds of x_1 ($d = 5$ in this case). The LMIs (28)-(33) are solved using the LMI toolbox of MatLab software for the control matrix $B = \begin{bmatrix} 5\,10^5 & 0 & 0 \end{bmatrix}^T$ and the tuning parameter $\alpha_1 = 10^{-4}$. After 41 iterations and for the initial conditions $x(0) = \begin{bmatrix} 0.5 & 0.5 & 0 \end{bmatrix}^T$ and $z(0) = \begin{bmatrix} -0.5 & -0.5 & 0 \end{bmatrix}^T$, the LMI constraints were found feasible. The matrix gain (34) is computed as $K = [-0.0821 \quad -0.2453 \quad 0.5948]$. Simulation results given by Fig.6 and Fig.7 prove that chaos synchronization is well achieved.

2. Modified Chua's circuits case study

Jiang and Zheng approach :

The matrices related to the master slave system (1) for the modified Chua's circuit described by (37)-(38) are given then by:

$$A = \begin{pmatrix} 0 & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & 0 \end{pmatrix} \quad g(x) = \begin{pmatrix} -\alpha g_2(x) \\ 0 \\ 0 \end{pmatrix} \quad g(z) = \begin{pmatrix} -\alpha g_2(z) \\ 0 \\ 0 \end{pmatrix}$$

The Lipchitz parameter given by the condition (2) is given by:

$$\rho = \left| -\alpha \frac{a-b}{2} \right|$$

The tuning parameters δ is fixed at 0.5 where the computed parameter is $\rho = 1.9286$. After 22 iterations, the LMIs constraints (4)-(5) were found infeasible.

Zhang, He and Wu approach:

The matrices related to the master slave system (7) for the modified Chua's circuit described by (37)-(38) are given by:

$$A = \begin{pmatrix} -\alpha b & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & 0 \end{pmatrix}, \quad H = \begin{pmatrix} -\alpha(a-b) \\ 0 \\ 0 \end{pmatrix}$$

$$C = D = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$$

$$\sigma(Dx) = 0.5(|x_1 + c| - |x_1 - c|)$$

$\sigma(Dz) = 0.5(|z_1 + c| - |z_1 - c|)$

The tuning parameters ε and γ are fixed at 10 and 20, respectively. Under the conditions (8) and (9) we have $h = 0.32$ and $W = 0.2593$. After 16 iterations and for the initial conditions $x(0) = [0.1 \quad 0.1 \quad 0.1]^T$ and $z(0) = [-1 \quad -1 \quad -1]^T$, the LMIs constraints (10)-(20) were found feasible. The state matrix gain (23) is computed as $K = [7.1656 \quad 1.8692 \quad -3.6126]^T$. Simulation results given by Fig. 8 prove that chaos synchronization is well achieved with realistic control laws.

Boubaker and Mkaouar approach:

The matrices related to the master slave system (24) for the modified Chua's circuit described by (37)-(38) are given by:

$$A_1 = A_3 = \begin{pmatrix} -b\alpha & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & 0 \end{pmatrix} \quad A_2 = \begin{pmatrix} -a\alpha & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & 0 \end{pmatrix}$$

$$b_1 = \begin{pmatrix} -\alpha(a-b)c \\ 0 \\ 0 \end{pmatrix} \quad b_2 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad b_3 = \begin{pmatrix} \alpha(a-b)c \\ 0 \\ 0 \end{pmatrix}$$

and the associate polytopic description (23)-(24) of the polyhedral cells are given by:

$$H_1 = H_2 = H_3 = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}^T \; h_1 = \begin{bmatrix} -d \\ c \end{bmatrix} \; h_2 = \begin{bmatrix} -c \\ -c \end{bmatrix} \; h_3 = \begin{bmatrix} c \\ -d \end{bmatrix}$$

Where d and $-d$ are respectively an upper and lower bounds of x_1 ($d = 5$ in this case).

The LMIs (28)-(33) are solved for the control matrix $B = [5\,10^3 \quad 0 \quad 0]^T$ and the tuning parameter $\alpha_1 = 10^{-4}$. After 5 iterations and for the initial conditions $x(0) = [0.1 \quad 0.1 \quad 0.1]^T$ and $z(0) = [-1 \quad -1 \quad -1]^T$, the LMI constraints were found feasible. The matrix gain (34) is computed as $K = 10^3[-0.6048 \quad -2.2343 \quad -0.0411]$. Simulation results given by Fig.9 and Fig.10 prove that chaos synchronization is well achieved.

3. Lur'e like circuits case study

Jiang and Zheng approach :

The matrices related to the master slave system (1) for Lur'e like circuit described by (39)-(40) are given then by:

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6.8 & -3.9 & -1 \end{pmatrix} \quad G(x) = \begin{pmatrix} 0 \\ 0 \\ 12g_3(x) \end{pmatrix} \quad G(z) = \begin{pmatrix} 0 \\ 0 \\ 12g_3(z) \end{pmatrix}$$

The Lipchitz parameter given by the condition (2) is computed $\rho = 12$. The tuning parameters δ is fixed at 0.5. After 11 iterations, the LMIs constraints (4)-(5) were found infeasible.

Zhang, He and Wu approach:

The matrices related to the master slave system (7) for Lur'e like circuit described by (39)-(40) are given then by:

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6.8 & -3.9 & -1 \end{pmatrix}, \quad H = \begin{pmatrix} 0 \\ 0 \\ 12 \end{pmatrix}$$

$$C = D = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$$

$$\sigma(Dx) = g_3(x)$$

$$\sigma(Dz) = g_3(z)$$

The tuning parameters ε and γ are fixed at 1 and 2, respectively. Under the conditions (8) and (9) we have $h = 0.1$ and $W = 2$. After 7 iterations and for the initial conditions $x(0) = [0.5 \quad 0.5 \quad 0]^T$ and $z(0) = [-0.5 \quad -0.5 \quad 0]^T$, the LMIs constraints (12)-(22) were found feasible. The state matrix gain (23) is computed as $K = [3.2801 \quad 3.5088 \quad 6.6266]^T$. Simulation results given by Fig.11 prove that chaos synchronization is well achieved with realistic control laws.

Boubaker and Mkaouar approach:

The matrices related to the master slave system (24) for the Lur'e like circuit described by (39)-(40) is given then by:

$$A_1 = A_3 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6.8 & -3.9 & -1 \end{pmatrix} \quad A_2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6.8+12\kappa & -3.9 & -1 \end{pmatrix}$$

$$b_1 = \begin{pmatrix} 0 \\ 0 \\ -12 \end{pmatrix} \quad b_2 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad b_3 = \begin{pmatrix} 0 \\ 0 \\ 12 \end{pmatrix}$$

and the associate polytopic description (23)-(24) of the polyhedral cells are given by:

$$H_1 = H_2 = H_3 = \begin{bmatrix} -1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}^T \; h_1 = \begin{bmatrix} -d \\ 1/\kappa \end{bmatrix} \; h_2 = \begin{bmatrix} -1/\kappa \\ -1/\kappa \end{bmatrix} \; h_3 = \begin{bmatrix} 1/\kappa \\ -d \end{bmatrix}$$

The LMIs (28)-(33) are solved for the control matrix $B = [1 \quad 0 \quad 0]^T$ and the tuning parameter $\alpha_1 = 10^{-4}$. After 7 iterations and for the initial conditions $x(0) = [0.5 \quad 0.5 \quad 0]^T$ and $z(0) = [-0.5 \quad -0.5 \quad 0]^T$, the LMI constraints were found feasible. The matrix gain (34) is computed as $K = [-8.2525 \quad -0.9880 \quad -0.0304]$. Simulation results given by Fig.12 and Fig.13 prove that chaos synchronization is well achieved.

4. Five scroll circuits case study

Jiang and Zheng approach :

The matrices related to the master slave system (1) for Lur'e like circuit described by (41)-(42) are given then by:

$$A = \begin{pmatrix} -\alpha m_5 & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & 0 \end{pmatrix} \quad g(x) = \begin{pmatrix} -\alpha g_4(x) \\ 0 \\ 0 \end{pmatrix} \quad g(z) = \begin{pmatrix} -\alpha g_4(z) \\ 0 \\ 0 \end{pmatrix}$$

The Lipchitz parameter given by the condition (2) is given by:

$$\rho = \max\left| -\alpha \frac{m_{i-1} - m_i}{2} \right|$$

The tuning parameters δ is fixed at 10^5 where the computed parameter is $\rho = 4.3071$. After 16 iterations and for the initial conditions $x(0) = [-1 \quad -1 \quad -1]^T$ and $z(0) = [0.2 \quad -0.2 \quad -0.5]^T$, the LMIs constraints (4)-(5) were found feasible. The state matrix gain (6) is computed as $K = 10^5[1.8613 \quad -0.0174 \quad 0]$. Simulation results given by Fig.14 prove that chaos synchronization is well achieved with realistic control laws.

Zhang, He and Wu approach:

The matrices related to the master slave system (7) for Lur'e like circuit described by (41)-(42) are given then by:

$$A = \begin{pmatrix} -\alpha m_5 & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & 0 \end{pmatrix}, \quad H = \begin{pmatrix} -0.5\alpha \\ 0 \\ 0 \end{pmatrix},$$

$$C = D = (1 \quad 0 \quad 0)$$

$$\sigma(x) = \sum_{i=1}^{2q-1} (m_{i-1} - m_i)(|x_i + c_i| - |x_1 - c_i|)$$

$$\sigma(z) = \sum_{i=1}^{2q-1} (m_{i-1} - m_i)(|z_1 + c_i| - |z_1 - c_i|)$$

The tuning parameters ε and γ are fixed at 0 and 0.02, respectively. Under the conditions (8) and (9) we have $h = 0.2$ and $W = 0.2593$. After 18 iterations and for the initial conditions $x(0) = [-1 \quad -1 \quad -1]^T$ and $z(0) = [0.2 \quad -0.2 \quad -0.5]^T$, the LMIs constraints (10)-(20) were found feasible. The state matrix gain (21) is computed as $K = [15.8034 \quad 0{,}9380 \quad -8.3458]^T$. Simulation results given by Fig.15 prove that chaos synchronization is well achieved with realistic control laws.

Boubaker and Mkaouar approach:

The matrices related to the master slave system (24) for the Lur'e like circuit described by (41)-(42) are given by:

$$A_j = \begin{pmatrix} \rho_j & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & 0 \end{pmatrix} \quad A_i = \begin{pmatrix} \rho_i & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & 0 \end{pmatrix} \quad b_j = \begin{pmatrix} \upsilon_j \\ 0 \\ 0 \end{pmatrix} \quad b_i = \begin{pmatrix} \upsilon_i \\ 0 \\ 0 \end{pmatrix}$$

where $\forall\, i, j \in \{1,2,\cdots,11\}$:

$\rho_1 = \rho_{11} = -\alpha m_5$,

$\rho_2 = \rho_{10} = -\alpha m_4$,

$\rho_3 = \rho_9 = -\alpha m_3$,

$\rho_4 = \rho_8 = -\alpha m_2$,

$\rho_5 = \rho_7 = -\alpha m_1$

$\rho_6 = -\alpha m_0$

$\upsilon_1 = -\upsilon_{11} = -\alpha((m_0 - m_1)c_1 + (m_1 - m_2)c_2 + (m_2 - m_3)c_3 + (m_3 - m_4)c_4 + (m_4 - m_5)c_5)$

$\upsilon_2 = -\upsilon_{10} = -\alpha((m_0 - m_1)c_1 + (m_1 - m_2)c_2 + (m_2 - m_3)c_3 + (m_3 - m_4)c_4)$,

$\upsilon_3 = -\upsilon_9 = -\alpha((m_0 - m_1)c_1 + (m_1 - m_2)c_2 + (m_2 - m_3)c_3)$,

$\upsilon_4 = -\upsilon_8 = -\alpha((m_0 - m_1)c_1 + (m_1 - m_2)c_2)$

$\upsilon_5 = -\upsilon_7 = -\alpha((m_0 - m_1)c_1)$

$\upsilon_6 = 0$

and the associate polytopic description (25)-(26) of the polyhedral cells are given by:

$$H_i = H_j = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}^T \quad \forall\, i, j \in \{1,2,\cdots,11\}$$

$$h_1 = \begin{bmatrix} -d \\ c_5 \end{bmatrix} \quad h_2 = \begin{bmatrix} -c_5 \\ c_4 \end{bmatrix} \quad h_3 = \begin{bmatrix} -c_4 \\ c_3 \end{bmatrix} \quad h_4 = \begin{bmatrix} -c_3 \\ c_2 \end{bmatrix} \quad h_5 = \begin{bmatrix} -c_2 \\ c_1 \end{bmatrix} \quad h_6 = \begin{bmatrix} -c_1 \\ -c_1 \end{bmatrix} \quad h_7 = \begin{bmatrix} c_1 \\ -c_2 \end{bmatrix}$$

$$h_8 = \begin{bmatrix} c_2 \\ -c_3 \end{bmatrix} \quad h_9 = \begin{bmatrix} c_3 \\ -c_4 \end{bmatrix} \quad h_{10} = \begin{bmatrix} c_4 \\ -c_5 \end{bmatrix} \quad h_{11} = \begin{bmatrix} c_5 \\ -d \end{bmatrix}$$

Where d and $-d$ are respectively an upper and lower bounds of x_1 ($d = 15$ in this case).

The LMIs (28)-(33) are solved for the control matrix $B=\begin{bmatrix}1 & 0 & 0\end{bmatrix}^T$ and the tuning parameter $\alpha_1=10$. After 49 iterations, the LMI constraints were found feasible. The matrix gain (33) is computed as $K=[-34.2574 \quad -9.3906 \quad -0.0016]$. Each one from the master and slave five scroll circuit switches between 11 polytopic cells. For the initial conditions $x(0)=\begin{bmatrix}-1 & -1 & -1\end{bmatrix}^T$ and $z(0)=\begin{bmatrix}0.2 & -0.2 & -0.5\end{bmatrix}^T$, we have observed that the master system switch only between the 7th to the 10th cells and the slave system switch only between the 6th to the 10th cells. The figure 17 shows the commutations of the master and slave systems between the involved polytopic cells. For other cells, values are remaining at zero.

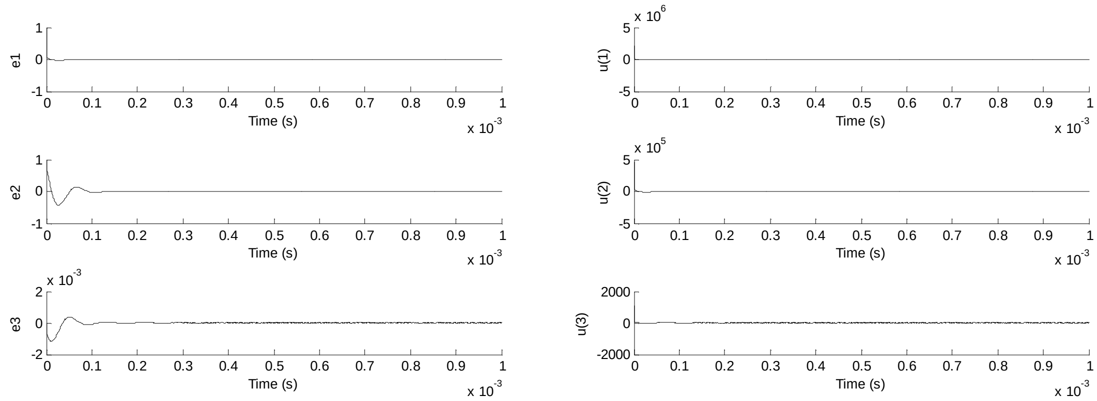


Figure 5. Synchronization of the original Chua's circuits: *Zhang and He* approach

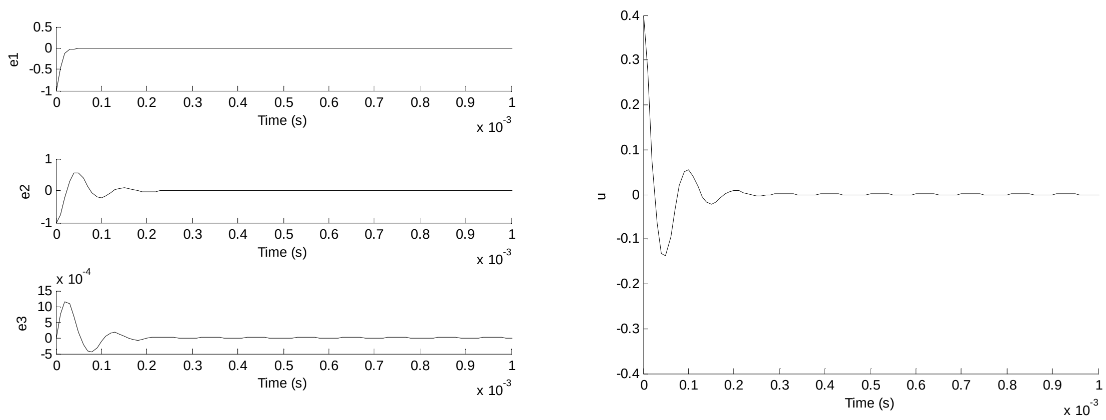
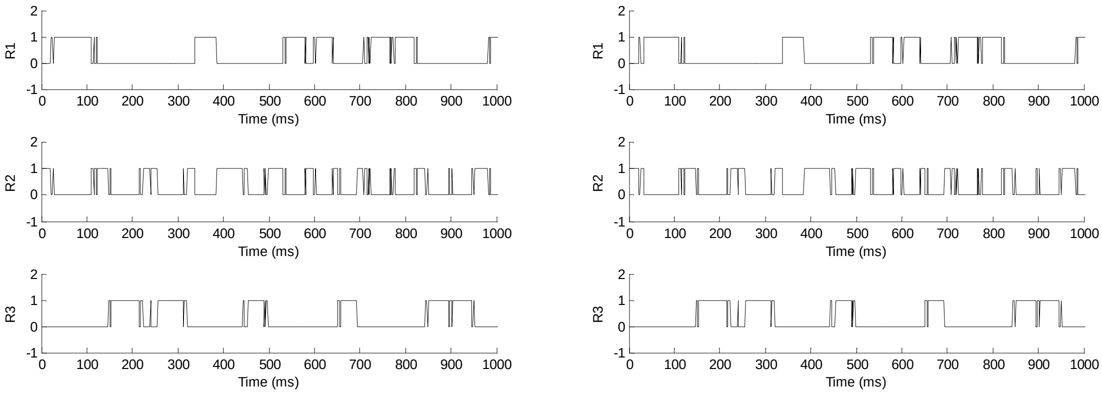


Figure 6. Synchronization of the original Chua's circuits: *Mkaouar and Boubaker* approach



(a) (b)

Figure 7. Switching dynamics between polytopic domains of the original Chua's circuits using *Mkaouar and Boubaker* approach: (a) master dynamics- (b) slave dynamics

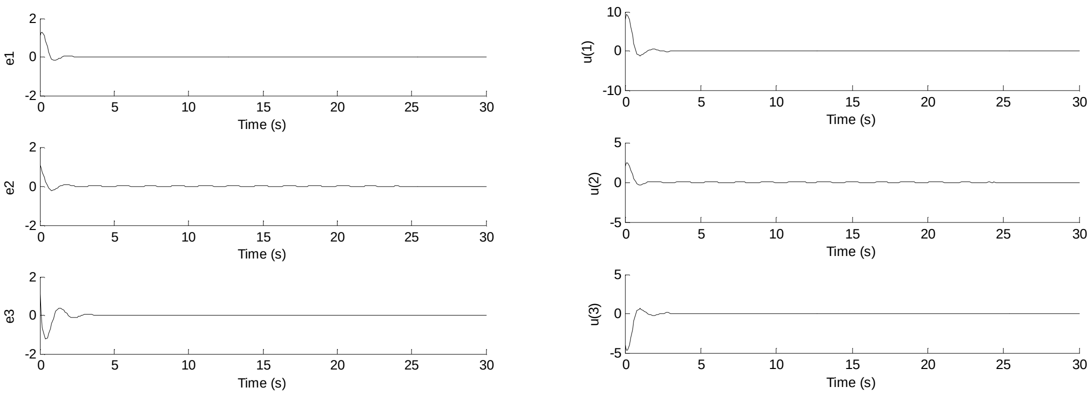


Figure 8. Synchronization of the modified Chua's circuits: *Zhang and He* approach

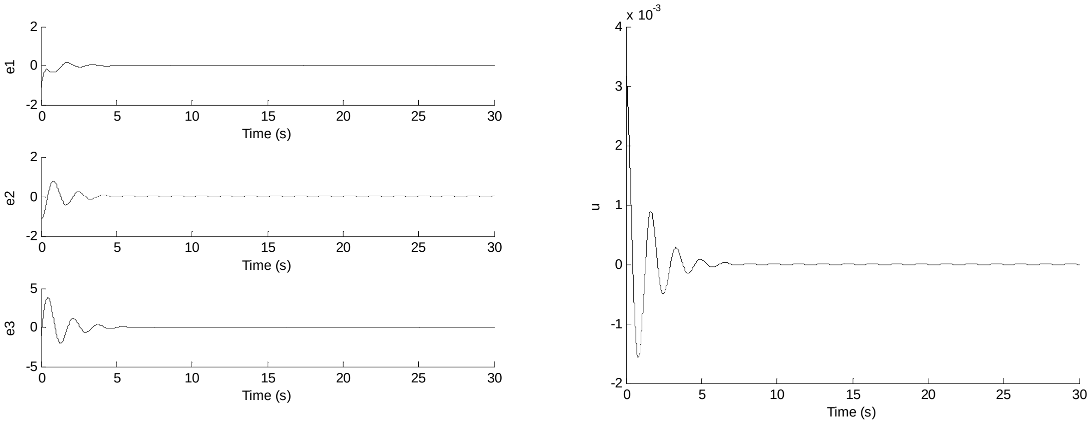
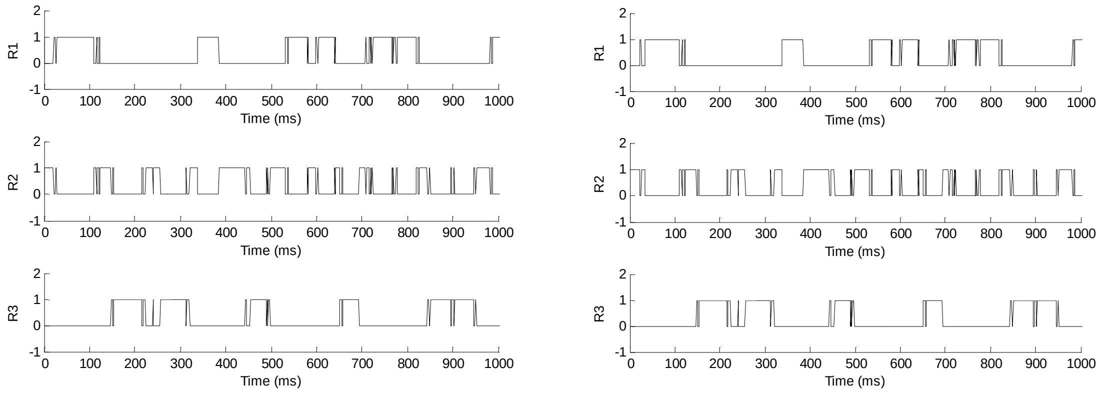


Figure 9. Synchronization of the modified Chua's circuits: *Mkaouar and Boubaker* approach



(a) (b)

Figure 10. Switching dynamics between polytopic domains of the modified Chua's circuits using *Mkaouar and Boubaker* approach: (a) master dynamics - (b) slave dynamics

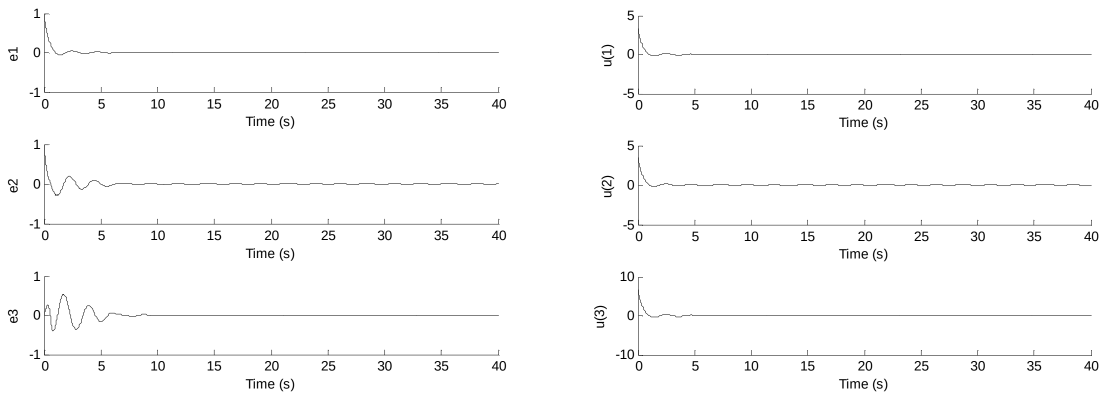


Figure 11. Synchronization of the Lur'e like circuits: *Zhang and He* approach

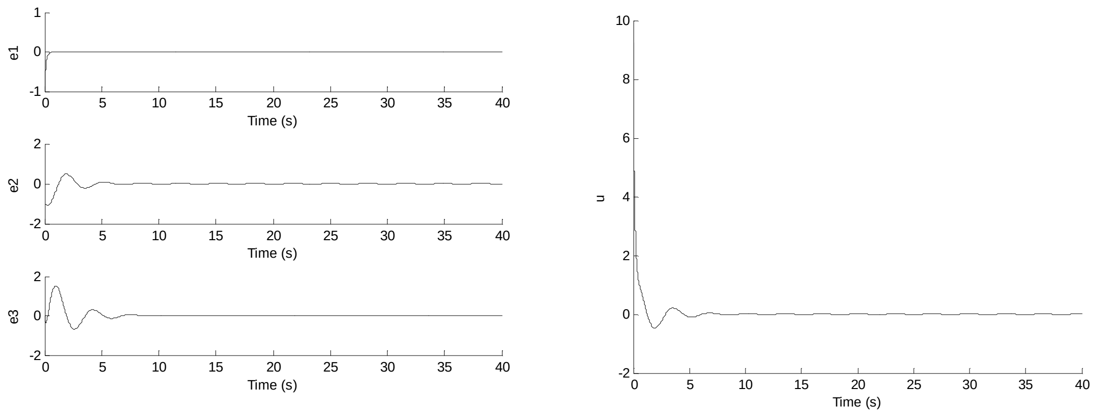


Figure 12. Synchronization of the Lur'e like circuits: *Mkaouar and Boubaker* approach

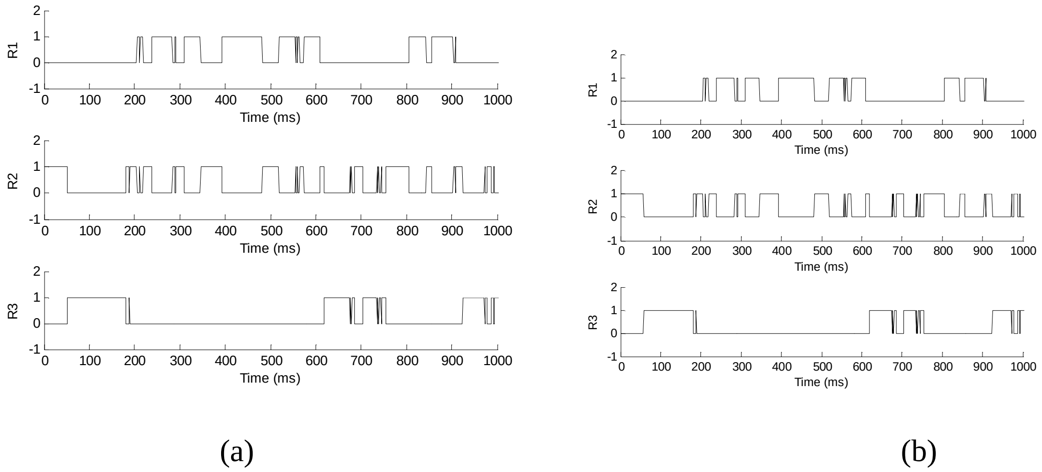


Figure 13. Switching dynamics between polytopic domains of the Lur'e like circuits using *Mkaouar and Boubaker* approach: (a) master dynamics - (b) slave dynamics

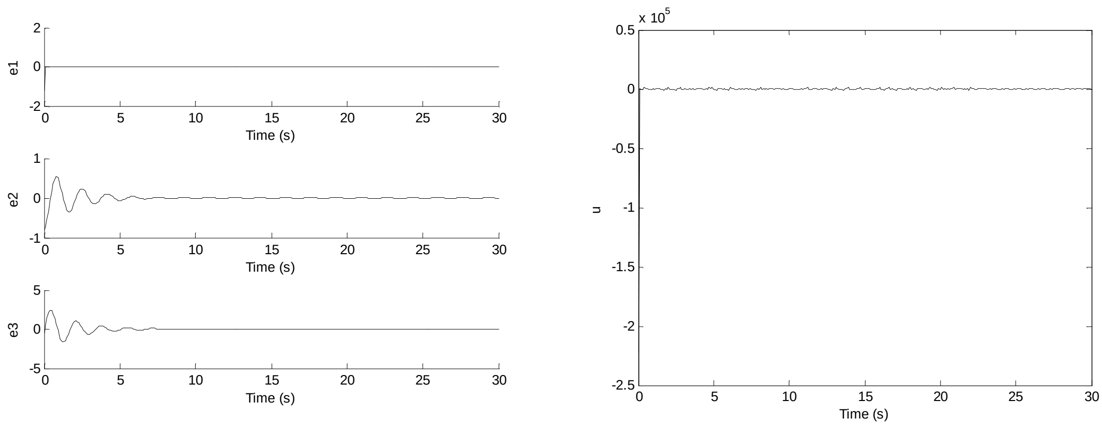


Figure 14. Synchronization of the Five-scroll circuits: *Jiang and Zheng* approach

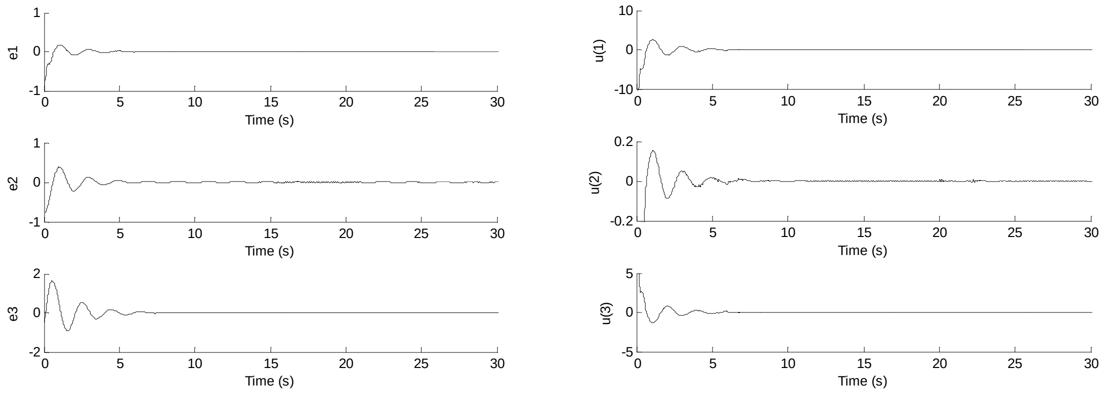


Figure 15. Synchronization of the Five-scroll circuits: *Zhang and He* approach

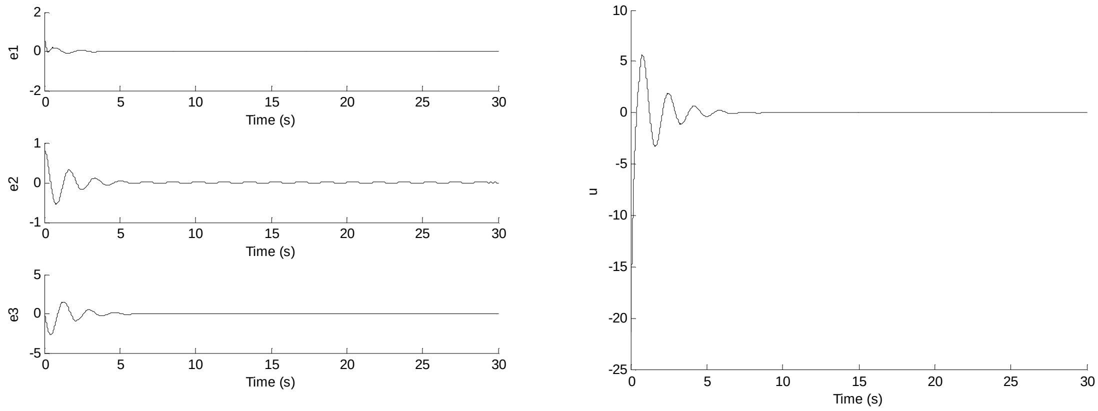
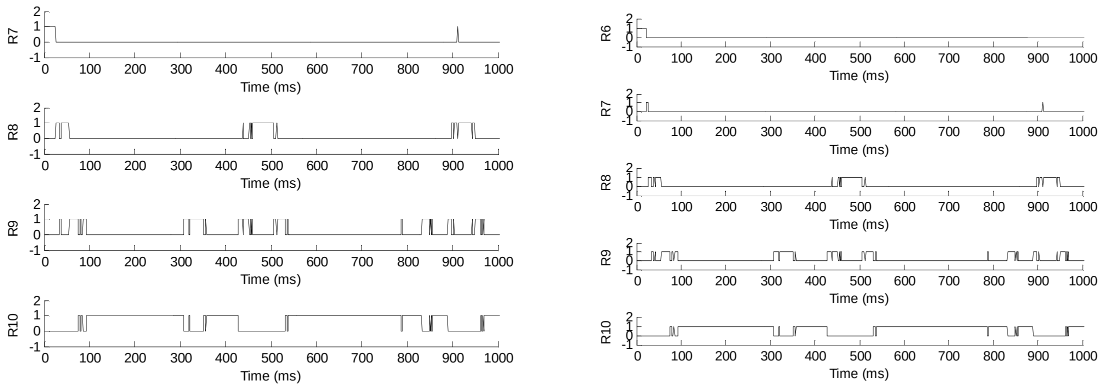


Figure 16. Synchronization of the Five-scroll circuits: *Mkaouar and Boubaker* approach



(a) (b)

Figure 17. Switching dynamics between polytopic domains of the Five-scroll circuits using *Mkaouar and Boubaker* approach: (a) master dynamics - (b) slave dynamics

V. COMPARATIVE ANALYSIS

To analyze the degree of the practical implementation of each synchronization approach we define first, as comparative criteria, the number of LMIs to be solved, the number of parameter solutions to be returned, the number of parameters to be computed before solving the LMIs and finally the number of tuning parameters of the control algorithm. As can be shown by Table 1, *Jiang and Zheng* approach [16] seems to be the simplest one; it has the lowest number of LMIs to be solved and the least number of tuning parameters. However, it always requires the pre-computation of the parameter ρ which can introduce some inherent calculation for some chaotic PWA systems.

Table 1. Comparative analysis of the of the LMI's implementation and their solving

Synchronization / Approach criterion	Jiang and Zheng [16]	Zhang, He and Wu [25]	Mkaouar and Boubaker [21]
LMIs to be solved	(4)-(5)	(12)-(22)	(28)-(33)
Tuning parameters	B, δ	h, ε, γ, W	B, α_1
Computed parameters	ρ	-	-
LMI's solutions	Q, Y	G, V, M, N, Λ, T, F, P, R, Z,X	S, R, E_{ij}, F_{ij}, β_{ij} ξ_{ij}, $i, j \in \{1,\dots,N\}$

On the other hand, the simplicity of implementation of *Mkaouar and Boubaker* approach [21] mainly depends on the number of polytopic cells. For lower or equal to 3 cells, *Mkaouar and Boubaker* approach will be always simpler than *Zhang and He* approach [25]. Since all synchronization approaches presented in this paper give sufficient conditions for global synchronization, the analysis of the LMI's feasibility on different master slave chaotic systems is required. Table 2 illustrates such comparative analysis. As can be shown, the simplest approach is the most conservative one.

Table 2. LMIs feasibility of each synchronization approach for every PWA system

PWA systems / Approach	Original Chua's circuits	Modified Chua's circuits	Lur'e like circuits	Five scroll circuits
Jiang and Zheng [16]	Infeasible	Infeasible	Infeasible	Feasible
Zhang, He and Wu [25]	Feasible	Feasible	Feasible	Feasible
Mkaouar and Boubaker [21]	Feasible	Feasible	Feasible	Feasible

For feasible solutions, the elapsed CPU time for solving the predefined LMIs for best tuning parameter's values can be considered as an important comparative criterion. As shown by Table 3, Mkaouar and Boubaker approach is the best one for the Chua's circuit, the modified Chua's circuit and lur'e like circuit but not for the five-scroll circuit. This is due to the important number of polytopic cells: 11 cells in this case.

Table 3. Elapsed CPU time in seconds for the best values of the tuning parameters for each synchronization approach for every PWA system

PWA Systems / Approach	Original Chua's circuits	Modified Chua's circuits	Lur'e like circuits	Five scroll circuits
Jiang and Zheng [16]	-	-	-	0.0168
Zhang, He and Wu [25]	7.4420	2.5026	1.1736	2.5262
Mkaouar and Boubaker [21]	0.5200	0.3160	0.3247	10.7407

The influence of the choice of the tuning parameters on the LMIs feasibility, on the global stability of the master slave system and on the synchronization time can be considered as the most important comparative criterion for this comparative analysis. The evaluation of such influence is established by practicing an intensive simulation and changing the values of tuning parameters in areas of sufficient size. We assume furthermore that the best values of the tuning parameter are obtained for the lowest synchronization time. Table 4 shows the result of this analysis study. As can be deduced *Mkaouar and Boubaker* approach is the less sensitive approach to the variation of the tuning parameters.

Table 4. Influence of the domain's belonging of the tuning parameters on the performances of each synchronization approach

Approach / Performances	Jiang and Zheng [16]	Zhang, He and Wu [25]	Mkaouar and Boubaker [21]
LMI's Feasibility	Yes	Yes	No
Stability of the error dynamics	Yes	Yes	No
Synchronization time	Yes	Yes	Yes

IV. CONCLUSION

In this paper three synchronization methods based on LMI tools for piecewise linear chaotic systems have been developed. The analysis study shows that *Zheng and He* [25], and *Mkaouar and Boubaker* [21] synchronization approaches are less conservative than *Jiang and Zhang* [16] one. However, *Mkaouar and Boubaker* approach achieve the best compromise between the practical implementation of LMIs and the influence of tuning parameters on the feasibility and the performances of the synchronization approach.

REFERENCES

[1] C. J. Luo, A theory for synchronization of dynamical systems, Communications in Nonlinear Science and Numerical Simulation. 14 (2009) 1901-1951.

[2] A. Pikovsky, M. Rosenblum, J. Kurths, B. Chirikov, P. Cvitanovic, Synchronization: A Universal Concept in Nonlinear Sciences, Second edition, United Kingdom, Cambridge University Press, 2003.

[3] L.M. Pecora, T.L. Carroll, Synchronization in chaotic system. Physical Review Letters. 64 (1990) 821-825.

[4] M. T .Yassen, Chaos synchronization between two different chaotic systems using active control, Chaos, Solitons and Fractals. 23 (2005) 131-140.

[5] W. Yu, Synchronization of three dimensional chaotic systems via a single state feedback, Communications in Nonlinear Science and Numerical Simulation, 16 (2011) 2880- 2886.

[6] U.E. Vincent, R. Guo, Finite-time synchronization for a class of chaotic and hyperchaotic systems via adaptive feedback controller, Physics Letters A, 375 (2009) 3925-3932.

[7] J. Zhang, C. Li, H. Zhang, J. Yu, Chaos synchronization using single variable feedback based on back-stepping method, Chaos, Solitons & Fractals. 21 (2004) 1183-1193.

[8] M. Hu, Y. Yang, Z. Xu, Impulsive control of projective synchronization in chaotic systems, Physics Letters A. 372 (2008) 3228-3233.

[9] T. Yang, L.O. Chua, Impulsive stabilization for control and synchronization of chaotic systems: theory and application to secure communication. IEEE Trans Circuits Systems I: Fundamentals Theory and Applications, 44 (1997) 976-988.

[10] M. Pourmahmood, S. Khanmohammadi, G. Alizadeh, Synchronization of two different uncertain chaotic systems with unknown parameters using a robust adaptive sliding mode controller, Communications in Nonlinear Science and Numerical Simulation. 16 (2011) 2853-2868.

[11] O. Boubaker, R. Dhifaoui, “Robust chaos synchronization for chua’s circuits via active sliding mode control,” In: S. Banerjee and Ş. Ş. Erçetin**,** Chaos, Complexity and Leadership 2012, (2014) 141-151.

[12] H. Yu, J. Wang, B. Deng, X. Wei, Y. Che, Y.K. Wong, W.L. Chan, K.M. Tsang, Adaptive back-stepping sliding mode control for chaos synchronization of two coupled neurons in the external electrical stimulation, Communications in Nonlinear Science and Numerical Simulation, 17 (2012)1344-1354,

[13] T. Zhang, G. Feng, Output tracking and synchronization of chaotic Chua’s circuit with disturbances via model predictive regulator. Chaos Solitons & Fractals, 9 (2009) 810-820.

[14] W. Xiaofeng, C. Guanrong, C. Jianping, Chaos synchronization of the master–slave generalized Lorenz systems via linear state error feedback control, Physica D: Nonlinear Phenomena, 229 (2007) 52-80.

[15] Q. Lin, X. Wu, The sufficient criteria for global synchronization of chaotic power systems under linear state-error feedback control, Nonlinear Analysis: Real World Applications, 12 (2011) 1500-1509.

[16] G.P. Jiang, W. X. Zheng, An LMI criterion for linear-state-feedback based chaos synchronization of a class of chaotic systems, Chaos, Solitons & Fractals. 26 (2005) 437-443.

[17] F. Chen, W. Zhang, LMI criteria for robust chaos synchronization of a class of chaotic systems, Nonlinear Analysis: Theory, Methods & Applications, 67 (2007) 3384-3393.

[18] S. Kuntanapreeda, Chaos synchronization of unified chaotic systems via LMI, Physics Letters A, 373 (2009) 2837-2840.

[19] M. M. Asheghan, M.T.H. Beheshti, An LMI approach to robust synchronization of a class of chaotic systems with gain variations, Chaos, Solitons & Fractals. 42 (2009) 1106-1111.

[20] Y. Chen, X. Wu, Z. Gui, Global synchronization criteria for a class of third-order non-autonomous chaotic systems via linear state error feedback control, Applied Mathematical Modelling. 34 (2010) 4161-4170.

[21] H. Mkaouar, O. Boubaker, Chaos synchronization for master slave piecewise linear systems: Application to Chua's circuit, Communications in Nonlinear Science and Numerical Simulation, 17 (2012) 1292-1302.

[22] J. Cao, H.X. Li, D.W.C. Ho, Synchronization criteria of Lur'e systems with time-delay feedback control. Chaos Soliton & Fractals 23 (2005) 1285-1298.

[23] Q.L. Han, On designing time-varying delay feedback controllers for master–slave synchronization of Lur'e systems. IEEE Transactions on Circuits and Systems I: Regular Papers 54 (2007) 1573-1583.

[24] J.L. Lu, D.J. Hill, Global asymptotical synchronization of chaotic Lur'e systems using sampled data: a linear matrix inequality approach, IEEE Transactions on Circuits and Systems II: Express Briefs. 55 (2008) 586-590.

[25] C.K. Zhang, Y. He, M. Wu, Improved global asymptotical synchronization of chaotic Lur'e systems with sampled-data control, IEEE Transactions on Circuits and Systems II: Express Briefs. 56 (2009) 320-324.

[26] M. Johansson, A. Rantzer, Computation of piecewise quadratic Lyapunov functions for hybrid systems, IEEE Transactions on Automatic Control, 43 (1998) 555-559.

[27] D. Kumar Ghara, D. Saha, K. Sengupta, Implementation of linear trace moisture sensor by nano porous thin film, International Journal of Smart Sensing and Intelligent Systems, 1 (2008) 955-969.

[28] B. Mulliez, E. Moutaye, H. Tap, L.Gatet, F. Gizard, Predistorsion system implementation based on analog neural networks for linearizing high power amplifiers transfer characteristics, International Journal of Smart Sensing and Intelligent Systems, 7 (2014) 400-422.

[29] A. Pavlov, A. Pogromsky, N. Van de Wouw, H. Nijmeijer, On convergence properties of piecewise affine systems. International Journal of Control, 80 (2007) 1233-1247.

[30] O. Boubaker, Gain scheduling control: an LMI approach, International Review of Electrical Engineering, 3 (2008) 378-385.

[31] T. Zhang, G. Feng, Output tracking of piecewise-linear systems via error feedback regulator with application to synchronization of nonlinear Chua's circuit, IEEE Transactions on Circuits and Systems I, 54 (2007) 1852-1863.

[32] Van de Wouw, N., Pavlov, A.: Tracking and synchronization for a class of PWA systems. Automatica, 44 (2008) 2909-2915.

[33] O. Boubaker, Master-slave synchronization for PWA systems, In Proceedings 3rd IEEE International Conference on Signals, Circuits and Systems, Medenine, Tunisia. 2009, 1-6.

[34] K. Kashima, Y. Kawamura, J.I. Imura, Oscillation analysis of linearly coupled piecewise affine systems: Application to spatio-temporal neuron dynamics, Automatica, 47 (2011) 1249-1254.

[35] H. Mkaouar, and O. Boubaker, Sufficient conditions for global synchronization of continuous piecewise affine systems, Lecture Notes in Computer Science, 6752 (2011) 199-211.

[36] A.C.J. Luo, Singularity and dynamics on discontinuous vector fields, Amsterdam, Elsevier, 2006

[37] A.S. Morse, Control using logic-based switching, Lecture notes in control and information sciences, London, Springer, 1997.

[38] D. Liberzon, A.S. Morse, Basic problems in stability and design of switched systems. IEEE Control Systems, 19 (1999) 59-70.

[39] Chua LO. Wu CW. Huang A. Zhong G. A universal circuit for studying and generating chaos-part I: routes to chaos, IEEE Transactions on circuits and systems. 40 (1993) 732-744.

[40] L. O. Chua, M. Komuro, T. Matsumto, The double scroll family, IEEE Transactions on Circuits and Systems. 33 (1986) 1072-1118.

[41] H. Mkaouar, O. Boubaker, On electronic design of the piecewise linear characteristic of the chua's diode: Application to chaos synchronization, In Proceedings 16th IEEE Mediterranean Electro-technical Conference, Yasmine Hammamet, Tunisia, 25-28 March 2012, 197-200.

[42] M.E. Yalcin, J.A.K. Suykens, J. Vandewalle, Experimental confirmation of 3-scroll and 5-scroll attractors for generalized Chua's circuit, IEEE Transactions on circuits and systems I: Fundamentals Theory and Applications. 47 (2000), 425-429.

[43] É. Gyurkovics, T. Takács, "Application of a multiplier method to uncertain Lur'e-like systems," Systems & Control Letters, 60 (2011) 854-862.

[44] G.P. Jiang, K.S. Tang, G.A. Chen, Simple global synchronization criterion for coupled chaotic systems, Chaos, Solitons & Fractals. 15 (2003) 925-935.