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CONDITION BASED PREVENTIVE MAINTENANCE CONTROL STRATEGY DESIGN

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Abstract- Compared with time based maintenance (TBM), Condition based maintenance (CBM) can improved the availability of the devices and reduced the examining maintenance cost. However, CBM possibly arouse an unexpected interference on production process due to an unplanned maintenance activity in advance so that the stable-state operation of the system was influenced. As the system scale was larger, this accidental disturbance should not be ignored. Based on the viewpoint, we first constructed a full-life cycle four-state model of the device, and then simplified it as three-state model based on some practical considerations in this paper. On the basis of it, the paper analyzed its reliability operation characteristics as the checking items being constants, and then proposed a dynamic real-time iterative control strategy on the checking items according to the practical state of the devices under CBM, and investigated its availability and adaptability. Moreover, the paper still performed devices state evaluation, and analyzed the control error and the selection of control timing, and etc, which

further expounded the availability of control strategy. In the end, the paper still conducted probability simulation on it, and some simulation experiments had been done. The related investigated results and the simulation results show that the proposed method in the paper is effective and available, and can ensure the system normal working not to be influenced by maintenance activities.

Index terms: Preventive maintenance, reliability, control strategy, estimation, Markov.

I. INTRODUCTION

Modern large-size wind power enterprises generally possess many same-type generators functioning simultaneously. It is significant for wind power plants security production whether these generators can normally operate or not, since it has a direct relationship with power grids stability [1]. Hence, based on the safe and economical consideration, modern large-size wind plants expect to adopt the condition based maintenance (CBM) to guide their maintenance and repairing practice to ensure the production process stability without exception [2]. For that each generator is installed a set of the monitoring cell to guard the status of the generator. If the failure of one cell occurs it can be detected out immediately, and the maintenance personnel may implement the timely repairing to restore it to good state [3]. Clearly, CBM not only improves the meantime to failure (MTTF) of the cell, but also reduces the maintenance cost, and is an ideal maintenance mode. However, compared with traditional time based maintenance (TBM), due to a unplanned maintenance activity beforehand, CBM possibly arouse a unexpected interruption for the generator operation. As a result, the stability operation of the generators is destroyed so as to give power grid interference. When the larger the wind plant scale is, the stronger such shock accidental is, such that it could not be ignored. And conversely, TBM is a planed maintenance activity, and can guarantee the stability operation of the wind plant due to a large number of substitutes in advance. But its cost is relatively high [4]. Clearly, it is important how to incorporate with the advantages of the two to ensure the safe production of the wind plant, and simultaneously, the maintenance cost is also reduced as soon as possible. The traditional solutions for this issue are to look for the best checking timing according to the real state of the devices, and incorporate with the life distribution characteristics of the devices during decision-making, such as proportional intensity model (PIM) [4], proportional hazards model (PHM) [5], delay time model (DTM) [6], Markov model (MM) [7], and etc. Indeed, these methods not only can reduce unnecessary checking maintenance activities but also ensure the security of production process. However, they ignore an important fact, i.e., a majority of such maintenance activity itself will result in unplanned downtime due to randomness of checking timing, which is possibly unsuitable for some production processes such as wind power plants. Particularly, for those large-scale production enterprises, this interruption will be more serious. In addition, the methods mentioned above seriously rely on life models or cost analysis data, but this is difficult to want to acquire precise mathematical models and life analysis data. Hence, in this paper, what is difference with traditional analysis methods, the checking rate of the preventive maintenance (PM), whether given or not beforehand, as a random variables, is considered in life model of the devices. And then though analyzing its role in life evolution process, the checking rate is selected as control variable, and system states act as state variables, and the aim what to do this is to possess a sufficient number of available units at stable state to ensure the security and stability of the production process under the condition of PM. Thus we will obtain the control strategy of the checking rate suitable to any life distribution models, and without consideration tedious and tiring cost computation. Although the increment of the number of PM checks times in the unit of time leads to a rising of expense, but reduces the necessity of corrective maintenance (CM), more expensive than PM, thus from long run, an effective thrift can be wined [8].

II. MODEL DESCRIPTION

To establish the life cycle model of such generators, the following assumptions require to be done.

Assumption1. The preventive maintenance, or the corrective maintenance, is independently expressed only using a state.

Assumption2. Whether at the operating or storage state, there are no devices the failures of which are no detectable.

Assumption3. Both the preventive maintenance and the corrective maintenance, the check and maintenance time of which follow exponential distribution.

Assumption4. Preventive maintenance can not change the natural failure rate of the equipments, i.e., the instantaneous failure rate after the equipment is repaired is same with one before it is repaired, and only the residual life is owned when it restores to work.

Assumption5. Preventive maintenance is perfect, i.e., if there are faults detected out during preventive maintenance, and then it would be able to get a timely repair.

Assumption6. Corrective maintenance is perfect, i.e., the equipment can restore to the original state after a corrective maintenance.

Based on the above assumptions, the life cycle model of the devices with four-state can be plotted using the state transition diagram as shown in Figure 1.

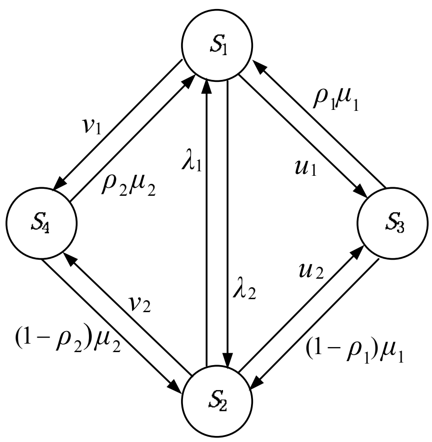


Figure 1. State transition diagram

In Figure1, the symbol S_i expresses the state of the equipment, where the subscript $i\in[1, 2, 3, 4]$ is applied to respectively express the equipment storage, work, preventive maintenance, and corrective maintenance of four kinds of states; and λ_1 expresses the storage rate of a functioning equipment; and λ_2 expresses the using rate of a stored equipment; and v_1 presents the failure rate of a stored equipment; and v_2 presents the failure rate of a functioning equipment; and u_1 presents the check rate of a stored equipment; and u_2 presents the check rate of a functioning equipment; and ρ_1 expresses the stored probability of the device after a preventive maintenance intervention; and 1-ρ_1 expresses the used probability of the device after a preventive maintenance intervention; and ρ_2 expresses the stored probability of the device after a corrective maintenance intervention; and 1-ρ_2 expresses the used probability of the device after a corrective maintenance intervention; and μ_1 expresses the repair risk rate function of the preventive maintenance; μ_2 expresses the repair risk rate function of the corrective maintenance. The physical meaning of the transfer rate is to ensure that it is the bounded and the nonnegative. Let $S(t)$ be the situated state of the device at time instant t, and then the $S(t)$ is a continuous time Markov process, namely, at arbitrary time instant t, as the specific numerical values of $S(t)$ are given, and the operation law after the time instant t of the process $S(t)$ has nothing to do with the history before t.

Based on model in Figure1 and assumption 5 and assumption 6, we further assume that PM can accomplish an eventual substitution before there is an upcoming failure or after a random damage happens, and thus the state S_4 can be eliminated from the life cycle model in Figure1. And then, we will obtain the following model.

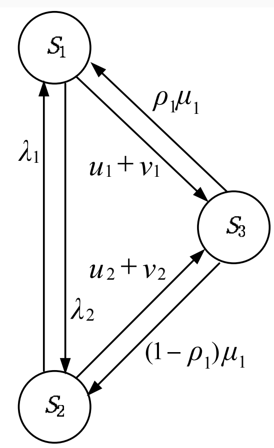


Figure 2. State transition diagram

The model in Figure2 only represents the active life of the devices, the infant mortality is ignored, and obsolescence ageing is not included due to an eventual substitution under PM. It is obvious that the process in Figure2 possesses Markov.

For ∀ t≥0, we use $x_i(t)$ to express the situated state of system $S(t)$ at time instant t, and then

$$x_i(t) = p\{S(t) = S_i\}, \quad i=1, 2, 3. \tag{1}$$

For convenient analysis, we firstly conduct the following definitions.

Deinition1. The availability degree $A_V(t)$ of the equipment is defined as the probability that it can function normally at the time instant t. According to the definition, the equipment can be thought to possess the availability if it is being in S_1 or S_2 at time instant t, and we can say it is unavailable at t if it is not being in S_1 or S_2 otherwise. And then, according to Figure2, we have

$$A_V(t) = x_1(t) + x_2(t) = 1^T \boldsymbol{x}(t) \tag{2}$$

where $\mathbf{1}^T = [1 \ \ 1]$, and $\boldsymbol{x}(t) = [x_1(t) \ \ x_2(t)]^T$.

Deinition2. The reliability degree of the equipment is defined as the probability that it can work normally within the scope of the time from zero to t. It does not allow failure, and so does not maintenance. According to the definition, the reliability degree $R(t)$ should have nothing to do with traversing what states before the device becomes the failure for the first time. In this system, since the natural failure or obsolescence due to ageing is eliminated, and moreover, based on consideration on assumption 4, we think although the preventive state S_3 could arouse machines halt, but not influence the equipments reliability. But when computing MTTF, the time that generator stays at the state S_3 should be deducted before it is possibly given an eventual substitution. Hence, according to [9], we have

$$\begin{aligned} R(t) &= P_r(T > t) = 1 - P_r(T \le t) = 1 - F(t) = 1 - \int_0^t f(t)\mathrm{d}t \\ &= \int_t^\infty f(t)\mathrm{d}t = F(\infty) - F(t) = R(t) - R(\infty) \\ &= x_1(t) + x_2(t) - x_1(\infty) - x_2(\infty) \end{aligned} \tag{3}$$

where $F(t)$ is the failure time distribution function, and whose probability density function(*pdf*) is $f(t)$ for $\forall t \ge 0$.

And then based on (3), MTTF can be defined by

$$\mathrm{MTTF} = \int_0^\infty R(t)\mathrm{d}t = 1^{\mathrm{T}} \int_0^\infty [\boldsymbol{x}(t) - \boldsymbol{x}(\infty)]\mathrm{d}t \tag{4}$$

where the definition $\mathbf{1}^{\mathrm{T}}$ and $\boldsymbol{x}(t)$ is same with definition 1, and $\boldsymbol{x}(\infty)=[x_1(\infty)\ \ x_2(\infty)]^{\mathrm{T}}$.

According to (4), the physical meaning of MTTF can be here understood as a mean time that the devices arrive at stable-state $x(\infty)$ from any non-zero initial state starting. On the scope of time interval, it is impossible for it to be given an eventually substitution even though PM exists. In other words, the devices can be substituted only when it is in stable-state.

III. RELIABILITY ANALYSIS

According to Fig. 2 and reliability mathematics theory [10, 11], and we then have the following Kolmogorov equation group.

$$\mathrm{d}x_1(t)/\mathrm{d}t = -(\lambda_2 + u_1 + v_1)\, x_1(t) + \lambda_1\, x_2(t) + \rho_1 \mu_1 x_3(t) \tag{5}$$

$$\mathrm{d}x_2(t)/\mathrm{d}t = \lambda_2\, x_1(t) - (\lambda_1 + u_2 + v_2)\, x_2(t) + (1 - \rho_1)\mu_1\, x_3(t) \tag{6}$$

$$\mathrm{d}x_3(t)/\mathrm{d}t = (u_1 + v_1)\, x_1(t) + (u_2 + v_2)\, x_2(t) - \mu_1 x_3(t) \tag{7}$$

The physical bounds of the transition rates satisfy

$$0\le\lambda_1\le\lambda_{1\max},\ 0\le\lambda_2\le\lambda_{2\max},\ 0\le v_1\le v_{1\max},\ 0\le v_2\le v_{2\max},\ 0\le u_1\le u_{1\max},\ 0\le u_2\le u_{2\max},$$

$$0\le\mu_1\le\mu_{1\max},\ 0\le\rho_1\le 1,\ \forall t\ge 0. \tag{8}$$

The system states are mutually exclusive, and therefore the following equation is bound to hold.

$$x_1(t)+x_2(t)+x_3(t)=1,\qquad \forall t\ge 0 \tag{9}$$

where $0\le x_1(t)\le 1$, $0\le x_2(t)\le 1$, $0\le x_3(t)\le 1$, $\forall t\ge 0$.

From (9), we can obtain

$$x_3(t)=1-[x_1(t)+x_2(t)] \tag{10}$$

Substituting (10) into (5), and (6), we obtain

$$\mathrm{d}x_1(t)/\mathrm{d}t=-(\lambda_2+\lambda_3+\rho_1\mu_1)\,x_1(t)+(\lambda_1-\rho_1\mu_1)\,x_2(t)+\rho_1\mu_1 \tag{11}$$

$$\mathrm{d}x_2(t)/\mathrm{d}t=[\lambda_2-(1-\rho_1)\,\mu_1]x_1(t)-(\lambda_1+\lambda_4+(1-\rho_1)\,\mu_1)\,x_2(t)+(1-\rho_1)\,\mu_1 \tag{12}$$

$$x_1(t)+x_2(t)\le 1,\qquad \forall t\ge 0 \tag{13}$$

where $\lambda_3=u_1+v_1$ and $\lambda_4=u_2+v_2$.

Writing (11) and (12) into matrix form, and then

$$\frac{\mathrm{d}\boldsymbol{x}(t)}{\mathrm{d}t}=\boldsymbol{A}\boldsymbol{x}(t)+\boldsymbol{\mu} \tag{14}$$

where

$$\boldsymbol{A}=\begin{bmatrix}-d_1 & a\\ b & -d_2\end{bmatrix},\ \boldsymbol{x}(t)=\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix},\ \boldsymbol{\mu}=\begin{bmatrix}\rho_1\mu_1\\ (1-\rho_1)\mu_1\end{bmatrix}$$

And

$$d_1=\lambda_2+\lambda_3+\rho_1\mu_1,\ a=\lambda_1-\rho_1\mu_1,\ d_2=\lambda_1+\lambda_4+(1-\rho_1)\,\mu_1,\ b=\lambda_2-(1-\rho_1)\,\mu_1.$$

From (8), we can known the system parameters satisfies $d_1>0$, $d_2>0$, $d_1d_2-ab>0$, and so that $\boldsymbol{A}$ is a stale matrix [12]. In addition, it is natural that (8) also shows that $d_1-b>0$, $d_2-a>0$, $d_1+a>0$ and $d_2+b>0$. Hence, $\boldsymbol{A}$ is also a strong diagonally dominant matrix. Thus, according to the ergodicity characteristics of homogeneous Markov process, the system (14) is bound to possess solely invariable steady-state solution [13, 14].

After Laplace(L) transformation on (14) we obtain

$$s\boldsymbol{x}(s)=\boldsymbol{A}\boldsymbol{x}(s)+\boldsymbol{x}(0)+\frac{\boldsymbol{\mu}}{s} \tag{15}$$

where $\boldsymbol{x}(0)=[x_1(0)\ \ x_2(0)]^{\mathrm{T}}$ is the initial state.

From (15), we obtain

$$\boldsymbol{x}(s)=\left[s\boldsymbol{I}-\boldsymbol{A}\right]^{-1}[\boldsymbol{x}(0)+\frac{\boldsymbol{\mu}}{s}] \tag{16}$$

And then

$$x_1(s)=\frac{(s+d_2)x_1(0)+ax_2(0)+(s+d_2)\frac{\rho_1\mu_1}{s}+a\frac{(1-\rho_1)\mu_1}{s}}{(s+d_1)(s+d_2)-ab} \tag{17}$$

$$x_2(s)=\frac{bx_1(0)+(s+d_1)x_2(0)+b\frac{\rho_1\mu_1}{s}+(s+d_1)\frac{(1-\rho_1)\mu_1}{s}}{(s+d_1)(s+d_2)-ab} \tag{18}$$

Theorem1. L transformation of system instantaneous availability degree can be expressed by

$$\boldsymbol{A}_{\mathrm{V}}(s)=1^{\mathrm{T}}\boldsymbol{x}(s)=1^{\mathrm{T}}\{\left[s\boldsymbol{I}-\boldsymbol{A}\right]^{-1}[\boldsymbol{x}(0)+\frac{\boldsymbol{\mu}}{s}]\} \tag{19}$$

where $\mathbf{1}^{\mathrm{T}}=[1\ \ 1]$, $\boldsymbol{x}(s)=[x_1(s)\ \ x_2(s)]^{\mathrm{T}}$.

Further, the steady-state availability exists and has nothing to do with the initial states.

$$A_{\mathrm{V}}=-1^{\mathrm{T}}\boldsymbol{A}^{-1}\boldsymbol{\mu}=\frac{(\lambda_1+\lambda_2)\mu_1+(1-\rho_1)\lambda_3\mu_1+\rho_1\lambda_4\mu_1}{\lambda_1\lambda_3++\lambda_2\lambda_4+\lambda_3\lambda_4+(\lambda_1+\lambda_2)\mu_1+(1-\rho_1)\lambda_3\mu_1+\rho_1\lambda_4\mu_1} \tag{20}$$

Proof. Conducting Laplace transformation on (2), and according to (16), we can have

$$A_{\mathrm{V}}(s)=x_1(s)+x_2(s)=1^{\mathrm{T}}\{\left[s\boldsymbol{I}-\boldsymbol{A}\right]^{-1}[\boldsymbol{x}(0)+\frac{\boldsymbol{\mu}}{s}]\}$$

Note that $(s\boldsymbol{I}\text{-}\boldsymbol{A})^{-1}$ is strictly true matrix, and so system steady-state availability is given by

$$A_{\mathrm{V}}=\lim_{s\to 0}sA_{\mathrm{V}}(s)=\lim_{s\to 0}[sx_1(s)+sx_2(s)]=1^{\mathrm{T}}\lim_{s\to 0}s(sI-A)^{-1}[\boldsymbol{x}(0)+\frac{\boldsymbol{\mu}}{s}]$$

Substituting the related values into the formula above, and (20) may be obtained immediately.

In fact, the steady-state probabilities being in S_1, and S_2 can be worked out as follows.

$$\boldsymbol{x}(\infty)=\lim_{s\to 0}s\boldsymbol{x}(s)=\lim_{s\to 0}s\left[s\boldsymbol{I}-\boldsymbol{A}\right]^{-1}[\boldsymbol{x}(0)+\frac{\boldsymbol{\mu}}{s}]=-\boldsymbol{A}^{-1}\boldsymbol{\mu} \tag{21}$$

Then

$$x_1(\infty)=\frac{\lambda_1\mu_1+\rho_1\lambda_4\mu_1}{\lambda_1\lambda_3++\lambda_2\lambda_4+\lambda_3\lambda_4+(\lambda_1+\lambda_2)\mu_1+(1-\rho_1)\lambda_3\mu_1+\rho_1\lambda_4\mu_1} \tag{22}$$

$$x_2(\infty)=\frac{\lambda_2\mu_1+(1-\rho_1)\lambda_3\mu_1}{\lambda_1\lambda_3++\lambda_2\lambda_4+\lambda_3\lambda_4+(\lambda_1+\lambda_2)\mu_1+(1-\rho_1)\lambda_3\mu_1+\rho_1\lambda_4\mu_1} \tag{23}$$

The system steady-state availability can also be obtained by $A_V=x_1(\infty)+x_2(\infty)$, we will get similar result with (20) thought substituting (22) and (23) into it.

Theorem2. L transformation of system instantaneous reliability degree can be expressed by

$$R(s)=1^{\mathrm{T}}\{[(s\boldsymbol{I}-\boldsymbol{A})^{-1}[\boldsymbol{x}(0)-\boldsymbol{x}(\infty)]\} \tag{24}$$

Further

$$R(t)=1^{\mathrm{T}}\mathrm{e}^{\boldsymbol{A}t}[\boldsymbol{x}(0)-\boldsymbol{x}(\infty)] \tag{25}$$

Proof. Performing Laplace transformation on (3), and then

$$\begin{aligned}R(s)&=x_1(s)+x_2(s)-\frac{x_1(\infty)}{s}-\frac{x_2(\infty)}{s}=1^{\mathrm{T}}[\boldsymbol{x}(s)+\frac{\boldsymbol{A}^{-1}\boldsymbol{\mu}}{s}]\\&=1^{\mathrm{T}}[(s\boldsymbol{I}-\boldsymbol{A})^{-1}(\boldsymbol{x}(0)+\frac{\boldsymbol{\mu}}{s})+\frac{\boldsymbol{A}^{-1}\boldsymbol{\mu}}{s}]=1^{\mathrm{T}}[(s\boldsymbol{I}-\boldsymbol{A})^{-1}(\boldsymbol{x}(0)+\boldsymbol{A}^{-1}\boldsymbol{\mu})]\\&=1^{\mathrm{T}}[(s\boldsymbol{I}-\boldsymbol{A})^{-1}(\boldsymbol{x}(0)-\boldsymbol{x}(\infty)]\end{aligned}$$

And so, we have

$$\begin{aligned}R(t)&=L^{-1}[R(s)]=1^{\mathrm{T}}L^{-1}[(s\boldsymbol{I}-\boldsymbol{A})^{-1}(\boldsymbol{x}(0)-\boldsymbol{x}(\infty)]\\&=1^{\mathrm{T}}e^{\boldsymbol{A}t}[\boldsymbol{x}(0)-\boldsymbol{x}(\infty)]\end{aligned}$$

End.

In the above equation, $R(t)=0$ only when $t\to\infty$, which is reliability definition in traditional meaning, or $\boldsymbol{x}(0)=\boldsymbol{x}(\infty)$, which means if system locates at $\boldsymbol{x}(\infty)$ from the start, and then its reliability is zero.

Theorem3. System availability $A_V(t)$ can be expressed by $R(t)$

$$A_V(t)=R(t)+1^{\mathrm{T}}\boldsymbol{x}(\infty) \tag{26}$$

And so

$$A_V(t)\geq R(t) \tag{27}$$

The equal sign holds in (27) only when $\boldsymbol{x}(\infty)=0$.

Proof. According to (19), and note that (21) and (25), we have

$$\begin{aligned}A_V(t)&=L^{-1}[A_V(s)]=1^{\mathrm{T}}L^{-1}\{\left[s\boldsymbol{I}-\boldsymbol{A}\right]^{-1}[\boldsymbol{x}(0)+\frac{\boldsymbol{\mu}}{s}]\}=1^{\mathrm{T}}[e^{\boldsymbol{A}t}[\boldsymbol{x}(0)-\boldsymbol{x}(\infty)]+\boldsymbol{x}(\infty)]\\&=R(t)+1^{\mathrm{T}}\boldsymbol{x}(\infty)\end{aligned}$$

Clearly, the formula (27) holds, and $A_V(t)\geq R(t)$.

End.

Seen from Theorem 2 and Theorem 3, the physical meaning of $R(t)$ is the required time of a motion track starting from arbitrary non-zero state to the stable state determined by state transition matrix $\boldsymbol{A}$. The terminated condition is determined by the expected stability state probability.

Theorem4. The system MTTF can be presented by

$$\begin{aligned}\text{MTTF} &= \frac{(d_2+b)(x_1(0)-x_1(\infty))+(d_1+a)(x_2(0)-x_2(\infty))}{d_1 d_2 - ab} \\ &= \text{MTTF}_{11} + \text{MTTF}_{21}\end{aligned} \tag{28}$$

where

$$\text{MTTF}_{11} = \frac{d_2(x_1(0)-x_1(\infty))+a(x_2(0)-x_2(\infty))}{d_1 d_2 - ab}$$

$$\text{MTTF}_{21} = \frac{b(x_1(0)-x_1(\infty))+d_1(x_2(0)-x_2(\infty))}{d_1 d_2 - ab}$$

Proof. According to (4), system MTTF can be calculated by

$$\text{MTTF} = \int_0^\infty R(t)\mathrm{d}t = 1^{\mathrm{T}}\int_0^\infty e^{\boldsymbol{A}t}(\boldsymbol{x}(t)-\boldsymbol{x}(\infty))\mathrm{d}t = -1^{\mathrm{T}}\boldsymbol{A}^{-1}[\boldsymbol{x}(0)-\boldsymbol{x}(\infty)]$$

And then, (28) can be obtained, immediately.

Differentiating on (11) with respect to *t,* and then substituting (12) into it we obtain

$$\frac{\mathrm{d}^2 x_1(t)}{\mathrm{d}t^2} + (d_1+d_2)\frac{\mathrm{d}x_1(t)}{\mathrm{d}t} + (d_1 d_2 - ab)x_1(t) = 0$$

Let p_1 and p_2 are two characteristic roots of its characteristic equation, and then

$$p_1 = -\frac{d_1+d_2}{2} - \frac{\sqrt{\Delta}}{2}$$

$$p_2 = -\frac{d_1+d_2}{2} + \frac{\sqrt{\Delta}}{2}$$

where

$$\sqrt{\Delta} = \sqrt{(d_1-d_2)^2+4ab} = p_2 - p_1$$

As $\Delta>0$, then

$$x_1(t) = c_1 e^{-p_1 t} + c_2 e^{-p_2 t}$$

where c_1 and c_2 are the two constants.

Differentiating on $x_1(t)$ with respect to *t,* and then we have

$$\frac{\mathrm{d}x_1(t)}{\mathrm{d}t} = -c_1 p_1 e^{-p_1 t} - c_2 p_2 e^{-p_2 t}$$

Note that (11), let t=0, we have

$$-c_1 p_1 - c_2 p_2 = -d_1 x_1(0) + a x_2(0)$$

Moreover

$$x_1(0) = c_1 + c_2$$

According to the two equations above, we can easily obtain c_1 and c_2, and then

$$x_1(t) = \frac{1}{p_2 - p_1}\{[(p_2 - d_1)x_1(0) + a x_2(0)]e^{-p_1 t} + [(d_1 - p_1)x_1(0) - a x_2(0)]e^{-p_2 t}\}$$

Likewise, we have

$$x_2(t) = \frac{1}{p_2 - p_1}\{[(p_2 - d_2)x_2(0) + b x_1(0)]e^{-p_1 t} + [(d_2 - p_1)x_2(0) - b x_1(0)]e^{-p_2 t}\}$$

And so it is easy that we obtain

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \frac{1}{p_2 - p_1}\begin{bmatrix} (p_2 - d_1)e^{-p_1 t} + (d_1 - p_1)e^{-p_2 t} & a(e^{-p_1 t} - e^{-p_2 t}) \\ b(e^{-p_1 t} - e^{-p_2 t}) & (p_2 - d_2)e^{-p_1 t} + (d_2 - p_1)e^{-p_2 t} \end{bmatrix}\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix}$$

And then based on the following equations

$$\mathrm{MTTF}_{11} = \int_0^\infty [x_1(t) - x_1(\infty)]\mathrm{d}t$$

$$\mathrm{MTTF}_{21} = \int_0^\infty [x_2(t) - x_2(\infty)]\mathrm{d}t$$

Theorem 4 can be proved, immediately. For $\Delta \le 0$, we may prove in the same way.

End.

Theorem5. Let $A_{\mathrm{V1}}(t)$ be system availability, and $R_1(t)$ be system reliability, and MTTF_1 be system mean time to failure without consideration on preventive maintenance checking, and then we have

$$A_{\mathrm{V1}} < A_{\mathrm{V}} \tag{29}$$

$$R_1(t) > R(t) \tag{30}$$

$$\mathrm{MTTF}_1 > \mathrm{MTTF} \tag{31}$$

Proof. If not considering preventive maintenance, we need to eliminate the arcs relevant to preventive maintenance practice activities. For this reason, we let the checking rate and time related to it are zero in system parameters matrix $\boldsymbol{A}$, i.e., $u_1 = u_2 = 0$, $\mu_1 = 0$. Then the $\boldsymbol{A}$ becomes

$$\boldsymbol{A}_{11}=\begin{bmatrix}-d_{11} & a_1\\ b_1 & -d_{22}\end{bmatrix}$$

wherein $d_{11}=\lambda_2+v_1$, $d_{22}=\lambda_1+v_2$, $a_1=\lambda_1$, $b_1=\lambda_2$, clearly, $d_{11}\,d_{22}-a_1b_1>0$.

Clearly, $\boldsymbol{A}\le\boldsymbol{A}_{11}$. Then system state equation can be written as

$$\frac{\mathrm{d}\boldsymbol{x}(t)}{\mathrm{d}t}=\boldsymbol{A}_{11}\boldsymbol{x}(t) \tag{32}$$

Then, we obtain

$$\boldsymbol{x}(t)=e^{\boldsymbol{A}_{11}t}\boldsymbol{x}(0)$$

Thus system state availability is

$$A_{\mathrm{V1}}(t)=x_1(t)+x_2(t)=1^{\mathrm{T}}\boldsymbol{x}(t)=1^{\mathrm{T}}e^{\boldsymbol{A}_{11}t}\boldsymbol{x}(0)$$

It is easy to know that $\boldsymbol{A}_{11}$ is Hurwitz stable matrix [15], and therefore system stable-state availability is

$$A_{\mathrm{V1}}=\lim_{t\to\infty}A_{\mathrm{V1}}(t)=\lim_{t\to\infty}1^{\mathrm{T}}\boldsymbol{x}(t)=\lim_{t\to\infty}1^{\mathrm{T}}e^{\boldsymbol{A}_{11}t}\boldsymbol{x}(0)=0$$

Clearly, system steady-state availability

$$\boldsymbol{A}_{\mathrm{V1}}=0\le\boldsymbol{A}_{\mathrm{V}}=-1^{\mathrm{T}}\boldsymbol{A}_{11}^{-1}\boldsymbol{\mu}>0$$

According to definition 2, the system reliability can be solved by

$$R_1(t)=x_1(t)+x_2(t)-x_1(\infty)-x_2(\infty)=1^{\mathrm{T}}[\boldsymbol{x}(t)-\boldsymbol{x}(\infty)]=1^{\mathrm{T}}e^{\boldsymbol{A}_{11}t}\boldsymbol{x}(0)$$

Clearly, at the time $A_{\mathrm{V1}}(t)=R_1(t)$.

According to [16], we have

$$e^{\boldsymbol{A}_{11}t}\approx(\boldsymbol{I}-\boldsymbol{A}_{11}\frac{t}{n})^{-n}=[(\boldsymbol{I}-\boldsymbol{A}_{11}\frac{t}{n})^{-1}]^n$$

$$e^{\boldsymbol{A}t}\approx(\boldsymbol{I}-\boldsymbol{A}\frac{t}{n})^{-n}=[(\boldsymbol{I}-\boldsymbol{A}\frac{t}{n})^{-1}]^n$$

And so,

$$(\boldsymbol{I}-\boldsymbol{A}_{11}\frac{t}{n})^{-1}\ge(\boldsymbol{I}-\boldsymbol{A}\frac{t}{n})^{-1}$$

Then

$$\frac{R_1(t)}{R(t)}=\frac{1^{\mathrm{T}}e^{A_{11}t}\boldsymbol{x}(0)}{1^{\mathrm{T}}e^{At}[\boldsymbol{x}(0)-\boldsymbol{x}(\infty)]}=\frac{1^{\mathrm{T}}e^{A_{11}t}[\boldsymbol{x}(0)-\boldsymbol{x}(\infty)]+1^{\mathrm{T}}e^{A_{11}t}\boldsymbol{x}(\infty)}{1^{\mathrm{T}}e^{At}[\boldsymbol{x}(0)-\boldsymbol{x}(\infty)]}\geq\frac{1^{\mathrm{T}}e^{A_{11}t}[\boldsymbol{x}(0)-\boldsymbol{x}(\infty)]}{1^{\mathrm{T}}e^{At}[\boldsymbol{x}(0)-\boldsymbol{x}(\infty)]}$$

$$\approx\frac{1^{\mathrm{T}}[(I-A_{11}\frac{t}{n})^{-1}]^n[\boldsymbol{x}(0)-\boldsymbol{x}(\infty)]}{1^{\mathrm{T}}[(I-A\frac{t}{n})^{-1}]^n[\boldsymbol{x}(0)-\boldsymbol{x}(\infty)]}\geq 1$$

And then

$$R_1(t)\geq R(t)$$

For MTTF_1, we have

$$\mathrm{MTTF}_1=\int_0^{\infty}R_1(t)\mathrm{d}t=1^{\mathrm{T}}\int_0^{\infty}e^{A_{11}t}[\boldsymbol{x}(0)-\boldsymbol{x}(\infty)]\mathrm{d}t=-1^{\mathrm{T}}\boldsymbol{A}_{11}^{-1}\boldsymbol{x}(0)$$

$$\frac{\mathrm{MTTF}_1}{\mathrm{MTTF}}=\frac{-1^{\mathrm{T}}\boldsymbol{A}_{11}^{-1}\boldsymbol{x}(0)}{-1^{\mathrm{T}}\boldsymbol{A}^{-1}[\boldsymbol{x}(0)-\boldsymbol{x}(\infty)]}=\frac{-1^{\mathrm{T}}\boldsymbol{A}_{11}^{-1}[\boldsymbol{x}(0)-\boldsymbol{x}(\infty)]-1^{\mathrm{T}}\boldsymbol{A}_{11}^{-1}\boldsymbol{x}(\infty)}{-1^{\mathrm{T}}\boldsymbol{A}^{-1}[\boldsymbol{x}(0)-\boldsymbol{x}(\infty)]}\geq\frac{-1^{\mathrm{T}}\boldsymbol{A}_{11}^{-1}[\boldsymbol{x}(0)-\boldsymbol{x}(\infty)]}{-1^{\mathrm{T}}\boldsymbol{A}^{-1}[\boldsymbol{x}(0)-\boldsymbol{x}(\infty)]}\geq 1$$

And therefore, we have

$$\mathrm{MTTF}_1\geq\mathrm{MTTF}$$

The equal sign in the above formula holds only when $u_1= u_2=0$, $\mu_1=0$.

Example 1: A wind power enterprise requires 500 sets of devices operating to satisfy the market demands at ordinary times. To ensure the stability of the production process, the enterprise implements the preventive maintenance for these cells, and arranges a checking-repairing center and spare parts warehouse. To verify the theorem above, a numerical computation is made based on the supplied real data of the enterprise below.

The transfer rates per month of the parameters are described by λ_1=1.5625, λ_2=1.8750, λ_3=0.9375, λ_4=0.9375, v_1=0.5210, v_2=0.7625, u_1=0.4165, u_2=0.1750, μ_1=1.8750, and ρ_1=2/3. In passive maintenance model, order u_1=u_2=0. Let $x_1(0)$ equal 0.5, and $x_2(0)$ equal 0.5, and $x_3(0)$ equal zero, and then we can obtain the following results.

According to Theorem 1, we easily work out the system steady-state probability and the system steady-state availability as follows.

$$\boldsymbol{x}(\infty)=-\boldsymbol{A}^{-1}\boldsymbol{\mu}=\begin{bmatrix}0.3333\\0.3333\end{bmatrix}$$

$$A_{\mathrm{V}}=-1^{\mathrm{T}}\boldsymbol{A}^{-1}\boldsymbol{\mu}=0.6667$$

From Theorem 3, we can obtain

$$A_V(t) = R(t) + 1^T \boldsymbol{x}(\infty) = R(t) + 0.6667$$

So, we can easily find

$$A_V(t) \geq R(t)$$

From Theorem 4 we have

$$\text{MTTF} = -1^T \boldsymbol{A}^{-1}[x(0) - x(\infty)] = 0.1185 \text{ month}$$

$$\text{MTTF}_{11} = \frac{d_2(x_1(0) - x_1(\infty)) + a(x_2(0) - x_2(\infty))}{d_1 d_2 - ab} = 0.0466 \text{ month}$$

$$\text{MTTF}_{21} = \frac{b(x_1(0) - x_1(\infty)) + d_1(x_2(0) - x_2(\infty))}{d_1 d_2 - ab} = 0.0720 \text{ month}$$

If not considering preventive maintenance, from Theorem 5, we have

$$\text{MTTF}_1 = -1^T \boldsymbol{A}_{11}^{-1} \boldsymbol{x}(0) = 1.5446 \text{ month}$$

Clearly, we have

$$\text{MTTF}_1 > \text{MTTF}$$

Seen from the above analysis, Theorem 3 and Theorem 5 are effective.

IV. CONTROL STRATEGY DESIGN

By a careful investigations and analysis [17], it is suitable that the checking rates are used to act as control variables since it is less expensive. The physical meaning of checking rate is the reciprocal of mean time between checks, by control it some scheduling plans may be arranged more reasonably.

When the checking rates are time-varying, the system (14) can be written as

$$\frac{\mathrm{d}\boldsymbol{x}(t)}{\mathrm{d}t} = \boldsymbol{A}(t)\boldsymbol{x}(t) + \boldsymbol{\mu} \tag{33}$$

where

$$\boldsymbol{A}(t) = \begin{bmatrix} -d_1 & a \\ b & -d_2 \end{bmatrix}, \ \boldsymbol{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}, \ \boldsymbol{\mu} = \begin{bmatrix} \rho_1 \mu_1 \\ (1-\rho_1)\mu_1 \end{bmatrix}$$

$d_1=\lambda_2+ u_1(t)+v_1+\rho_1\mu_1$, $a=\lambda_1-\rho_1\mu_1$, $d_2=\lambda_1+ u_2(t)+v_2+(1-\rho_1)\mu_1$, $b=\lambda_2-(1-\rho_1)\mu_1$.

Based on (33), we separate the control variables $\boldsymbol{U}(t)$ from $\boldsymbol{A}(t)$, and then

$$\frac{\mathrm{d}\boldsymbol{x}(t)}{\mathrm{d}t} = \boldsymbol{A}_1\boldsymbol{x}(t) - \boldsymbol{U}(t)\boldsymbol{x}(t) + \boldsymbol{\mu} \tag{34}$$

where

$$\boldsymbol{A}_1(t) = \begin{bmatrix} -d_1^{'} & a \\ b & -d_2^{'} \end{bmatrix},\ \boldsymbol{U}(t) = \begin{bmatrix} u_1(t) & 0 \\ 0 & u_2(t) \end{bmatrix}$$

And

$$d_1^{'} = \lambda_2 + v_1 + \rho_1\mu_1,\quad d_2^{'} = \lambda_1 + v_2 + (1-\rho_1)\mu_1 .$$

Note that

$$\boldsymbol{U}(t)\boldsymbol{x}(t) = \begin{bmatrix} u_1(t) & 0 \\ 0 & u_2(t) \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} x_1(t) & 0 \\ 0 & x_2(t) \end{bmatrix}\begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} = \boldsymbol{X}(t)\boldsymbol{u}(t)$$

where

$$\boldsymbol{X}(t) = \begin{bmatrix} x_1(t) & 0 \\ 0 & x_2(t) \end{bmatrix},\ \boldsymbol{u}(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}$$

Hence, (34) can be rewritten as

$$\frac{\mathrm{d}\boldsymbol{x}(t)}{\mathrm{d}t} = \boldsymbol{A}_1\boldsymbol{x}(t) - \boldsymbol{X}(t)\boldsymbol{u}(t) + \boldsymbol{\mu} \tag{35}$$

To prevent $\boldsymbol{u}(t)$ unbounded when x_1 or x_2 tends to zero, we divide the area of $x_1(t)$ and $x_2(t)$ into four sets presented in Fig. 3, where $0<\gamma<1$.

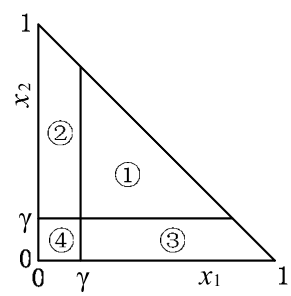


Figure 3. Valid areas of control variable

Theorem6. In the area ①, $\gamma \le x_1 \le 1$, $\gamma \le x_2 \le 1$, and $x_1+x_2 \le 1$, we can conduct the following control law

$$\boldsymbol{u}(t) = \begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix} = [\boldsymbol{X}(t)]^{-1}[(\boldsymbol{A}_1 + \boldsymbol{K})\boldsymbol{x}(t) + \boldsymbol{\mu} - \boldsymbol{K}\overline{\boldsymbol{x}}] \tag{36}$$

to make system (36) to arrive at the expected stable-state aim

$$A_{\mathrm{v}} = x_1(\infty) + x_2(\infty) = \bar{x}_1 + \bar{x}_2$$

where

$$\boldsymbol{K} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}, \quad \bar{\boldsymbol{x}} = \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix}$$

wherein $\bar{\boldsymbol{x}}$ is the expected steady-state probability in the area ①, $\boldsymbol{K}$ matches that $k_{11}>0$, $k_{22}>0$, $k_{11}k_{22}-k_{12}k_{21}>0$. At the moment, $-\boldsymbol{K}$ asymptotically stable. Moreover, to ensure the input $\boldsymbol{u}(t)\geq 0$, $\boldsymbol{A}_1+\boldsymbol{K}\geq 0$, $\boldsymbol{\mu}-\boldsymbol{K}\bar{\boldsymbol{x}}\geq 0$.

Proof. Known from Theorem 1, when $\boldsymbol{A}(t)$ is time-invariant, the system possesses the desired stable- state probability. Hence, the expected state equation should be

$$\frac{\mathrm{d}\boldsymbol{x}_{\mathrm{m1}}(t)}{\mathrm{d}t} = \boldsymbol{A}_{\mathrm{m1}}\boldsymbol{x}_{\mathrm{m1}}(t) + \boldsymbol{\mu}_{\mathrm{m1}}$$

And so, if $\boldsymbol{A}_{\mathrm{m1}}$ asymptotically stable, we then have

$$E[\boldsymbol{x}_{\mathrm{m1}}(t)] = \boldsymbol{x}_{\mathrm{m1}}(\infty) = \bar{\boldsymbol{x}} = (\bar{x}_1, \bar{x}_2)^{\mathrm{T}} = \bar{\boldsymbol{x}}_{\mathrm{m1}}$$

Let $\boldsymbol{A}_{\mathrm{m1}}=-\boldsymbol{K}$, and $\boldsymbol{\mu}_{\mathrm{m1}}=\boldsymbol{K}\bar{x}$, and then the expected state equation becomes

$$\frac{\mathrm{d}\boldsymbol{x}_{\mathrm{m1}}(t)}{\mathrm{d}t} = -\boldsymbol{K}\boldsymbol{x}_{\mathrm{m1}}(t) + \boldsymbol{K}\bar{\boldsymbol{x}}$$

Let the above formula be subtracted by (35), we have

$$\frac{\mathrm{d}[\boldsymbol{x}(t) - \boldsymbol{x}_{\mathrm{m1}}(t)]}{\mathrm{d}t} = \boldsymbol{A}_1\boldsymbol{x}(t) + \boldsymbol{K}\boldsymbol{x}_{\mathrm{m1}}(t) - \boldsymbol{X}(t)\boldsymbol{u}(t) + \boldsymbol{\mu} - \boldsymbol{K}\bar{x}$$

In theoretical, if $\boldsymbol{x}(t)$ can fully track the $\boldsymbol{x}_{\mathrm{m1}}(t)$ by controlling $\boldsymbol{u}(t)$, and then the left side of the above equation would be zero, and thus, we can have $\boldsymbol{x}(t)= \boldsymbol{x}_{\mathrm{m1}}(t)$ under the same initial states. Hence, (36) can be derived out, immediately.

End.

In the area ②, $0\leq x_1\leq\gamma$, $\gamma\leq x_2\leq 1$, and $x_1+ x_2 \leq 1$, To ensure that $u_1(t)$ is bounded in (36), we let $u_1(t)$ be constant, but $u_2(t)$ remain unchanged. To solve the $u_1(t)$, let the first equation equal zero in (35), thus we have

$$\frac{\mathrm{d}x_1(t)}{\mathrm{d}t}\Big|_{t\to\infty} = 0$$

Then, the control law can be revised by

$$\begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix} = \begin{pmatrix} \dfrac{\rho_1\mu_1 + a\bar{x}_2}{\bar{x}_1} - d_1' \\ \dfrac{(k_{21}+b)x_1 + (k_{22}-d_2')x_2 + [(1-\rho_1)\mu_1 - k_{21}\bar{x}_1 - k_{22}\bar{x}_2)]}{x_2} \end{pmatrix} \tag{37}$$

where the meaning of $\bar{\boldsymbol{x}}$ is analogous to theorem 6, and the positive definite conditions of input $\boldsymbol{u}(t)$ are met naturally according to theorem 6. At the moment, the dynamic equation of system can be corrected by

$$\frac{\mathrm{d}\boldsymbol{x}(t)}{\mathrm{d}t} = \boldsymbol{A}_{\mathrm{m}2}\boldsymbol{x}(t) + \boldsymbol{\mu}_{\mathrm{m}2}$$

where

$$\boldsymbol{A}_{\mathrm{m}2} = \begin{pmatrix} -\dfrac{\rho_1\mu_1 + a\bar{x}_2}{\bar{x}_1} & a \\ -k_{21} & -k_{22} \end{pmatrix},\ \boldsymbol{\mu}_{\mathrm{m}2} = \begin{pmatrix} \rho_1\mu_1 \\ k_{21}\bar{x}_1 + k_{22}\bar{x}_2 \end{pmatrix}$$

To ensure the asymptotical stability of the system, the condition $(\rho_1\mu_1 + a\bar{x}_2)k_{22} + a\bar{x}_1 k_{21} > 0$ must be satisfied. The steady-state solution can be expressed by

$$\boldsymbol{x}(\infty) = -\boldsymbol{A}_{\mathrm{m}2}^{-1}\boldsymbol{\mu}_{\mathrm{m}2} = \bar{\boldsymbol{x}} = \begin{bmatrix} \bar{x}_1 & \bar{x}_2 \end{bmatrix}^{\mathrm{T}} = \bar{\boldsymbol{x}}_{\mathrm{m}2}$$

Likewise, in the area ③, $\gamma \le x_1 \le 1$, $0 \le x_2 \le \gamma$, and $x_1 + x_2 \le 1$, at the time, it is possible for $u_2(t)$ to become unbounded in (36), and so we let $u_2(t)$ be constant, but $u_1(t)$ still remain unchanged. Similarly, To solve the $u_2(t)$, let the second equation equal zero in (35), and then, we have

$$\frac{\mathrm{d}x_2(t)}{\mathrm{d}t}\Big|_{t\to\infty} = 0$$

Then, the control law can be revised by

$$\begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix} = \begin{pmatrix} \dfrac{(k_{11}-d)x_1 + (k_{12}+a)x_2 + ((1-\rho_1)\mu_1 - k_{11}\bar{x}_1 - k_{12}\bar{x}_2)}{x_2} \\ \dfrac{(1-\rho_1)\mu_1 + b\bar{x}_1}{\bar{x}_2} - d_2' \end{pmatrix} \tag{38}$$

where the meaning of $\bar{\boldsymbol{x}}$ is analogous to theorem 6, and the positive definite conditions of input $\boldsymbol{u}(t)$ are met naturally according to theorem 6. At the moment, the dynamic equation of system can be corrected by

$$\frac{\mathrm{d}\boldsymbol{x}(t)}{\mathrm{d}t} = \boldsymbol{A}_{\mathrm{m}3}\boldsymbol{x}(t) + \boldsymbol{\mu}_{\mathrm{m}3}$$

where

$$\boldsymbol{A}_{\mathrm{m3}}=\begin{bmatrix} -k_{11} & -k_{12} \\ b & -\dfrac{(1-\rho_1)\mu_1+b\bar{x}_1}{\bar{x}_2} \end{bmatrix},\ \boldsymbol{\mu}_{\mathrm{m3}}=\begin{bmatrix} k_{11}\bar{x}_1+k_{12}\bar{x}_2 \\ (1-\rho_1)\mu_1 \end{bmatrix}$$

To ensure the asymptotical stability of the system, the condition $[(1-\rho)\mu_1+b\bar{x}_1]k_{11}+b\bar{x}_2k_{12}>0$ must be satisfied. The steady-state solution can be expressed by

$$\boldsymbol{x}(\infty)=-\boldsymbol{A}_{\mathrm{m3}}^{-1}\boldsymbol{\mu}_{\mathrm{m3}}=\bar{\boldsymbol{x}}=\begin{bmatrix}\bar{x}_1 & \bar{x}_2\end{bmatrix}^{\mathrm{T}}=\bar{\boldsymbol{x}}_{\mathrm{m3}}$$

Eventually, in the area ④, $\gamma\le x_1\le 1$, $0\le x_2\le\gamma$, the control law can be updated by

$$\begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix}=\begin{pmatrix} \dfrac{\rho_1\mu_1+a\bar{\boldsymbol{x}}_2}{\bar{\boldsymbol{x}}_1}-d_1' \\ \dfrac{(1-\rho_1)\mu_1+b\bar{\boldsymbol{x}}_1}{\bar{\boldsymbol{x}}_2}-d_2' \end{pmatrix} \tag{39}$$

And then, the dynamic behavior of the system becomes

$$\frac{\mathrm{d}\boldsymbol{x}(t)}{\mathrm{d}t}=\boldsymbol{A}_{\mathrm{m4}}\boldsymbol{x}(t)+\boldsymbol{\mu}_{\mathrm{m4}}$$

where

$$\boldsymbol{A}_{\mathrm{m4}}=\begin{bmatrix} -\dfrac{\rho_1\mu_1+a\bar{x}_2}{\bar{x}_1} & a \\ b & -\dfrac{(1-\rho_1)\mu_1+b\bar{x}_1}{\bar{x}_2} \end{bmatrix},\ \boldsymbol{\mu}_{\mathrm{m4}}=\begin{bmatrix} \rho_1\mu_1 \\ (1-\rho_1)\mu_1 \end{bmatrix}$$

To ensure the asymptotical stability, the condition $(\rho_1\mu_1+a\bar{x}_2)((1-\rho_1)\mu_1+b\bar{x}_1)-ab\bar{x}_1\bar{x}_2>0$ must be satisfied. The steady-state solution can be expressed by

$$\boldsymbol{x}(\infty)=-\boldsymbol{A}_{\mathrm{m4}}^{-1}\boldsymbol{\mu}_{\mathrm{m4}}=\bar{\boldsymbol{x}}=\bar{\boldsymbol{x}}_{\mathrm{m4}}$$

Theorem7. Under the role of control law (36) to (39), the motion of the controlled system tends to $\bar{x}$ starting from arbitrary initial location. In addition, the trajectories starting from the region ① never leave such set, and those starting from the region ② leave such set to enter in region ①, and those starting from the region ③ leave such set to enter in region ①, and those starting from the region ④ leave such set to enter in set ①, and set ②, and set ③. In addition, when $t\to\infty$, $\boldsymbol{x}(t)$ converges to $\bar{\boldsymbol{x}}$ according to exponential rate.

Proof. The state equation of the four systems above can be summarized as

$$\frac{\mathrm{d}\boldsymbol{x}(t)}{\mathrm{d}t} = A_{\mathrm{m}i}\boldsymbol{x}(t) + \mu_{\mathrm{m}i}, \qquad i = 1,2,3,4\,.$$

Solving this equation, we obtain

$$\boldsymbol{x}(t) = e^{\boldsymbol{A}_{\mathrm{m}i}t}x(0) + \int_0^t e^{\boldsymbol{A}_{\mathrm{m}i}(t-\tau)}\boldsymbol{\mu}_{\mathrm{m}i}d\tau$$

Known from the former analysis, the $\boldsymbol{A}_{\mathrm{m}i}$ is a steady matrix, and so we have

$$x(t) = e^{A_{\mathrm{m}i}t}[x(0) + A_{\mathrm{m}i}{}^{-1}\mu_{\mathrm{m}i}] - A_{\mathrm{m}i}{}^{-1}\mu_{\mathrm{m}i}$$

As t tends to infinity, we have

$$\boldsymbol{x}(\infty) = -\boldsymbol{A}_{\mathrm{m}i}{}^{-1}\boldsymbol{\mu}_{\mathrm{m}i}$$

Then

$$\boldsymbol{x}(t) = e^{\boldsymbol{A}_{\mathrm{m}i}t}[\boldsymbol{x}(0) - \boldsymbol{x}(\infty)] + \boldsymbol{x}(\infty)$$

Then

$$\boldsymbol{x}(t) - \boldsymbol{x}(\infty) = e^{\boldsymbol{A}_{\mathrm{m}i}t}[\boldsymbol{x}(0) - \boldsymbol{x}(\infty)] \tag{40}$$

And therefore,

$$\begin{aligned}\left\|\boldsymbol{x}(t) - \boldsymbol{x}(\infty)\right\| &= \left\|e^{\boldsymbol{A}_{\mathrm{m}i}t}[\boldsymbol{x}(0) - \boldsymbol{x}(\infty)]\right\| \le \| e^{\boldsymbol{A}_{\mathrm{m}i}t} \| \| \boldsymbol{x}(0) - \boldsymbol{x}(\infty) \| \\ &\approx \| (I + A_{\mathrm{m}_i}\frac{t}{n})^n \| \| \boldsymbol{x}(0) - \boldsymbol{x}(\infty) \| \\ &\le \| (I + A_{\mathrm{m}_i}\frac{t}{n}) \|^n \| \boldsymbol{x}(0) - \boldsymbol{x}(\infty) \| \\ &\le \left\|\boldsymbol{x}(0) - \boldsymbol{x}(\infty)\right\|\end{aligned}$$

where n is a larger sufficient, and the approximation method on $\exp(\boldsymbol{A}_{\mathrm{m}i}t)$ can be seen in [16], and ||•|| is the matrix operator norm. In addition, the desired value $\boldsymbol{x}(\infty)$ generally locates at set ①. Clearly, the motion of $\boldsymbol{x}(t)$ in area (13) tends to $\boldsymbol{x}(\infty)$ starting from $\boldsymbol{x}(0)$ in exponential law convergence. Let $\overline{\boldsymbol{x}} = \boldsymbol{x}(\infty)$, Theorem7 then holds. End

Theorem8. Under the condition of starting from the region ① to ④, and respectively, the role of control rules (36) to (39), the MTTF of the system can be calculated as follows.

1) If the system starts from the area ①, and then

$$\mathrm{MTTF}_1[x_1(0), x_2(0)] = \int_0^\infty R(t)dt = 1^{\mathrm{T}}\boldsymbol{K}^{-1}[x(0) - \bar{x}] = T_1[x_1(0), x_2(0)]$$

2) If the system starts from the area ②, and then

$$\mathrm{MTTF}_2[x_1(0), x_2(0)] = T_2[x_1(0), x_2(0)]$$

3) If the system starts from the area ③, and then

$$\mathrm{MTTF}_3[x_1(0), x_2(0)] = T_3[x_1(0), x_2(0)]$$

4) If the system starts from the area ④, and then

$$\mathrm{MTTF}_4[x_1(0), x_2(0)] = \begin{cases} T_{41}[x_1(0), x_2(0)] & if \quad \bar{t}_{41} < \bar{t}_{42} \\ \bar{t}_4 + T_1(\gamma, \gamma) & if \quad \bar{t}_{41} = \bar{t}_{42} = \bar{t}_4 \\ T_{42}[x_1(0), x_2(0)] & if \quad \bar{t}_{41} > \bar{t}_{42} \end{cases}$$

where

$$T_1(\xi, \eta) = \frac{(k_{22} - k_{21})(\xi - \bar{x}_1) + (k_{11} - k_{12})(\eta - \bar{x}_2)}{k_{11}k_{22} - k_{12}k_{21}}$$

$$T_2(\xi, \eta) = t_2(\xi, \eta) + T_1(\gamma, x_{22}(t_2(\xi, \eta), \xi, \eta))$$

$$T_3(\xi, \eta) = t_3(\xi, \eta) + T_1(x_{13}(t_3(\xi, \eta), \xi, \eta), \gamma)$$

$$T_{41}(\xi, \eta) = t_{41}(\xi, \eta) + T_1(\gamma, x_{22}(t_{41}(\xi, \eta), \xi, \eta))$$

$$T_{42}(\xi, \eta) = t_{42}(\xi, \eta) + T_1(x_{13}(t_{42}(\xi, \eta), \xi, \eta), \gamma)$$

$t_2(\xi,\eta)$ be the smallest solution of the following equation

$$f_{21}(t_2)[\xi - \bar{x}_1] + f_{22}(t_2)[\eta - \bar{x}_2] + \bar{x}_1 = \gamma$$

$t_3(\xi,\eta)$ be the smallest solution of the following equation

$$f_{33}(t_3)[\xi - \bar{x}_1] + f_{34}(t_2)[\eta - \bar{x}_2] + \bar{x}_2 = \gamma$$

$t_{41}(\xi,\eta)$ and $t_{42}(\xi,\eta)$ be the smallest solution of the following equations

$$f_{41}(t_{41})[\xi - \bar{x}_1] + f_{42}(t_{42})[\eta - \bar{x}_2] + \bar{x}_1 = \gamma$$

$$f_{43}(t_{42})[\xi - \bar{x}_1] + f_{44}(t_{42})[\eta - \bar{x}_2] + \bar{x}_2 = \gamma$$

$$\bar{t}_{41} = t_{41}[x_1(0), x_2(0)], \ \bar{t}_{42} = t_{42}[x_1(0), x_2(0)]$$

$$x_{22}(t, \xi, \eta) = f_{23}[\xi - \bar{x}_1] + f_{24}(t)[\eta - \bar{x}_2] + \bar{x}_2$$

$$x_{13}(t, \xi, \eta) = f_{31}(t)[\xi - \bar{x}_1] + f_{32}[\eta - \bar{x}_2] + \bar{x}_1$$

where f_{ij} is defined by matrix exponent function $e^{A_{\mathrm{mi}}t}$, and expresses the transition rate.

Proof. Known from the former descriptions, under the condition of starting from the region ① to ④, and respectively, the role of control rules (36) to (39), the state equation possesses the following form.

$$\frac{\mathrm{d}\boldsymbol{x}(t)}{\mathrm{d}t}=\boldsymbol{A}_{\mathrm{m}i}\boldsymbol{x}(t)+\boldsymbol{b}_{\mathrm{m}i}\qquad (i=1,2,3,4)$$

And then, solving the above equation, we get

$$\boldsymbol{x}(t)=\exp(\boldsymbol{A}_{\mathrm{m}i}t)\boldsymbol{x}(0)+\int_0^t\exp\left(\boldsymbol{A}_{\mathrm{m}i}(t-\tau)\right)\mathrm{d}\tau\cdot\boldsymbol{b}_{\mathrm{m}i}$$

According to the definition of $\boldsymbol{A}_{\mathrm{m}i}$ we know that the system is asymptotically steady, and so

$$\boldsymbol{x}(t)=\exp(\boldsymbol{A}_{\mathrm{m}i}t)\left(\boldsymbol{x}(0)+\boldsymbol{A}_{\mathrm{m}i}^{-1}\boldsymbol{b}_{\mathrm{m}i}\right)-\boldsymbol{A}_{\mathrm{m}i}^{-1}\boldsymbol{b}_{\mathrm{m}i}$$

Then, the stable-state value of the system is

$$\boldsymbol{x}(\infty)=-\boldsymbol{A}_{\mathrm{m}i}^{-1}\boldsymbol{b}_{\mathrm{m}i}=\overline{\boldsymbol{x}}_{\mathrm{m}i}\ .$$

Hence, we have

$$\boldsymbol{x}(t)=\exp(\boldsymbol{A}_{\mathrm{m}i}t)\left(\boldsymbol{x}(0)-\overline{\boldsymbol{x}}_{\mathrm{m}i}\right)+\overline{\boldsymbol{x}}_{\mathrm{m}i}$$

Under the condition that the initial state $\boldsymbol{x}(0)$ and the expected steady-state value $\overline{\boldsymbol{x}}_{\mathrm{m}i}$ are given out, it is quite obvious that the motion equation of the system is only related to $\exp(\boldsymbol{A}_{\mathrm{m}i}t)$. So, we use the method of resolvent matrix to solve $\exp(\boldsymbol{A}_{\mathrm{m}i}t)$ [18]. Thus, we have

$$\exp(\boldsymbol{A}_{\mathrm{m}i}t)=L^{-1}\left((s\boldsymbol{I}-\boldsymbol{A}_{\mathrm{m}i})^{-1}\right)$$

Firstly, we conduct the following definitions.

$$p_1(s)=s^2+(d_1'+d_1')s+(d_1'd_2'-ab)$$

$$p_2(s)=s^2+\left(\frac{\rho_1\mu_1+a\overline{x}_2}{\overline{x}_1}+k_{22}\right)s+\left(\frac{\rho_1\mu_1+a\overline{x}_2}{\overline{x}_1}k_{22}+ak_{21}\right)$$

$$q_2(s)=s+\frac{\rho_1\mu_1+a\overline{x}_2}{\overline{x}_1}$$

$$p_3(s)=s^2+\left(\frac{(1-\rho_1)\mu_1+b\overline{x}_1}{\overline{x}_2}+k_{11}\right)s+\left(\frac{(1-\rho_1)\mu_1+b\overline{x}_1}{\overline{x}_2}k_{11}+bk_{12}\right)$$

$$q_3(s)=s+\frac{(1-\rho_1)\mu_1+b\overline{x}_1}{\overline{x}_2}$$

$$p_4(s)=s^2+\left(\frac{\rho_1\mu_1+a\overline{x}_2}{\overline{x}_1}+\frac{(1-\rho_1)\mu_1+b\overline{x}_1}{\overline{x}_2}\right)s+\left(\frac{\rho_1\mu_1+a\overline{x}_2}{\overline{x}_1}\times\frac{(1-\rho_1)\mu_1+b\overline{x}_1}{\overline{x}_2}-ab\right)$$

$$e^{A_{\mathrm{m1}}t}=\begin{pmatrix} f_{11}(t) & f_{12}(t) \\ f_{13}(t) & f_{14}(t) \end{pmatrix}=L^{-1}\begin{pmatrix} \dfrac{s+d_2^{'}}{p_1(s)} & \dfrac{a}{p_1(s)} \\ \dfrac{b}{p_1(s)} & \dfrac{s+d_1^{'}}{p_1(s)} \end{pmatrix}$$

$$e^{A_{\mathrm{m2}}t}=\begin{pmatrix} f_{21}(t) & f_{22}(t) \\ f_{23}(t) & f_{24}(t) \end{pmatrix}=L^{-1}\begin{pmatrix} \dfrac{s+k_{22}}{p_2(s)} & \dfrac{a}{p_2(s)} \\ \dfrac{-k_{21}}{p_2(s)} & \dfrac{q_2(s)}{p_2(s)} \end{pmatrix}$$

$$e^{A_{\mathrm{m3}}t}=\begin{pmatrix} f_{31}(t) & f_{32}(t) \\ f_{33}(t) & f_{34}(t) \end{pmatrix}=L^{-1}\begin{pmatrix} \dfrac{q_3(s)}{p_3(s)} & \dfrac{-k_{12}}{p_3(s)} \\ \dfrac{b}{p_3(s)} & \dfrac{s+k_{11}}{p_3(s)} \end{pmatrix}$$

$$e^{A_{\mathrm{m4}}t}=\begin{pmatrix} f_{41}(t) & f_{42}(t) \\ f_{43}(t) & f_{44}(t) \end{pmatrix}=L^{-1}\begin{pmatrix} \dfrac{q_2(s)}{p_4(s)} & \dfrac{a}{p_3(s)} \\ \dfrac{b}{p_4(s)} & \dfrac{q_1(s)}{p_4(s)} \end{pmatrix}$$

Based on the above discussions, under the condition that the initial value $\boldsymbol{x}(0)$ and the expected steady-state value $\overline{\boldsymbol{x}}_{\mathrm{m}i}$ are given out, we can get the motion equation $\boldsymbol{x}(t)$ of the system by combining $\boldsymbol{x}(\infty)=-\boldsymbol{A}_{\mathrm{m}i}^{-1}\boldsymbol{b}_{\mathrm{m}i}=\overline{\boldsymbol{x}}_{\mathrm{m}i}$ and $\exp(\boldsymbol{A}_{\mathrm{m}i}t)$.

Example 2: The primary data are same with ones in *Example 1*. To verify theorem 6, we let γ=0.1, and then respectively define the reference matrixes as follows: $\boldsymbol{A}_{\mathrm{m1}}$=[-4.3625 0.3125; 1.2500 -3.5250], $\boldsymbol{A}_{\mathrm{m2}}$=[-4.0625 0.3125; 1.2500 -3.5250], $\boldsymbol{A}_{\mathrm{m3}}$=[-4.3625 0.3125; 1.2500 -3.1250], $\boldsymbol{A}_{\mathrm{m4}}$=[-4.0625 0.3125; 1.2500 -3.1250] , and select the desired steady-state value as $\overline{\boldsymbol{x}}_{\mathrm{m1}}=\overline{\boldsymbol{x}}_{\mathrm{m2}}=\overline{\boldsymbol{x}}_{\mathrm{m3}}=\overline{\boldsymbol{x}}_{\mathrm{m4}}=\left[\frac{1}{3}\ \frac{1}{3}\ \frac{1}{3}\right]^{\mathrm{T}}$. In the area ①, let the initial state probabilities of the system satisfy $x_1(0)$=0.5, $x_2(0)$=0.5, and $x_3(0)$=0, and in the area ②, the initial state probabilities satisfy

$x_1(0)$=0, $x_2(0)$=1 and $x_3(0)$=0, and in the area ③ the initial state probabilities satisfy $x_1(0)$=1, $x_2(0)$=0 and $x_3(0)$=0, and in the area ④ the initial state probabilities satisfy $x_1(0)$=0.05, $x_2(0)$=0.05 and $x_3(0)$=0.9. Based on these data, we can conduct the following analysis.

Under the action of the control laws (36), (37), (38) and (39), we clearly have

$$\boldsymbol{x}(\infty) = \bar{\boldsymbol{x}}_{\mathrm{m1}} = \bar{\boldsymbol{x}}_{\mathrm{m2}} = \bar{\boldsymbol{x}}_{\mathrm{m3}} = \bar{\boldsymbol{x}}_{\mathrm{m4}} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}^{\mathrm{T}}$$

In the area ①, from (36), we have

$$\lim_{t\to\infty} \boldsymbol{u}(t) = \lim_{t\to\infty} [\boldsymbol{X}(t)]^{-1}[(\boldsymbol{A}_1 + \boldsymbol{K})\boldsymbol{x}(t) + \boldsymbol{\mu} - \boldsymbol{K}\bar{\boldsymbol{x}}] = [0.4165 \quad 0.1750]^{\mathrm{T}}$$

In the area ②, from (37), we have

$$\lim_{t\to\infty} \boldsymbol{u}(t) = \lim_{t\to\infty} \begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix} = \lim_{t\to\infty} \begin{pmatrix} \dfrac{\rho_1\mu_1 + a\bar{x}_2}{\bar{x}_1} - d_1' \\ \dfrac{(k_{21}+b)x_1 + (k_{22} - d_2')x_2 + [(1-\rho_1)\mu_1 - k_{21}\bar{x}_1 - k_{22}\bar{x}_2)]}{x_2} \end{pmatrix} = \begin{pmatrix} 0.4165 \\ 0.1750 \end{pmatrix}$$

In the area ③, from (38), we have

$$\lim_{t\to\infty} \boldsymbol{u}(t) = \lim_{t\to\infty} \begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix} = \lim_{t\to\infty} \begin{pmatrix} \dfrac{(k_{11}-d)x_1 + (k_{12}+a)x_2 + ((1-\rho_1)\mu_1 - k_{11}\bar{x}_1 - k_{12}\bar{x}_2)}{x_2} \\ \dfrac{(1-\rho_1)\mu_1 + b\bar{x}_1}{\bar{x}_2} - d_2' \end{pmatrix} = \begin{pmatrix} 0.4165 \\ 0.1750 \end{pmatrix}$$

In the area ④, from (39), we have

$$\lim_{t\to\infty} \boldsymbol{u}(t) = \lim_{t\to\infty} \begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix} = \begin{pmatrix} \dfrac{\rho_1\mu_1 + a\bar{\boldsymbol{x}}_2}{\bar{\boldsymbol{x}}_1} - d_1' \\ \dfrac{(1-\rho_1)\mu_1 + b\bar{\boldsymbol{x}}_1}{\bar{\boldsymbol{x}}_2} - d_2' \end{pmatrix} = \begin{pmatrix} 0.4165 \\ 0.1750 \end{pmatrix}$$

Figure 4 shows the evolutions of the system states $x_1(t)$, and $x_2(t)$, and $x_3(t)$ with time, and Figure 5 shows the evolutions of the checking rates $u_1(t)$ and $u_2(t)$ with time under the condition that the system starts from the area ①. As the system starts from the area ②, same demonstrations are made in Figure 6 and Figure 7 with Figure 4 and Figure 5. As the system starts from the area ③, same demonstrations are made in Figure 8 and Figure 9. Likewise, as the system starts from the area ④, same demonstrations are conducted in Figure 10 and Figure 11.

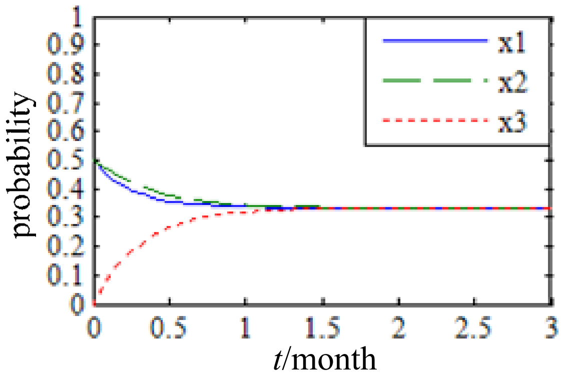


Figure 4. State evolution starting in area ①

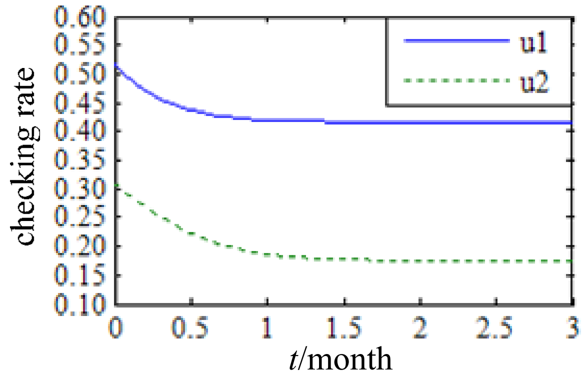


Figure 5. Check rates evolution starting in area ①

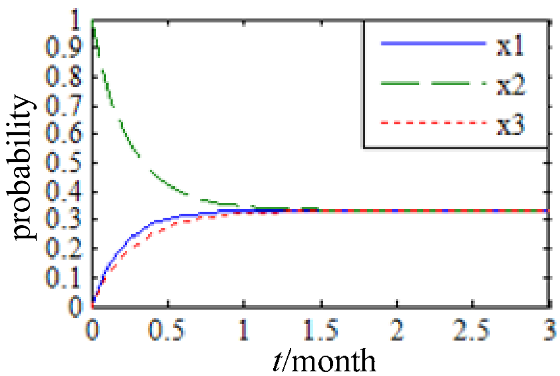


Figure 6. State evolution starting in area ②

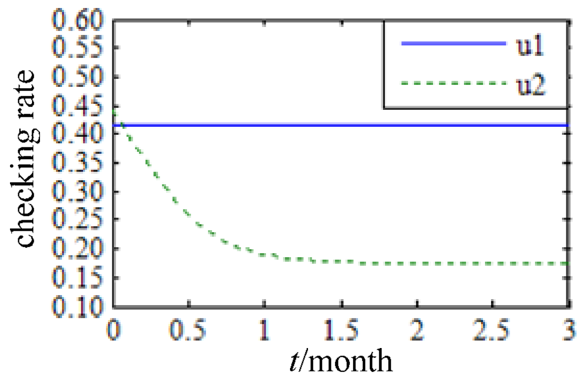


Figure 7. Check rates evolution starting in area ②

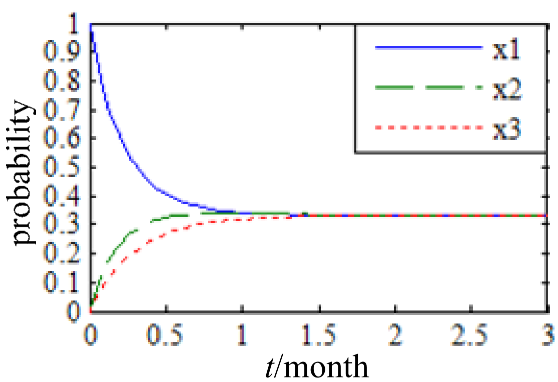


Figure 8. State evolution starting in area ③

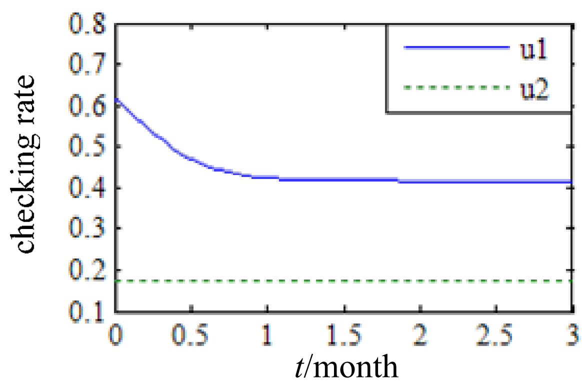
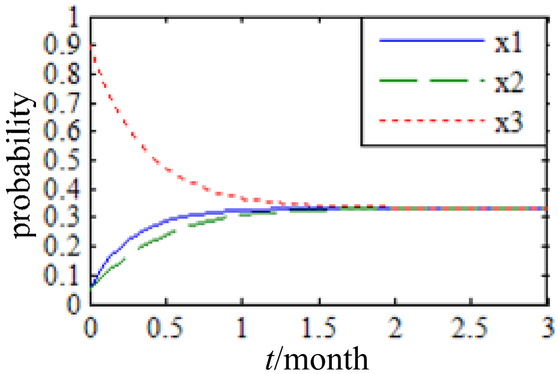


Figure 9. Check rates evolution starting in area ③



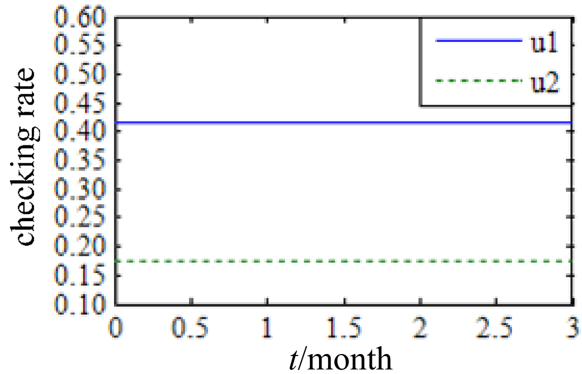


Figure 10. State evolution starting in area ④ Figure 11. Check rates evolution starting in area ④

From Figure 4, and Figure 6, and Figure 8, and Figure 10, we can know that the system steady-state value is $x_1(\infty) = x_2(\infty) = x_3(\infty) = 0.3333$, which is consistent with the theoretical calculation results. From Figure 5 we can know that the checking rates $u_1(t)$ and $u_2(t)$ are variable and the steady-state checking rates are $u_1(\infty) = 0.4165$ and $u_2(\infty) = 0.1750$, and in Figure 7 we can know that the checking rate $u_1(t)$ is constant and $u_2(t)$ is variable, where $u_1(t)$=0.4165 and the steady-state checking rate $u_2(\infty) = 0.1750$, and from Figure 9 we can know that the checking rate $u_1(t)$ is variable and $u_2(t)$ is constant, where $u_2(t)$=0.1750 and the steady-state checking rate $u_1(\infty) = 0.4165$, and from Figure 11 we can know that the checking rates $u_1(t)$ and $u_2(t)$ are constant, where $u_1(t)$=0.4165 and $u_2(t)$=0.1750, which are consistent with the theoretical calculation results, and show that the proposed method is suitable and effective. And from $\boldsymbol{x}(t) = \exp(\boldsymbol{A}_{\mathrm{m}i}t)\left(\boldsymbol{x}(0) + \boldsymbol{A}_{\mathrm{m}i}{}^{-1}\boldsymbol{b}_{\mathrm{m}i}\right) - \boldsymbol{A}_{\mathrm{m}i}{}^{-1}\boldsymbol{b}_{\mathrm{m}i}$ we can obtain $x_1(t)$=0.3340, $x_2(t)$=0.3355 and $x_3(t)$=0.3305 at time t=1.5month starting form the area ①, and $x_1(t)$=0.3334, $x_2(t)$=0.3354 and $x_3(t)$=0.3312 at time t=1.5 month starting form the area ②, and $x_1(t)$=0.3345, $x_2(t)$=0.3354 and $x_3(t)$=0.3301 at time t=1.5 month starting form the area ③, and as well as $x_1(t)$=0.3314, $x_2(t)$=0.3269 and $x_3(t)$=0.3417 at time t=1.5 month starting form the area ④, which show that the system will approximately enter steady-state at time instant t=1.5 month when the system starts from different areas.

According to Theorem 8, we easily obtain

1) As the system starts from the area ①, we can get

$$\mathrm{MTTF}_1[x_1(0), x_2(0)] = T_1[x_1(0), x_2(0)] = 0.1051 \text{ month}$$

2) As the system starts from the area ②, we have

$$\mathrm{MTTF}_2[x_1(0), x_2(0)] = T_2[x_1(0), x_2(0)] = 0.0951 \text{ month}$$

3) As the system starts from the area ③, we have

$$\mathrm{MTTF}_3[x_1(0), x_2(0)] = T_3[x_1(0), x_2(0)] = 0.1026 \text{ month}$$

4) As the system starts from the area ④, we have

$$\mathrm{MTTF}_4[x_1(0), x_2(0)] = 0.2015 \text{ month}$$

V. PROBABILITY ESTIMATION

Definition 3 According to [19], the probability of the system being in S_i at time t may be expressed by

$$\tilde{x}_i(t) = n_i(t) / n(t) \tag{41}$$

where $n_i(t)$ represents the number of the device being in S_i at time t, and $n(t)$ presents the total number of the same device at the same time.

Note that

$$\tilde{x}_i(t) = x_i(t) + \tilde{\delta}_i(t) \tag{42}$$

where $\tilde{\delta}_i(t)$ expresses the difference between $x_i(t)$ and $\tilde{x}_i(t)$, which is a bounded random variable. For $n(t)$ large enough, it follows normal distribution $N(0, \sigma_{xi})$.

And then

$$E(\tilde{x}_i(t)) = x_i(t)$$

The significance of the above formula lies in that $\tilde{x}_i(t)$ possesses same evolution law with $\boldsymbol{x}(t)$ even though it is a probability statistics variable. As a fact, due to initial values and system arguments given, it is necessary that there exists one connection between both.

Let $\hat{\boldsymbol{x}}(t)$ be an estimate of $\boldsymbol{x}(t)$ determined by system (34), and then we obtain

$$\frac{\mathrm{d}\hat{\boldsymbol{x}}(t)}{\mathrm{d}t} = \boldsymbol{A}_1 \hat{\boldsymbol{x}}(t) - \boldsymbol{U}(t)\hat{\boldsymbol{x}}(t) + \boldsymbol{\mu} \tag{43}$$

The difference between them is

$$\hat{\boldsymbol{\delta}}(t) = \hat{\boldsymbol{x}}(t) - \boldsymbol{x}(t)$$

With initial conditions satisfying

$$\hat{\boldsymbol{x}}(0)=\tilde{\boldsymbol{x}}(0)=\boldsymbol{x}(0)+\hat{\boldsymbol{\delta}}(0)=\boldsymbol{x}(0)+\tilde{\boldsymbol{\delta}}(0) \tag{44}$$

Applying $\hat{\boldsymbol{x}}(t)$ to replace $\boldsymbol{x}\,(t)$ in (36), we obtain

$$\boldsymbol{u}(t)=\begin{bmatrix}u_1(t)\\u_2(t)\end{bmatrix}=[\widehat{\boldsymbol{X}}(t)]^{-1}[(\boldsymbol{A}_1+\boldsymbol{K})\hat{\boldsymbol{x}}(t)+\boldsymbol{\mu}-\boldsymbol{K}\bar{\boldsymbol{x}}] \tag{45}$$

Known from the former analysis, the system (43) is stable under the role of (45).

Let (34) be subtracted by (43), we obtain

$$\frac{\mathrm{d}[\hat{\boldsymbol{x}}(t)-\boldsymbol{x}(t)]}{\mathrm{d}t}=(\boldsymbol{A}_1-\boldsymbol{U}(t))[\hat{\boldsymbol{x}}(t)-\boldsymbol{x}(t)]$$

That is

$$\frac{\mathrm{d}[\hat{\boldsymbol{x}}(t)-E(\tilde{\boldsymbol{x}}(t))]}{\mathrm{d}t}=(\boldsymbol{A}_1-\boldsymbol{U}(t))[\hat{\boldsymbol{x}}(t)-E(\tilde{\boldsymbol{x}}(t))]$$

Thus, we have

$$\frac{\mathrm{d}\hat{\boldsymbol{\delta}}(t)}{\mathrm{d}t}=(\boldsymbol{A}_1-\boldsymbol{U}(t))\hat{\boldsymbol{\delta}}(t) \tag{46}$$

From (46) we obtain

$$\frac{\mathrm{d}\hat{\boldsymbol{\delta}}(t)}{\mathrm{d}t}=(\boldsymbol{A}_1-\boldsymbol{U}(t))\hat{\boldsymbol{\delta}}(t) \tag{47}$$

It is possible to select a suitable scalar matrix $\boldsymbol{M}$ that matches $m_{ii}=\max\{[\boldsymbol{A}_1-\boldsymbol{U}(t)]_{ii}\}$, and $m_{ij}=\max\{|[\boldsymbol{A}_1-\boldsymbol{U}(t)]_{ij}|, i\neq j\}$, and thus, for $\forall t>0$, then $\boldsymbol{A}_1-\boldsymbol{U}(t)\leq \boldsymbol{M}, \boldsymbol{M}=(m_{ij})$. According to [20, 21], we have

$$|\hat{\boldsymbol{\delta}}(t)|\leq e^{\boldsymbol{M}t}\,|\hat{\boldsymbol{\delta}}(0)|=e^{-\boldsymbol{\beta}t}\,|\hat{\boldsymbol{\delta}}(0)| \tag{48}$$

where $\boldsymbol{\beta}=-\boldsymbol{M}$.

From (48) we can say that (46) is asymptotically stable with zero solution. In effect, due to the boundedness of $\boldsymbol{U}(t)$ determinated by (45), the condition above is met easily of course.

Theorem9. In area ①, under the role of the control law (45), the system state $\boldsymbol{x}\,(t)$ satisfies

$$\left\|\boldsymbol{x}(t)-\bar{\boldsymbol{x}}\right\|\leq\left\|\boldsymbol{x}(0)-\bar{\boldsymbol{x}}\right\|\|\exp(-\boldsymbol{K}t)\|+\theta\left\|\hat{\boldsymbol{\delta}}(0)\right\|\left\|\frac{\exp(-\boldsymbol{K}t)-\exp(-\boldsymbol{\beta}t)}{\boldsymbol{\beta}-\boldsymbol{K}}\right\| \tag{49}$$

where $\theta = \|\boldsymbol{\beta} - \boldsymbol{K}\| = \|\boldsymbol{A}_1 + \boldsymbol{K} - \boldsymbol{U}(t)\|_{\max} = \max\{\|\boldsymbol{A}_1 + \boldsymbol{K} - \boldsymbol{U}(t)\|\}$

After a maximum time T_1, the second item in (49) decrease rapidly

$$T_1 = \max\{\frac{n(\boldsymbol{I} - \boldsymbol{R}^{\frac{1}{n}})}{\boldsymbol{K}(\boldsymbol{I} - \boldsymbol{R})}\} \approx m_{11}\Delta t \tag{50}$$

where n is given sufficient large, and Δt is a small but infinite time interval for an effective implementation, and $\boldsymbol{R}$=$\boldsymbol{K}/\boldsymbol{\beta}$, and n and m_{11} is a suitable integer.

Proof. Substituting (45) into (43), we have

$$\frac{\mathrm{d}\hat{\boldsymbol{x}}(t)}{\mathrm{d}t} = -\boldsymbol{K}\hat{\boldsymbol{x}}(t) + \boldsymbol{K}\bar{\boldsymbol{x}}$$

Resolving the equation, then

$$\hat{\boldsymbol{x}}(t) - \bar{\boldsymbol{x}} = [\hat{\boldsymbol{x}}(0) - \bar{\boldsymbol{x}}]\exp(-\boldsymbol{K}t)$$

And then

$$\begin{aligned}\|\boldsymbol{x}(t) - \bar{\boldsymbol{x}}\| &= \|[\hat{\boldsymbol{x}}(0) - \bar{\boldsymbol{x}}]\exp(-\boldsymbol{K}t) - \hat{\boldsymbol{\delta}}(t)\| \\ &\le \|[\boldsymbol{x}(0) - \bar{\boldsymbol{x}}]\exp(-\boldsymbol{K}t)\| + \|\hat{\boldsymbol{\delta}}(0)\exp(-\boldsymbol{K}t) - \hat{\boldsymbol{\delta}}(t)\| \\ &\le \|\boldsymbol{x}(0) - \bar{\boldsymbol{x}}\|\|\exp(-\boldsymbol{K}t)\| + \|\boldsymbol{\beta} - \boldsymbol{K}\|\|\hat{\boldsymbol{\delta}}(0)\|\|\frac{\exp(-\boldsymbol{K}t) - \exp(-\boldsymbol{\beta}t)}{\boldsymbol{\beta} - \boldsymbol{K}}\| \\ &= \|\boldsymbol{x}(0) - \bar{\boldsymbol{x}}\|\|\exp(-\boldsymbol{K}t)\| + \theta\|\hat{\boldsymbol{\delta}}(0)\|\|\frac{\exp(-\boldsymbol{K}t) - \exp(-\boldsymbol{\beta}t)}{\boldsymbol{\beta} - \boldsymbol{K}}\|\end{aligned}$$

where

$$\theta = \|\boldsymbol{\beta} - \boldsymbol{K}\| = \|\boldsymbol{A}_1 + \boldsymbol{K} - \boldsymbol{U}(t)\|_{\max} = \max\{\|\boldsymbol{A}_1 + \boldsymbol{K} - \boldsymbol{U}(t)\|\}$$

To prove (50), we let

$$\boldsymbol{f}(t) = \exp(-\boldsymbol{K}t) - \exp(-\boldsymbol{\beta}t)$$

And then

$$\boldsymbol{f}'(t) = -\boldsymbol{K}\exp(-\boldsymbol{K}t) + \boldsymbol{\beta}\exp(-\boldsymbol{\beta}t)$$

Let $\boldsymbol{f}'(t) = 0$, and then

$$\frac{\boldsymbol{\beta}}{\boldsymbol{K}} = \frac{\exp(-\boldsymbol{K}t)}{\exp(-\boldsymbol{\beta}t)}$$

For n large enough, the matrix exponent function exp($\boldsymbol{\beta}t$) and exp($\boldsymbol{K}t$) can be approximated by

$$\exp(-\boldsymbol{K}t) \approx (\boldsymbol{I} - \boldsymbol{K}\frac{t}{n})^n$$

$$\exp(-\boldsymbol{\beta} t) \approx (\boldsymbol{I} - \boldsymbol{\beta}\frac{t}{n})^n$$

The following formula can be then obtained by simple substitution.

$$\boldsymbol{t} = \frac{n(\boldsymbol{K}^{\frac{1}{n}} - \boldsymbol{\beta}^{\frac{1}{n}})}{\boldsymbol{K}\boldsymbol{K}^{\frac{1}{n}} - \boldsymbol{\beta}\boldsymbol{\beta}^{\frac{1}{n}}}$$

Put $\boldsymbol{R}$=$\boldsymbol{K}/\boldsymbol{\beta}$, and we then have

$$\boldsymbol{t} = \frac{n(\boldsymbol{I} - \boldsymbol{R}^{\frac{1}{n}})}{\boldsymbol{K}(\boldsymbol{I} - \boldsymbol{R}^{\frac{n+1}{n}})}$$

As $n\to\infty$, then $(n+1)/n\to 1$, and so

$$T_1 = \max\{\boldsymbol{t}\} \approx \max\{\frac{n(\boldsymbol{I} - \boldsymbol{R}^{\frac{1}{n}})}{\boldsymbol{K}(\boldsymbol{I} - \boldsymbol{R})}\} \approx m_{11}\Delta t$$

End.

The control strategy is to apply (45) for time interval larger than $m_{12}T_1$ to control the course of $|\hat{\boldsymbol{x}}(t) - \tilde{\boldsymbol{x}}(t)|$, where m_{12} is a suitable integer. If the difference can not remain bounded, then the estimator (43) must be reinitialized with

$$\hat{\boldsymbol{x}}(m_1\Delta t) = \tilde{\boldsymbol{x}}(m_1\Delta t) \tag{51}$$

where $m_1=m_{12}\times m_{11}$.

Theorem10. Let us first define the following random variables.

$$\chi_i = \max_{t\in \mathrm{R}^+}\{|\tilde{x}_i(t) - \hat{x}_i(t)|\} \tag{52}$$

$$\varepsilon_i(t) = \max_{\tau<t}\{|\tilde{x}_i(\tau) - \hat{x}_i(\tau)|\} \tag{53}$$

Clearly, $\forall t\geq 0$, $\varepsilon_i(t)\leq\chi_i$. For a sufficient large $n(t)$, there exists $c_i(t)$

$$c_i(t) = \sqrt{-\frac{1}{2n(t)}\ln\frac{1-\gamma_i(t)}{2}} \tag{54}$$

satisfies

$$P(\varepsilon_i(t) \leq c_i(t)) = \gamma_i(t) \tag{55}$$

Proof. From (41), we can have

$$|\tilde{\boldsymbol{x}}_i(t)-\hat{\boldsymbol{x}}_i(t)|=|\frac{n_i(t)}{n(t)}-\hat{\boldsymbol{x}}_i(t)|=|\hat{\boldsymbol{x}}_i(t)-\frac{n_i(t)}{n(t)}|$$

where $p_i(t)=\dfrac{n_i(t)}{n(t)}$ is a statistic probability of $x_i(t)$ at time *t*, and $\hat{x}_i(t)$ is a estimate of state $x_i(t)$ at time *t* determined by (53). Hence, put $p_i(t)$ as the occurring probability of the event $\zeta_i(t)=|\tilde{x}_i(t)-\hat{x}_i(t)|\le c_i(t)$ at time *t*, and thus $\hat{\boldsymbol{x}}(t)$ can be seen as binomial distribution with parameters $n(t)$ and $\boldsymbol{p}(t)$, and $0<\boldsymbol{p}(t)<1$, and remembered as

$$\zeta_i(t)\sim B(n(t),p_i(t))$$

And so, as $n(t)\to\infty$, the following variable converges to standard normal $N(0,1)$.

$$\xi_i(t)=\frac{\sum_k \zeta_i(t)-n(t)p_i(t)}{\sqrt{n(t)p_i(t)q_i(t)}}\sim N(0,1)$$

where $q_i(t)=1-p_i(t)$.

From

$$P(|\zeta_i(t)|\le c_i(t))=\gamma_i(t)$$

which is equivalent to

$$P(|\frac{\sum_k \xi_i(t)-n(t)p_i(t)}{\sqrt{n(t)p_i(t)q_i(t)}}|\le c_i(t)\sqrt{\frac{n(t)}{p_i(t)q_i(t)}})=\gamma_i(t)$$

Therefore

$$P(\xi_i(t)>c_i(t)\sqrt{\frac{n(t)}{p_i(t)q_i(t)}})=\frac{1-\gamma_i(t)}{2}$$

On the other hand, since $\xi_i(t)$ follows $N(0, 1)$, and then its moment-generating function(MGF) is

$$M_{\xi_i}(t)=\exp(t^2/2)$$

According to Chernoff bounds [22-25], we have

$$P(\xi_i(t)>c_i(t)\sqrt{\frac{n(t)}{p_i(t)q_i(t)}})\le e^{-c_i(t)\sqrt{\frac{n(t)}{p_i(t)q_i(t)}}t}M_{\xi_i}(t)=e^{-c_i(t)\sqrt{\frac{n(t)}{p_i(t)q_i(t)}}t+\frac{t^2}{2}}$$

Note that $p_i(t)q_i(t)\le 1/4$ for $0<p_i(t), q_i(t)<1$, and so we have

$$e^{-2c_i(t)\sqrt{n(t)}t+\frac{t^2}{2}}\ge e^{-c_i(t)\sqrt{\frac{n(t)}{p_i(t)q_i(t)}}t+\frac{t^2}{2}}\ge\frac{1-\gamma_i(t)}{2}$$

The minimum of the left-hand side is obtained by differentiating with respect to t, and of course, when

$$t = T_2 = 2c_i(t)\sqrt{n(t)} \approx m_{21}\Delta t \tag{56}$$

We have

$$c_i(t) \le \sqrt{-\frac{1}{2n(t)}\ln\frac{1-\gamma_i(t)}{2}} \tag{57}$$

Seen from (57), intuitively, for given confidence $\gamma_i(t)$, all $c_i(t)$ corresponding to it can satisfy (55). The more small $c_i(t)$ means confidence level high, and conversely, the larger $c_i(t)$ means a low confidence value. The largest $c_i(t)$ satisfying (57) is a critical value that it enable to do (53) hold. According to (53), we can obtain

$$\tilde{x}_i(t) \in [\hat{x}_i(t) - c_i(t), \hat{x}_i(t) + c_i(t)].$$

If $n(t)$ and $\gamma_i(t)$ do not change with time, and then $c_i(t)$ becomes a constant c. After the time interval $m_2\Delta t$, if one has $\varepsilon_i(t)>c$, then the following formula should be reinitialized, where $m_2=m_{22}\times m_{21}$, and m_{21}, m_{22} are suitable integer.

$$\hat{\boldsymbol{x}}(m_2\Delta t) = \tilde{\boldsymbol{x}}(m_2\Delta t) \tag{58}$$

Note that (51) and (58), we select T as reinitialization time by

$$T = T_1 \wedge T_2 \tag{59}$$

The symbol "$\wedge$" means the minimum of T_1 and T_2, where T_1 is necessary to keep stable-state stability, and T_2 is to keep transient stability.

Theorem11. The probability distribution of the checking times x_{u1} and x_{u1} are the minimal time instants which satisfy the following equations.

$$\int_0^{x_{u1}(N)} u_1(\tau)d\tau = -\ln(1-\delta_N). \tag{60}$$

$$\int_0^{x_{u2}(M)} u_2(\tau)d\tau = -\ln(1-\delta_M). \tag{61}$$

where δ_N and δ_M are independent random numerical sequences with uniform distribution in [0, 1], and $\boldsymbol{N}$ and $\boldsymbol{M}$ are the two indexes.

Proof. Let the distribution of the checking time of the stored device be $G_1(x)$, and then

$$G_1(x) = 1 - \exp[-\int_0^x u_1(\tau)d\tau]$$

And so,

$$\int_0^x u_1(\tau)d\tau = -\ln[1 - G_1(x)]$$

Since the stochastic variable Y=$G_1(x)$ follows the uniform distribution in [0, 1], as long as to generate the stochastic samples which follows uniform distribution in [0, 1], and then through X= $G_1^{-1}(Y)$ the samples of stochastic variable for arbitrary distribution function $G_1(x)$ can be obtained. Let δ_N be the nth random sample in [0, 1], then we have

$$\int_0^{x_{u1}(N)} u_1(\tau)d\tau = -\ln(1 - \delta_N)$$

Similarly, we may prove (61).

Example 3: The basic data are same with ones in *Example 2*. Based on probability estimation method in this section, we order $\hat{\boldsymbol{x}}(0) = \tilde{\boldsymbol{x}}(0) = \begin{bmatrix} 0.55 & 0.45 \end{bmatrix}^{\mathrm{T}}$ and $\boldsymbol{x}(0) = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix}^{\mathrm{T}}$. In Theorem 9, it is easy for us to select $\boldsymbol{\beta}= -\boldsymbol{M}$=[4.0625 0.3125; -1.2500 3.1250], $\boldsymbol{K}= -\boldsymbol{A}_{m1}$=[4.3625 -0.3125; -1.2500 3.5250], n=100, Δt=0.0001 month, and m_{12}=1. In Theorem 10, let $n(t)$=500, $\gamma_i(t)$=0.2, Δt=0.0001 month, and m_{22}=1, and then we have

$$\theta = \|\boldsymbol{\beta} - \boldsymbol{K}\| = 0.5$$

$$T_1 = \max\{\frac{n(\boldsymbol{I} - \boldsymbol{R}^{\frac{1}{n}})}{\boldsymbol{K}(\boldsymbol{I} - \boldsymbol{R})}\} = \max\begin{Bmatrix} 0.2265 & 0.0191 \\ 0.0742 & 0.2732 \end{Bmatrix} = 0.2732 \text{ month}$$

$$c_i(t) = \sqrt{-\frac{1}{2n(t)}\ln\frac{1-\gamma_i(t)}{2}} = 0.0303$$

$$T_2 = 2c_i(t)\sqrt{n(t)} = 1.3551 \text{ month}$$

Clearly, $T_1 < T_2$. Hence, according to (59), we select T_1 as re-initialization time.

and then according to (42), (43), (45) and (47), we can obtain the following results.

$$\lim_{t\to\infty}\hat{\boldsymbol{x}}(t) = \lim_{t\to\infty}\tilde{\boldsymbol{x}}(t) = \lim_{t\to\infty}\boldsymbol{x}(t) = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}^{\mathrm{T}}$$

$$\lim_{t\to\infty}\boldsymbol{u}(t) = \lim_{t\to\infty}[\hat{\boldsymbol{X}}(t)]^{-1}[(\boldsymbol{A}_1 + \boldsymbol{K})\hat{\boldsymbol{x}}(t) + \boldsymbol{\mu} - \boldsymbol{K}\bar{\boldsymbol{x}}] = [0.4165 \quad 0.1750]^{\mathrm{T}}$$

$$\lim_{t\to\infty}\bar{\boldsymbol{\delta}}(t)=\lim_{t\to\infty}\hat{\boldsymbol{\delta}}(t)=\lim_{t\to\infty}\tilde{\boldsymbol{\delta}}(t)=\begin{bmatrix}0 & 0 & 0\end{bmatrix}^{\mathrm{T}}$$

where $\bar{\boldsymbol{\delta}}(t)=\hat{\boldsymbol{x}}(t)-\tilde{\boldsymbol{x}}(t)$.

Figure 12 shows the evolutions of the estimated states probability $\hat{x}_1(t)$, and $\hat{x}_2(t)$, and $\hat{x}_3(t)$ with time, and respectively marked using xe1, and xe2, and xe3. The evolutions of the system actual states probability $x_1(t)$, and $x_2(t)$ and $x_3(t)$ are described in Figure 13, and the ones of the system statistics states probability $\tilde{x}_1(t)$, and $\tilde{x}_2(t)$ and $\tilde{x}_3(t)$ is showed in Figure 14, and respectively marked using xs1, and xs2, and xs3. Figure 15 shows the error $\bar{\boldsymbol{\delta}}(t)$ between $\hat{\boldsymbol{x}}(t)$ and $\tilde{\boldsymbol{x}}(t)$, and respectively marked using em1, and em2, and em3. Figure 16 shows the system error $\hat{\boldsymbol{\delta}}(t)$ between $\hat{\boldsymbol{x}}(t)$ and $\boldsymbol{x}(t)$, and respectively marked using en1, and en2, and en3. Figure 17 shows the evolutions of the checking rates $u_1(t)$ and $u_2(t)$, and Figure 18 describes the system the error $\tilde{\boldsymbol{\delta}}(t)$ between $\tilde{\boldsymbol{x}}(t)$ and $\boldsymbol{x}(t)$, and respectively marked using ep1, and ep2, and ep3. Figure 19 describes the modulus $|\bar{\boldsymbol{\delta}}(t)|$ of $\bar{\boldsymbol{\delta}}(t)$, and marked using mo1, and mo2, and mo3, respectively.

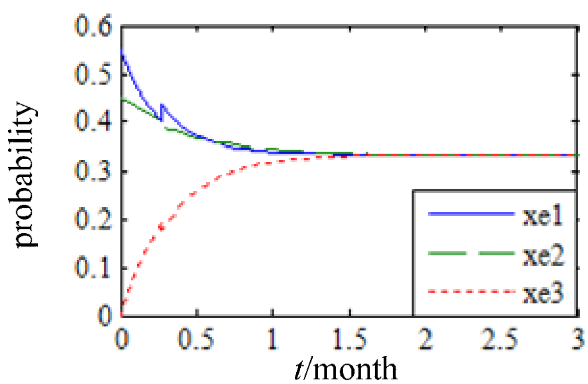


Figure 12. Estimation states evolution diagram

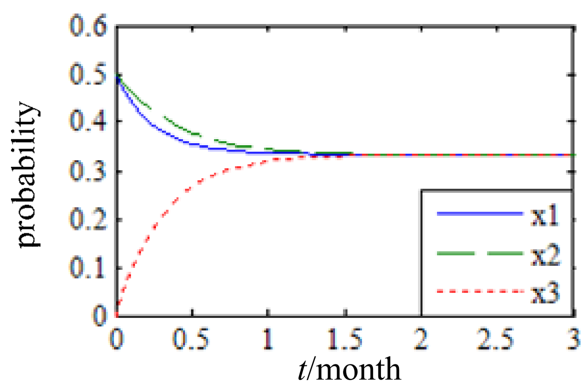
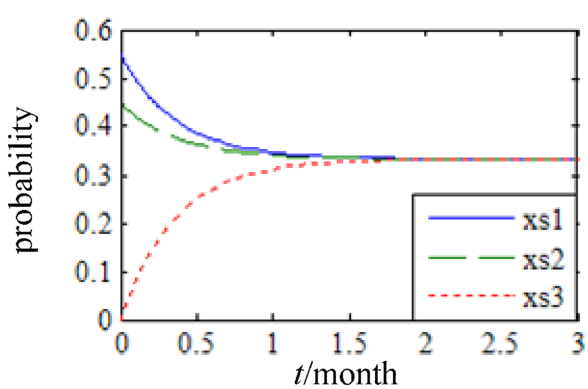


Figure 13. Actual state evolution diagram



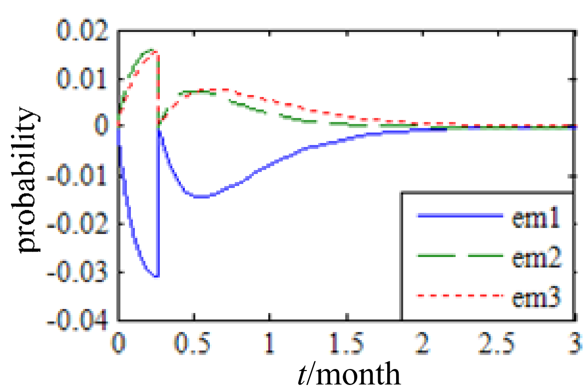


Figure 14. Statistics state evolution diagram

Figure 15. Error $\bar{\boldsymbol{\delta}}(t)$ between $\hat{\boldsymbol{x}}(t)$ and $\tilde{\boldsymbol{x}}(t)$

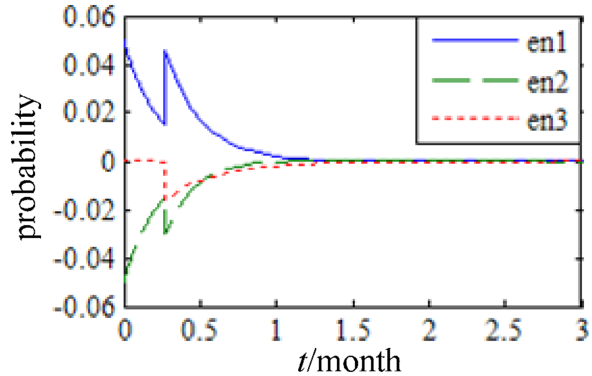


Figure 16 Error $\hat{\boldsymbol{\delta}}(t)$ between $\hat{\boldsymbol{x}}(t)$ and $\boldsymbol{x}(t)$

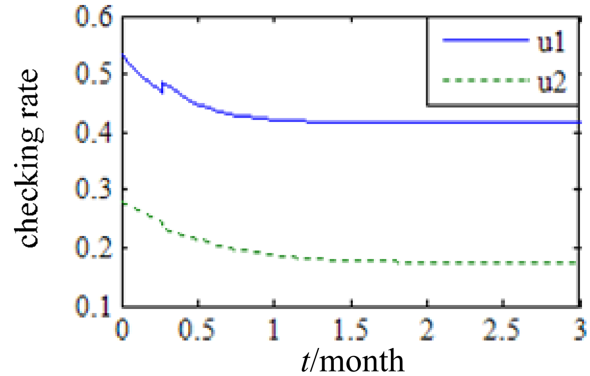


Figure 17. Checking rates diagram

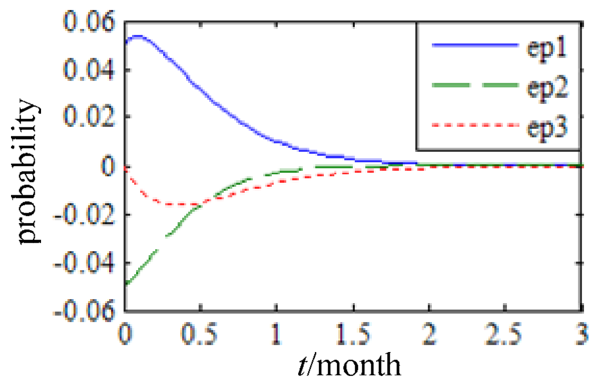


Figure 18 Error $\tilde{\boldsymbol{\delta}}(t)$ between $\tilde{\boldsymbol{x}}(t)$ and $\boldsymbol{x}(t)$

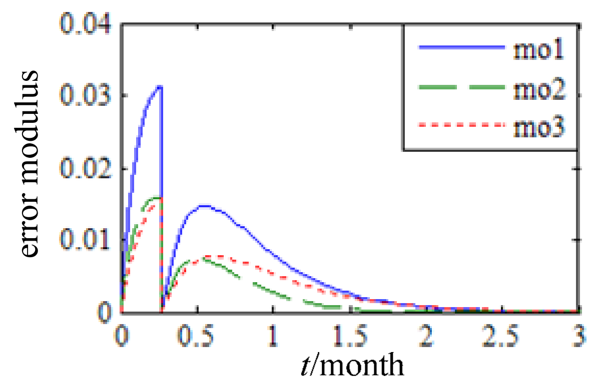


Figure 19. The modulus $\left|\bar{\boldsymbol{\delta}}(t)\right|$

Known from Figure 19, the modulus of the error $\left|\bar{\delta}_1(t)\right| = 0.0313$ is larger than the threshold c at time t=0.2732 month. Hence, we need to reinitialize $\hat{\boldsymbol{x}}(t) = \tilde{\boldsymbol{x}}(t)$ at time t=0.2732 month. Clearly, seen from Figure 12 $\hat{\boldsymbol{x}}(t)$ is reinitialized at time t=0.2732 month and $\hat{x}_1(t) = 0.4036$, $\hat{x}_2(t) = 0.4038$ and $\hat{x}_3(t) = 0.1926$ before reinitializing $\hat{\boldsymbol{x}}(t) = \tilde{\boldsymbol{x}}(t)$ at time t=0.2732 month and $\hat{x}_1(t) = 0.4348$, $\hat{x}_2(t) = 0.3883$ and $\hat{x}_3(t) = 0.1769$ after reinitializing $\hat{\boldsymbol{x}}(t) = \tilde{\boldsymbol{x}}(t)$ at time t=0.2732 month. From Figure 15 and Figure 19 we also can know that the initial error $\bar{\boldsymbol{\delta}}(t)$ between $\hat{\boldsymbol{x}}(t)$ and $\tilde{\boldsymbol{x}}(t)$ is zero, and then gradually increase in modulus, but the errors suddenly reduce to zero at time t=0.2732 month, which clearly is due to a re-initialization with $\hat{\boldsymbol{x}}(t) = \tilde{\boldsymbol{x}}(t)$ at time t=0.2732 month, since then the errors modulus are lower than the constant c and eventually tend to zero, which are consistent with the theoretical analysis results. In accordance with Figure 12 and Figure 13, and as well as Figure 14 we can know the steady-state values of the systems $\hat{\boldsymbol{x}}(\infty)=\tilde{\boldsymbol{x}}(\infty)=\boldsymbol{x}(\infty)=\begin{bmatrix}0.3333 & 0.3333 & 0.3333\end{bmatrix}^{\mathrm{T}}$, and from Figure 15, and Figure 16, and as well as Figure 18 we can know $\bar{\boldsymbol{\delta}}(\infty)=\hat{\boldsymbol{\delta}}(\infty)=\tilde{\boldsymbol{\delta}}(\infty)=\begin{bmatrix}0 & 0 & 0\end{bmatrix}^{\mathrm{T}}$, and from Figure 17 we can know $u_1(\infty)=0.4165$ and $u_2(\infty)=0.1750$, which are consistent with the theoretical calculation results and show that the proposed control method is very effective.

VI. CONCLUSIONS

Preventive maintenance strategy deigned in this paper effectively improves the availability, and reduces the correction maintenance cost and MTTF, and provides theory support and decision-making reference for the maintenance management for those large complex equipments. The proposed control strategy corrects the probability distribution of random inspection in this paper, and effectively saves the inspection maintenance cost and ensures the stability of production process. In the end, some simulation experiments have been done and the simulation results show that the control strategy proposed in this paper is correct and effective. Thus, applying this method to guide the maintenance practice activities of the enterprises would be more objective and effective.

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