ORIENTATION OF A TRIAXIAL ACCELEROMETER USING A HOMOGENEOUS TRANSFORMATION MATRIX AND KALMAN FILTERS

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Abstract- The evolution in the development of manufacturing techniques of electronic components, including accelerometers, has allowed access to a new field of research and applications in consumer electronics. The aim of this work is to present a method for aligning triaxial accelerometers, finding the parameters of the rotation, the translation and the scale of the homogeneous transformation matrix. In principle, it is necessary to acquire six points to build the frame of reference of the accelerometer and ensure the consistency of the measurements, in order to check the angle between the axis and the magnitude. Subsequently, using spatial geometry, the intersection of the system of reference is estimated, to determine the extent of translation in the homogeneous transformation matrix. In a further step, the rotation values of the matrix are generated by taking the orientation of the z-axis into account and, finally, the resulting factor is scaled to normalize the magnitude value of gravity. Using the transformation matrix, it is possible to align the original reference system of the accelerometer to another coordinate system. The satisfactory results of this experiment show the need of implementing the here described method to enable the use of variable tilt measurements.

Index terms: Accelerometer, alignment, tilt, translation, triaxial, rotation
I. INTRODUCTION

Accelerometers are sensors designed to measure the change in the velocity of an object over time. Currently, the conditions of manufacturing, and especially the reduction in size, have allowed their occurrence in many everyday applications. Due to the need for tools that promote the interaction of man and machine more effectively, a whole field of research has developed to study sensors that improve user-friendliness and allow a more direct interaction. In this context, accelerometers are now present in almost all mobile communication devices, and in combination with magnetometers and gyroscopes they are used in the construction or improvement of navigation systems. Accelerometers have been used in manifold applications, e.g. as shown in [1], as a method for assessing dental hygiene, using a low-cost triaxial accelerometer, in human identification [2], or to classify basic human movements using data collected from an accelerometer [3–5], in space applications [6], also for estimating the angle of the leg segments [7] or for general measurements of the spatial inclination, as well for measurements on vehicles using Kalman filters [8], or RLS filters using Lattice Algorithm [9], as for estimating the path of a motorcycle [10]. In further applications, a combination of accelerometers and gyroscopes was used to reconstruct paths [11], to position mobile robots in two dimensions [12], to measure structural displacements [13], and last but not least, to facilitate the human-computer interaction for people with cerebral palsy [14], among others. A common problem in the development of these systems is presented by the need of a physical model of the sensor [15], to obtain particular design characteristics [16], and for an adequate estimation of the measurement error [17]; considering that most of the operations are performed cumulatively as a discrete integration to obtain the position or velocity, error reduction is crucial. However, it is not easy to produce low-cost inertial measurement systems, which allow adjusting and aligning spatial coordinates in standard measurement formats. The authors of [18,19] present an algorithm to adjust the orientation of a complete inertial system, although the offset and the scale adjustment were not taken into account.

This paper presents a method that uses the elements of a homogeneous transformation matrix (rotation, translation and scaling) to align a general-purpose triaxial accelerometer. To accomplish this, it was necessary, mainly, to implement a Kalman filter [20–22] with the aim of reducing the noise of the measurement, as recommended in [7,10,15]; while studying the signals of the observed accelerometers allowed to determine the nature of the noise. Then using a robot manipulator KR5


*KUKA arc HW* (as a reference system) repeated measures were taken in the eight basic orientations for each accelerometer. The robot positioning repeatability is 0.04 mm so that the reliability is ensured in successive tests. With the data package of each sensor the data consistency is given and then the homogeneous transformation matrix is built to move the focus and the scale of each accelerometer. Finally, the implementation of this method in some low-cost commercial accelerometers will be presented and some comparisons will be drawn to demonstrate the need for improvement. The mathematical study of methods that improve measurement quality allows a wider range of applications for accelerometers. In Section 2 of this document the equipment, used in the experiments, will be presented, in Section 3 the mathematical techniques will be described, in Section 4 the methodological solution of the problem of alignment will be shown, and finally, in Section 5 the results of applying the alignment to a set of accelerometers will be revealed.

II. MATERIALS

a. Accelerometers

In order to evaluate the here presented alignment method, four different triaxial accelerometers were used, namely, two digital accelerometer, the ADXL345 from Analog Devices and the LSM303DLH (3-axis accelerometer and 3-axis magnetometer) from ST Microelectronics, as well as two analog accelerometers, the MMA7260QT from Freescale Semiconductor and the ADXL335 from Analog Devices. All accelerometers were used in the lower measurement range. Table 1 presents some relevant features of the devices.

<table>
<thead>
<tr>
<th></th>
<th>MMA7260</th>
<th>ADXL335</th>
<th>ADXL345</th>
<th>LSM303</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Type</strong></td>
<td>Analog</td>
<td>Analog</td>
<td>Digital</td>
<td>Digital</td>
<td>NA</td>
</tr>
<tr>
<td><strong>Resolution</strong></td>
<td>−</td>
<td>−</td>
<td>10</td>
<td>16</td>
<td>[bit]</td>
</tr>
<tr>
<td><strong>Measurement Range</strong></td>
<td>±1.5/±2/±4/±6</td>
<td>±3</td>
<td>±2/±4/±8/±16</td>
<td>±2/±4/±8</td>
<td>[g]</td>
</tr>
<tr>
<td><strong>Package Alignment Error</strong></td>
<td>±1.0</td>
<td></td>
<td></td>
<td></td>
<td>[Degrees]</td>
</tr>
<tr>
<td><strong>Interaxis Alignment Error</strong></td>
<td>±0.1</td>
<td>±0.1</td>
<td></td>
<td></td>
<td>[Degrees]</td>
</tr>
<tr>
<td><strong>Bandwidth X Y</strong></td>
<td>350¹</td>
<td>1600¹</td>
<td>1600²</td>
<td></td>
<td>[Hz]</td>
</tr>
<tr>
<td><strong>Bandwidth Z</strong></td>
<td>150¹</td>
<td>500¹</td>
<td></td>
<td></td>
<td>[Hz]</td>
</tr>
</tbody>
</table>

¹Bandwidth without external filter
²Bandwidth is −3 dB frequency and is half the output data rate, bandwidth = ODR/2
b. KR5 KUKA arc HW

The KR5 KUKA arc HW is a robot specially designed for arc welding. It has special features like an opening in the arm and the wrist to protect cable packages or tubes through the manipulator. Table 2 presents the major characteristics of the manipulator.

Table 2: Features of the manipulator

<table>
<thead>
<tr>
<th>Reference</th>
<th>KR5 arc HW</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load</td>
<td>5 Kg.</td>
</tr>
<tr>
<td>Scope</td>
<td>1423 mm.</td>
</tr>
<tr>
<td>Axes</td>
<td>6</td>
</tr>
<tr>
<td>Repeatability</td>
<td>&lt; ±0.04 mm.</td>
</tr>
<tr>
<td>Weight</td>
<td>120 Kg.</td>
</tr>
<tr>
<td>Control Unity</td>
<td>KRC2</td>
</tr>
</tbody>
</table>

Figure 1 shows the arrangement of the six axes. For the experiment the accelerometers were located between the axis A5 and A6.

Figure 1: Axes of KR5 KUKA arc HW
III. SOLUTION

a. Kalman Filter 1D

The output signal of the accelerometer has a high noise component that can be characterized as Gaussian noise. Therefore it was necessary to implement a Kalman filter 1D. The equations are shown in Equation (1) where $\hat{X}_k$ the estimated value is, $Z_k$ is the measured value and $K_k$ is the Kalman gain.

$$\hat{X}_k = K_kZ_k + (1 - K_k)\hat{X}_{k-1}$$  \hspace{1cm} (1)

To calculate $K_k$ it is necessary to know or estimate the error covariance of the process $P$, the error covariance of the observation $R$ that in the case of the accelerometers can be obtained with the probability density function of the data while the accelerometer is static. If the value of $R$ is known, the process shown in Equation (2) will be used, where $m$ is a vector of previously acquired raw samples to obtain the nature of the noise. Note that in this case, since the application of the filter in a one-dimensional signal is scaled, the matrix $A = 1$, and the deviation $Q$ should be adjusted to the speed of the system.

$$P_k = \frac{P_k}{P_k + R}$$

$$\hat{X}_k = \hat{X}_{k-1} + K_k(Z_k - \hat{X}_{k-1})$$

$$P_k = (1 - K_k)P_{k-1}$$  \hspace{1cm} (2)

$$X_{(0)} = mean(m)$$

$$P_{(0)} = mean(m)$$

$$R = \sigma^2(m)$$

To apply the linear Kalman filter, it must be absolutely assured that the nature of the noise is Gaussian. Figure 2 shows the PDF of the static signal of $ADXL345$ on the x-, y- and z-axis before and after applying the Kalman filter. These results were obtained by placing the accelerometer on an anti-vibration table with a sampling frequency of 150 Hz.
Figure 2: (a) Signal $X$ axis before, and after applying the Kalman filter; (b) PDF $X$ axis, before and after applying the Kalman filter; (c) Signal $Y$ axis before, and after applying the Kalman filter; (d) PDF $Y$ axis before, and after applying the Kalman filter; (e) Signal $Z$ axis before, and after applying the Kalman filter; (f) PDF $Z$ axis before, and after applying the Kalman filter
b. Representation of the coordinate system

Initially it is necessary to demonstrate that the coordinate system of the accelerometer is correctly aligned. In order to obtain this, the accelerometer is oriented in six possible directions using an industrial manipulator KR5 KUKA arc HW. Using a static accelerometer in each of the six directions 500 readings are acquired. With the average of those values the vectors $P_{x+}, P_{x-}, P_{y+}, P_{y-}, P_{z+}, P_{z-}$ are constructed, as shown in Figure 3. Each point was referenced to the corresponding axis $(P_{x+}, P_{x-}, P_{y+}, P_{y-}, P_{z+}, P_{z-})$ of the accelerometer in a direction opposite to gravity.

![Figure 3: Representation of the coordinate system](image)

Equation (3) shows the notation that will have the points representing the original coordinate system of the accelerometer.

$$
P_{x+} = \begin{bmatrix} a_{x1} \\ a_{y1} \\ a_{z1} \end{bmatrix}; P_{x-} = \begin{bmatrix} a_{x2} \\ a_{y2} \\ a_{z2} \end{bmatrix}; P_{y+} = \begin{bmatrix} b_{x1} \\ b_{y1} \\ b_{z1} \end{bmatrix}; P_{y-} = \begin{bmatrix} b_{x2} \\ b_{y2} \\ b_{z2} \end{bmatrix}$$

$$
P_{z+} = \begin{bmatrix} c_{x1} \\ c_{y1} \\ c_{z1} \end{bmatrix}; P_{z-} = \begin{bmatrix} c_{x2} \\ c_{y2} \\ c_{z2} \end{bmatrix}
$$

(3)

Now, using the corresponding points enables to obtain the parametric equations representing each of the axes as shown in Equation (4), where $v_x, v_y$ and $v_z$ represent the direction vectors.
Before performing the process that finds the parameters of the homogeneous transformation matrix, the data consistency has to be validated. First one must find the magnitude of the axes \( M_x, M_y \) and \( M_z \) and the angles between the axes. With these values the correspondence of the scale and the internal orthogonality can be obtained. Equation (5) shows how to obtain the magnitude values and the angles between the axes.

\[
\begin{align*}
M_x &= \sqrt{(a_{x_2} - a_{x_1})^2 + (a_{y_2} - a_{y_1})^2 + (a_{z_2} - a_{z_1})^2} \\
M_y &= \sqrt{(b_{x_2} - b_{x_1})^2 + (b_{y_2} - b_{y_1})^2 + (b_{z_2} - b_{z_1})^2} \\
M_z &= \sqrt{(c_{x_2} - c_{x_1})^2 + (c_{y_2} - c_{y_1})^2 + (c_{z_2} - c_{z_1})^2}
\end{align*}
\]

\[
\begin{align*}
A_{XY} &= \cos \left( \frac{\vec{v}_x \cdot \vec{v}_y}{|\vec{v}_x| |\vec{v}_y|} \right) \\
A_{YZ} &= \cos \left( \frac{\vec{v}_y \cdot \vec{v}_z}{|\vec{v}_y| |\vec{v}_z|} \right) \\
A_{XZ} &= \cos \left( \frac{\vec{v}_x \cdot \vec{v}_z}{|\vec{v}_x| |\vec{v}_z|} \right)
\end{align*}
\]
Ideally there should be a point where the three axes are crossing, but due to factors such as noise and measurement tolerance this point must be estimated. Initially, with the parametric equations that describe the axes it is possible to determine the point on the $X$, $Y$ plane where $A_x$ and $A_y$ intersect. Equation (6) describes how to obtain the parameters $k_{1i}$ and $k_{2i}$. Via substituting $k_{1i}$ into the equation of $A_x$, or $k_{2i}$ into the equation of $A_y$, it is possible to determine the point $x_i$, $y_i$ where the axes cross, as shown in Equation (7).

$$
\begin{bmatrix}
  k_{1i} \\
  k_{2i}
\end{bmatrix}
= \begin{bmatrix}
  a_{x_2} - a_{x_1} & -(b_{x_2} - b_{x_1}) \\
  a_{y_2} - a_{y_1} & -(b_{y_2} - b_{y_1})
\end{bmatrix}
\begin{bmatrix}
  b_{x_1} - a_{x_1} \\
  b_{y_1} - a_{y_1}
\end{bmatrix}
$$

(6)

Now, to find the coordinate $z_i$, the midpoint is obtained, as shown in Equation (7). The point $(x_i, y_i, z_i)$, represents an approximation of the offset of the accelerometer and hence this is the value that will be used in the homogeneous transformation matrix to move the coordinate system.

$$
x_i = \left[ (a_{x_2} - a_{x_1})k_{1i} + a_{x_1} \right] = \left[ (b_{x_2} - b_{x_1})k_{2i} + b_{x_1} \right]$$

$$
y_i = \left[ (a_{y_2} - a_{y_1})k_{1i} + a_{y_1} \right] = \left[ (b_{y_2} - b_{y_1})k_{2i} + b_{y_1} \right]$$

$$
z_i = \frac{\left[ (a_{z_2} - a_{z_1})k_{1i} + a_{z_1} \right] - \left[ (b_{z_2} - b_{z_1})k_{2i} + b_{z_1} \right]}{2}
$$

(7)

To ensure the validity of the point $(x_i, y_i, z_i)$, the distance of the point to the line $A_z$ is calculated. The distance should be negligible compared to the scale of representation of the accelerometer. It is calculated using Equation (8).

$$
\bar{v}_z = \begin{bmatrix}
  c_{x_2} - c_{x_1} \\
  c_{y_2} - c_{y_1} \\
  c_{z_2} - c_{z_1}
\end{bmatrix};

P_i = (x_i, y_i, z_i); P_{z+} = (c_{x_1}, c_{y_1}, c_{z_1})
$$

$$
dist(P_i, A_z) = \frac{|P_{z+}P_i \times \bar{v}_z|}{\bar{v}_z}
$$

(8)

If the test shown in Equation (8) is successful, the next step is to adjust the rotation matrix. Equation (9) presents the rotation matrix on the $x$-, $y$- and $z$-axis, individually.

$$
R_x = \begin{bmatrix}
  1 & 0 & 0 \\
  0 & c(\alpha) & s(\alpha) \\
  0 & -s(\alpha) & c(\alpha)
\end{bmatrix};

R_y = \begin{bmatrix}
  c(\beta) & 0 & -s(\beta) \\
  0 & 1 & 0 \\
  s(\beta) & 0 & c(\beta)
\end{bmatrix};

R_z = \begin{bmatrix}
  c(\gamma) & s(\gamma) & 0 \\
  -s(\gamma) & c(\gamma) & 0 \\
  0 & 0 & 1
\end{bmatrix}
$$

(9)
Equation (10) shows the result of multiplying $R_x$, $R_y$ and $R_z$.

$$RR = \begin{bmatrix}
c(\beta)c(\gamma) & c(\beta)s(\gamma) & -s(\beta) \\
s(\alpha)s(\beta)c(\gamma) - c(\alpha)s(\gamma) & c(\alpha)c(A_3) + s(\alpha)s(\beta)s(\gamma) & s(\alpha)c(\beta) \\
c(\alpha)s(\beta)c(\gamma) + s(\alpha)s(\gamma) & c(\alpha)s(\beta)s(\gamma) - s(\alpha)c(\gamma) & c(\alpha)c(\beta)
\end{bmatrix} \quad (10)$$

To align the coordinate system, taking into account that the alignment was evaluated between the axes, it is sufficient to align one axis, in this case the $z$-axis. By pre-multiplying the rotation matrix $RR$ for the ideal value orientation, the direction vector of the line representing the axis $A_z$ can be obtained. Equation (11) shows the resulting relation.

$$\begin{bmatrix} m_x \\ m_y \\ m_z \end{bmatrix} = \begin{bmatrix} c_{x_2} - c_{x_1} \\ c_{y_2} - c_{y_1} \\ c_{z_2} - c_{z_1} \end{bmatrix} = RR \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -s(\beta) \\ s(\alpha)c(\beta) \\ c(\alpha)c(\beta) \end{bmatrix} \quad (11)$$

The relation presented in Equation (11), can be solved for the values, $\alpha$ and $\beta$. The result is shown in Equation (12)

$$\alpha = \text{atan2} \left( \frac{m_y}{m_z} \right); \quad \beta = \text{atan2} \left( -\frac{m_x}{\sqrt{m_y^2 + m_z^2}} \right) \quad (12)$$

With the values found, the homogeneous transformation matrix shown in Equation (13) is constructed.

$$MT = \begin{bmatrix} -x_i & -y_i & -z_i \\ R(\alpha, -\beta, 0) \\ 0 \\ 0 \end{bmatrix} \quad (13)$$
IV. EXPERIMENTAL RESULTS

Table 3 shows the values found after applying the procedure described in Eq. (5). The misalignment between the axes is less than 1 degree in all cases and the values of magnitude maintain an acceptable correspondence. Section 3.2 explains the method to obtain the coordinate axes, presented in Table 1 as $M_X$, $M_Y$ and $M_Z$.

**Table 3:** Comparison of accelerometers

<table>
<thead>
<tr>
<th></th>
<th>MMA7260</th>
<th>ADXL335</th>
<th>ADXL345</th>
<th>LSM303</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_X$</td>
<td>496.4487</td>
<td>204.3814</td>
<td>531.3149</td>
<td>3.2866E + 04</td>
<td>NA</td>
</tr>
<tr>
<td>$M_Y$</td>
<td>501.4278</td>
<td>208.1339</td>
<td>525.3441</td>
<td>3.2272e + 04</td>
<td>NA</td>
</tr>
<tr>
<td>$M_Z$</td>
<td>506.1498</td>
<td>207.1296</td>
<td>499.5091</td>
<td>3.2043e + 04</td>
<td>NA</td>
</tr>
<tr>
<td>$A_{XY}$</td>
<td>90.0678</td>
<td>90.0568</td>
<td>90.1798</td>
<td>90.6674</td>
<td>[Degrees]</td>
</tr>
<tr>
<td>$A_{YZ}$</td>
<td>90.1932</td>
<td>89.1871</td>
<td>89.9305</td>
<td>89.4960</td>
<td>[Degrees]</td>
</tr>
<tr>
<td>$A_{XZ}$</td>
<td>89.4585</td>
<td>90.3971</td>
<td>90.6987</td>
<td>90.8070</td>
<td>[Degrees]</td>
</tr>
</tbody>
</table>

If the data for the internal characteristics of the accelerometer are acceptable the techniques can be applied to find the alignment parameters. Table 4 shows the results with the accelerometers, here used.

**Table 4:** Value alignment

<table>
<thead>
<tr>
<th></th>
<th>MMA7260</th>
<th>ADXL335</th>
<th>ADXL345</th>
<th>LSM303</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_i$</td>
<td>521.5730</td>
<td>503.8008</td>
<td>−3.1926</td>
<td>81.4372</td>
<td>NA</td>
</tr>
<tr>
<td>$y_i$</td>
<td>485.7294</td>
<td>524.9800</td>
<td>14.7078</td>
<td>−173.2843</td>
<td>NA</td>
</tr>
<tr>
<td>$z_i$</td>
<td>519.5430</td>
<td>511.5891</td>
<td>−27.2574</td>
<td>514.6308</td>
<td>NA</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>−3.1337</td>
<td>−3.1334</td>
<td>3.1101</td>
<td>−3.1414</td>
<td>[Degrees]</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.0068</td>
<td>0.0094</td>
<td>0.0044</td>
<td>0.0043</td>
<td>[Degrees]</td>
</tr>
<tr>
<td>$dist$</td>
<td>1.1708</td>
<td>0.5706</td>
<td>0.9595</td>
<td>14.0875</td>
<td>NA</td>
</tr>
</tbody>
</table>

The data presented shows the need for the alignment algorithm, and that the problems associated with the binary representation are corrected for the acquisition of digital analog converters x and the system is moved to 0. Figure 4 shows the processing carried out for each of the accelerometers used in the experiments. In Figure 4, the yellow Cartesian axes represent the location and
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orientation of the Cartesian axes after applying the here described algorithm. The axes shown in cyan, magenta and blue represent the position and original orientation for each type of accelerometer.

Figure 4. Axis transformation. (a) ADXL335, (b) MMA7260, (c) ADXL345, (d) LSM303

V. CONCLUSION

In this work, we presented a methodology for using a homogeneous transformation matrix in order to rotate, translate and scale the axes of the coordinate system of a general-purpose accelerometer, and transform it into a standard reference system defined with an industrial manipulator robot. Tests showed that for the four analyzed accelerometers alignment is required. Additionally, it was shown that the used method is easy to implement, because the parameters are calculable in the
offline mode. Although in this paper we used an industrial manipulator robot to define the values of the reference system, ensuring the location of the accelerometer in an orthogonal parallelepiped (cuboid) and supporting it on a level surface when performing the measurements to construct the axes, probably would have shown the same results. Given the current interest in implementing algorithms that use information from inertial measurements, it is of utmost importance to improve the mathematical tools that are available for the data processing. The future work should be aimed at reducing the computational cost of the algorithms, while at the same time improving the quality of the measurements.

Appendix 1

The following explains the procedure for the Kalman filter construction described in Section 1

1. First the validity of the use of a Kalman filter on the system described in Equation (14) is defined.

\[ X_k = AX_{k-1} + Bu_k + w_{k-1} \]

\[ Z_k = HX_k + v_k \]  

(14)

The signal \( X_k \) must be a linear combination of previous values of \( X \) in addition to the control signal \( u \) plus a Gaussian noise source \( w \). In the output equation \( Z \) is the measure and \( v \) is a Gaussian noise source. The process noise \( w \) and measurement noise \( v \) are statistically independent. \( A \) is the state transition matrix, and the measurement matrix is \( H \).

2. The estimation process is performed first in the update phase (prediction) as shown in Equation (15). Where \( P_k^- \) is the covariance error of the process, and \( Q \) is

\[ \hat{X}_k = AX_{k-1} + Bu_k \]

\[ P_k^- = AP_{k-1}A^T + Q \]  

(15)

3. Now it is necessary to develop the phase measurement update (correction). See Equation (16).
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\[ K_k = P_k^{-1}H^T + (HP_k^{-1}H^T + R)^{-1} \]

\[ P_k = (I - K_kH)P_k^{-1} \] (16)

To calculate \( K_k \) it is necessary to know or estimate the error covariance of the process \( P \), and the error covariance of the observation \( R \). If the value of \( R \) is known, we use the process shown in Equation (17), where \( m_i \) \((i = 1, 2, 3, \ldots \)) is a vector of previously acquired raw samples to identify the nature of the noise.

\[ X_{(0)} = \text{mean}(m) \]

\[ R = \sigma^2(m) \] (17)

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REFERENCES


