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INVESTIGATION ON PHOTOELECTRIC THEODOLITE DATA PROCESSING AND RANDOM ERRORS MODEL

Xiang Hua, Jinjin Zhang and Bin Lei

School of Electronic Information Engineering, Xi'an Technological University, Xi'an, 710021, China

Email: huaxiang79@126.com, xixiangxiang@gmail.com, gropenind@163.com

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Abstract- ***Measure error of photoelectric theodolite would influence result precision when tracking flight target. This paper researches error problem of testing. It analyses all the causes of error forming, divides them to several sorts: system error, random error and outlier error, then provides resolution method to each sort. Especially for random error, it builds an error model, analyses the properties of unbiased, equal variance and uncorrelated, conducts best error estimate, discuss the relationship between choosing and effect of kernel function and smooth parameter. While it researches measure theory of coplanar intersection and dis-coplanar intersection of photoelectric theodolite, derives a series of measure formulas, builds random error model respectively, and analyses the relationship of effect actors. By comparing simulation of model with experiment measurement, the result shows the error model and processing method is correct.***

Index terms: photoelectric theodolite; data process; error model; random error; error estimate

I. INTRODUCTION

Photoelectric theodolite has many functions as dynamic tracking and image reappearing, which take on the features of high precision and quick response. It's suitable for using accurate measure movment trail of dynamic target, so it can be widely applied in aerospace experiment and low-attitude flyer test in scientific research [1, 2, 3, 4].

With the measure technology maturing, the measure method of photoelectric theodilite gets changing to add associated measures and mix data fuse, which from the simple optics technology to combining electronics, communication and multiple technologies[5]. For the classical photoelectric theodilite can just measure the azimuth and elevation between the target and the occupied station, it needs two or more measure stations working together to ascertain the target's space position by computing [6]. While the modern photoelectric theodilite installing laser range measurement device can afford single occupied station mode, double occupied station mode or multiple occupied station mode, especially multi-occupied station mode provides quick tracking capability and fine accuracy [7, 8]. So it would affect the measure result directly of the precision of device that the location designing of occupied station and the error of data fuse.

This paper researches test problem in photoelectric theodolite, analyses error cause, builds error model, discusses random error estimation, measures coplanar intersection and dis-coplanar intersection principle of double occupied station, works out geometry relationship between angle and distance, builds error model, simulates error outcome, proves model correctness.

In 1950s, the "L" formula and "K" formula was used in optical to measure by using geometry projection to work out dynamic position parameters in intersection measure target of photoelectric theodolites[9,10,11]. In 1960s, least squares estimation was used to work out multi-occupied station intersection to measure dynamic target position parameters, and then polynomial optimal filter was used to differentially solve velocity parameters. Nowadays, there are many of wireless measure systems appearing with high precision and long distance performance, for example use "EMBET" self-adjust offset error method and continue wave system intersection Markoff estimation to solve parameters[12,13,14].

Least squares estimation is not related to observed data's distribution performance, the principle is not too complex, the mathematics model and the compute method are also bridfness, and programme is easy to achieve, specially for the data obeying normal distribution rigorously the least squares estimation takes on optimal unbiased and variance mimimum[15,16]. It is proved that most data follow Gaussian distribution except wild error condition, thus least squares estimation receives seriously valued and rapid application. Up to now it is the base in estimation theory [17].

II. ERROR MODEL OF PHOTOELECTRIC THEODOLITE

A. Regression error model

The paper is under the condition of making time sequence as united reference[18], in which all the parameters of photoelectric theodolite just have means, such as azimuth, elevation and position, etc.

If the time begins from *T0*, equal sample interval is $\triangle t$, sample sequence can be expressed: $Ti = \{T0, T0+\triangle t, T0+2*\triangle t, T0+3*\triangle t, \cdots\}$

Each sample points' measure information is *g(Ti)*, all the sample points' information form a full sequence of measure data, the data assemblage can be expressed as:

$$G = \sum_{i=0}^{n} g(T0 + i * \Delta t) \tag{1}$$

When a certain photoelectric theodolite tracking a space movement, consecutive tracing measure information *g(t)* can be divided to ideal output value *gT (t)* and error sum *ε(t)*.

$$g(t) = g_T(t) + \varepsilon(t) \tag{2}$$

$g_T(t)$ is the ideal output value.

$\varepsilon(t)$ is the sum of errors.

As *gT(t)* is continuous smooth intergral function in sample space, by using approximation theorem in every closed interval, the true value function *gT(t)* can be infinite approximated in the interval through algebraic polynomial.

Thus it can be made very small number $\delta > o$, approximate polynomial would be:

$$g(t) = \sum_{i=0}^{n} d_i t_i \tag{3}$$

Given:

$$\max_i \left| g_T(t) - g(t) \right| < \delta \qquad \{i=0,\cdots,n\} \tag{4}$$

Sum error includes system error, random error and outlier error:

$$\varepsilon(t)=\varepsilon_s(t)+\varepsilon_r(t)+\varepsilon_w(t) \tag{5}$$

$\varepsilon_s(t)$: system error

$\varepsilon_r(t)$: random error

$\varepsilon_w(t)$: outlier error

System error $\varepsilon_s(t)$ is inherence for a certain group of sample data. Usually it is produced by hardware manufacture error, mainly including axis error and location error, such as vertical axis slope error, horizontal axis slope error, collimation axis error, coding orientation error, level of zero potential difference, etc[19]. These system errors would be evaluated and corrected with specific hardware.

Random error $\varepsilon_r(t)$ always exists in uncertain value without regularity, under a certain observing condition to make several times and repeat measure in time sequence[20]. While in the mass it obeys some statistics characteristic error on mean value or variance distribution. The random error observed by photoelectric theodolite is white noise in time sequence, that can be presented on random error sequence $\{\varepsilon r(Ti),\ i=0,\cdots,n\}$, and takes on properties of unbiased , equal variance and uncorrelated.

$$E(\varepsilon_{ri})=0 \tag{6}$$

$$E(\delta_{ri},\delta_{rj}) = \begin{bmatrix} \delta^2 & 0 & 0 & \cdots & 0 \\ 0 & \delta^2 & 0 & \cdots & 0 \\ 0 & 0 & \delta^2 & \cdots & 0 \\ & & \cdots & & \\ 0 & \cdots & 0 & 0 & \delta^2 \end{bmatrix} \tag{7}$$

Outlier error $\varepsilon_w(t)$ is very bigger or smaller obviously than true value, also called wild-value, jump spot or wrong error. It reflects that some unusual factors cause the observed data to deflects normal trend and shows individual saltus[21]. Usually it brought by abrupt factor disturbing as bit error, electromagnetic pulse, etc. The outlier error is obviously irrational and disadvantageous to observed result. Because of being differentiated easy, customarily it may be wiped off by some mathematics method to avoid influence error manage during data pre-processing phase. Its common value is performance zero, while occasionally is very greater than standard deviation of random error.

$$\left|\varepsilon_w(t)\right| = \begin{cases} \approx 0 & \text{when sample data is normal} \\ >> \delta^2 & \text{when sample data obviously deflects whole trend} \end{cases} \quad (8)$$

In brief, random error can't be avoided because of unregular and unforecast. It's need to research on model build and best estimate for random error. The following is aimed at random error model to settle statistics estimate method. The model can be simplified to:

$$g(t)= g_T(t)+ \varepsilon_r(t) \quad (9)$$

B. Random error estimate model

Suppose g(T_i) is an independent distributed random sample from the model above.

$$g(T_i)= g_T(T_i)+\varepsilon_i\ ，\ i=1,\cdots,n \quad (10)$$

Error sequence $\{\varepsilon_i\}$ is satisfied that:

$$E(\varepsilon_i)=0，\ Var(\varepsilon_i)= \delta^2 < \infty，\quad i=1,\cdots,n \quad (11)$$

On every t_0 consider the estimate of $g_T(t_0)$, Talor formula is used to make $g_T(t_0)$ first order expansion on t_0 . It may approximate expression to :

$$g_T(t) \approx g_T(t_0)+ g_T(t_0)\ (t - t_0) \quad (12)$$

By minimizing object function, first-order approximation can be get[22]. When second-order matrix existing, by the formula following the data can be processed liner and smooth well.

$$\sum_{j=1}^{m}\left[\sum_{i=1}^{n}\left(g_i - a_k - b(t_i - t_0)\right)^2 K\left(\frac{t_i - t_0}{h}\right)\right] \quad (13)$$

When second-order matrix can't existing, this estimate method would not agree, and should adopt fractile regression method[23], that select m of points $\{\tau_j = j/(m+1), j=1,2,\cdots,m\}$, checking the loss function corresponding on τ_j is $\rho_{\tau j}(r) = \tau_j - rI(r<0), j=1,2,\cdots,m$. So the local equal-weight cumulative sum of m loss functions with m points may be presented the object function:

$$\sum_{j=1}^{m}\left[\sum_{i=1}^{n}\rho_{\tau j}\{g_i-a_j-b(t_i-t_0)\}K\left(\frac{t_i-t_0}{h}\right)\right] \tag{14}$$

Computing the optimum solution, as $(\hat{a}_1,\hat{a}_2\cdots,\hat{a}_m,\hat{b})$. Then $g(t_0)$ would be estimated to:

$$\hat{g}(t_0)=\frac{1}{m}\sum_{j=1}^{m}\hat{a}_j=\frac{1}{m}E^T\hat{a} \tag{15}$$

Thereinto *E* is the m-dimension column vector at which element is *1*.

$$\hat{a}=(\hat{a}_1,\hat{a}_2,\cdots,\hat{a}_m)^T \tag{16}$$

$K(\cdot)$ is kernel function, it can be choosen with Boxcar kernel, Gaussian kernel, Epanechnikov kernel or Tricube kernel, ect. During theoretical reducing, estimate results are very closed whatever choosing different kernel function[22,23]. Namely estimate result is not sensitive to the kernel function chosen.

h is smooth parameter of bandwidth chosen, that control the smooth degree. If using smaller bandwidth it would get estimate with accurate but rough, else using bigger bandwidth it would get estimate with smooth but inaccurate[24,25]. Thus, a perfect choice of *h* is rigorous to obtain a good smooth estimation. It may be selected for reference as below.

$$K(x)=1/2I(x) \tag{17}$$

$$h=\left(\frac{1}{n}\right)^{1/5}\left\{\frac{\delta^2\int K^2(x)dx\int dx/f(x)}{\left[\int x^2K^2(x)dx\right]^2\int\left[r''(x)+2r'(x)\frac{f'(x)}{f(x)}\right]^2dx}\right\}^{1/5} \tag{18}$$

III. ERROR ESTIMATE OF PHOTOELECTRIC THEODOLITE

A. Error estimate of coplanar intersection

The photoelectric theodolite without laser range measurement can only provide the azimuth A and elevation E of the target relatively to the occupied station. For the target is three-dimensional in space, it should use two or more photoelectric theodolites intersecting to ascertain the target location[26,27,28]. Coplanar intersection uses the method that two lines intersect in one point to ensure space location. The principle is as the figure 1.

Suppose two occupied stations coordinate are $O_1(x_1, y_1, z_1)$ and $O_2(x_2, y_2, z_2)$ respectively, A_1 is the azimuth of P relatively to O_1, A_2 is the azimuth of P relatively to O_2, E_1 is the elevation of P relatively to O_1, E_2 is the elevation of P relatively to O_2, L_1 is made by P and O_1, L_2 is made by P and O_2. By means of space geometry theory, we can compute the coordinate of space target as following.

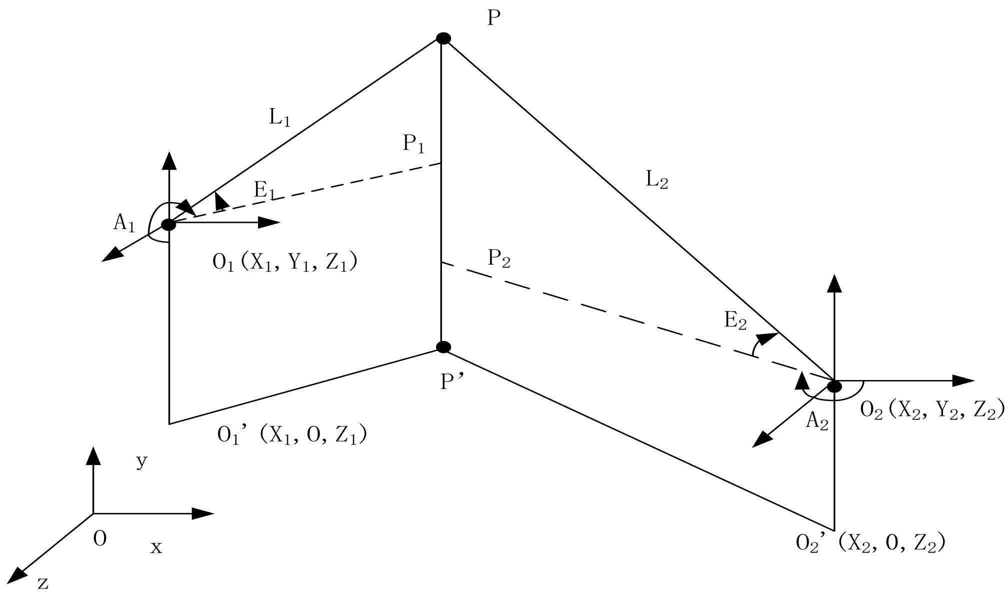

Figure 1. Measurement theory of coplanar intersection

$$X_p=x_1+\Delta X_1$$
$$Y_p=y_1+\Delta X_1 \tan E_1/\cos A_1$$
$$Z_p=z_1+\Delta X_1 \tan A_1 \tag{19}$$

Or:

$$X_p=x_2+\Delta X_2$$
$$Y_p=y_2+\Delta X_2 \tan E_2/\cos A_2$$
$$Z_p=z_2+\Delta X_2 \tan A_2 \tag{20}$$

There into:

$$\Delta X_1=[(x_2-x_1)\tan A_2-(z_2-z_1)]/(\tan A_2-\tan A_1)$$

$$\Delta X_2=[(x_2\text{-}x_1)\tan A_1\text{-}(z_2\text{-}z_1)]/(\tan A_2\text{-}\tan A_1) \tag{21}$$

After computing, the space target can be derived:

$$\begin{cases} X = X_1 + \dfrac{(X_1 - X_2)\tan A_2 - (Z_1 - Z_2)}{\tan A_1 - \tan A_2} \\ Y = Y_1 + \dfrac{(X_1 - X_2)\tan A_2 - (Z_1 - Z_2)}{\tan A_1 - \tan A_2}\sec A_1 \tan E_1 \\ Z = Z_1 + \dfrac{(X_1 - X_2)\tan A_2 - (Z_1 - Z_2)}{\tan A_1 - \tan A_2}\tan A_1 \end{cases} \tag{22}$$

By error propagation rule, it can be conducted the variance of random error on X、Y、Z:

$$\begin{cases} \sigma_X = \left[(\dfrac{\partial X}{\partial A_1})^2 + (\dfrac{\partial X}{\partial A2})^2 \right]^{\frac{1}{2}} \sigma \\ \sigma_Y = \left[(\dfrac{\partial Y}{\partial A_1})^2 + (\dfrac{\partial Y}{\partial A_2})^2 + (\dfrac{\partial Y}{\partial E_1})^2 \right]^{\frac{1}{2}} \sigma \\ \sigma_Z = \left[(\dfrac{\partial Z}{\partial A_1})^2 + (\dfrac{\partial Z}{\partial A_2})^2 \right]^{\frac{1}{2}} \sigma \end{cases} \tag{23}$$

Because the two angle measure systems of photoelectric theodolite are uncorrelated, that means error according to $A_1, A_{2,} E_1, E_2$ separately are mutually independence, the random noise can be reduced:

$$C = \begin{bmatrix} \sigma_x^2 & 0 & 0 \\ 0 & \sigma_y^2 & 0 \\ 0 & 0 & \sigma_z^2 \end{bmatrix} \tag{24}$$

In the formula 23, there into:

$$\frac{\partial X}{\partial A_1} = \frac{-(X_1 - X_2)\sin A_2 \cos A_2 + (Z_1 - Z_2)\cos^2 A_2}{\sin^2(A_1 - A_2)}$$

$$\frac{\partial X}{\partial A_2} = \frac{(X_1 - X_2)\sin A_1 \cos A_1 + (Z_1 - Z_2)\cos^2 A_1}{\sin^2(A_1 - A_2)}$$

$$\frac{\partial Y}{\partial A_1} = \frac{-\cos(A_1 - A_2)[(X_1 - X_2)\sin A_2 - (Z_1 - Z_2)\cos A_2]}{\sin^2(A_1 - A_2)}\tan E_1$$

$$\frac{\partial Y}{\partial A_2}=\frac{(X_1-X_2)\sin A_1-(Z_1-Z_2)\cos A_2}{\sin^2(A_1-A_2)}\tan E_1$$

$$\frac{\partial Y}{\partial E_1}=\frac{(X_1-X_2)\sin A_1-(Z_1-Z_2)\cos A_2}{\sin^2(A_1-A_2)}\sec^2 E_1$$

$$\frac{\partial Z}{\partial A_1}=\frac{-(X_1-X_2)\sin^2 A_2+(Z_1-Z_2)\sin A_2\cos A_2}{\sin^2(A_1-A_2)}$$

$$\frac{\partial Z}{\partial A_2}=\frac{(X_1-X_2)\sin^2 A_1-(Z_1-Z_2)\sin A_1\cos A_1}{\sin^2(A_1-A_2)} \qquad (25)$$

B. Error estimate of dis-coplanar intersection

When two photoelectric theodolites measure the same target *P(X,Y,Z)*, usually there primary optical axes cannot intersect and be on different coplanar for many reasons[29]. $O_1 P_1$, O_2P_2 are two optical axes of each photoelectric theodolite. $P_1 P_2$ is the common perpendicular, that formed by two plane $O_1 P_1 P'_1 O'_1$ and $O_2 P_2 P'_2 O'_2$.

According to the *O-XYZ* space rectangular coordinates system, O_1 is one photoelectric theodolite, (X_1, Y_1, Z_1) is the coordination, A_1 is the azimuth relatively to P_1, E_1 is the elevation, L_1 is formed in space. O_2 is another photoelectric theodolite, (X_2, Y_2, Z_2) is the coordination, A_2 is the azimuth relatively to P_2, E_2 is the elevation, L_2 is formed in space. By using space geometry, there would be conducted:

$$L_1=\frac{X_{P1}-X_{o1}}{\cos A_1}=\frac{Y_{P1}-Y_{o1}}{tgE_1}=\frac{Z_{P1}-Z_{o1}}{\sin A_1}$$

$$L_2=\frac{X_{P2}-X_{o2}}{\cos A_2}=\frac{Y_{P2}-Y_{o2}}{tgE_2}=\frac{Z_{P2}-Z_{o2}}{\sin A_2} \qquad (25)$$

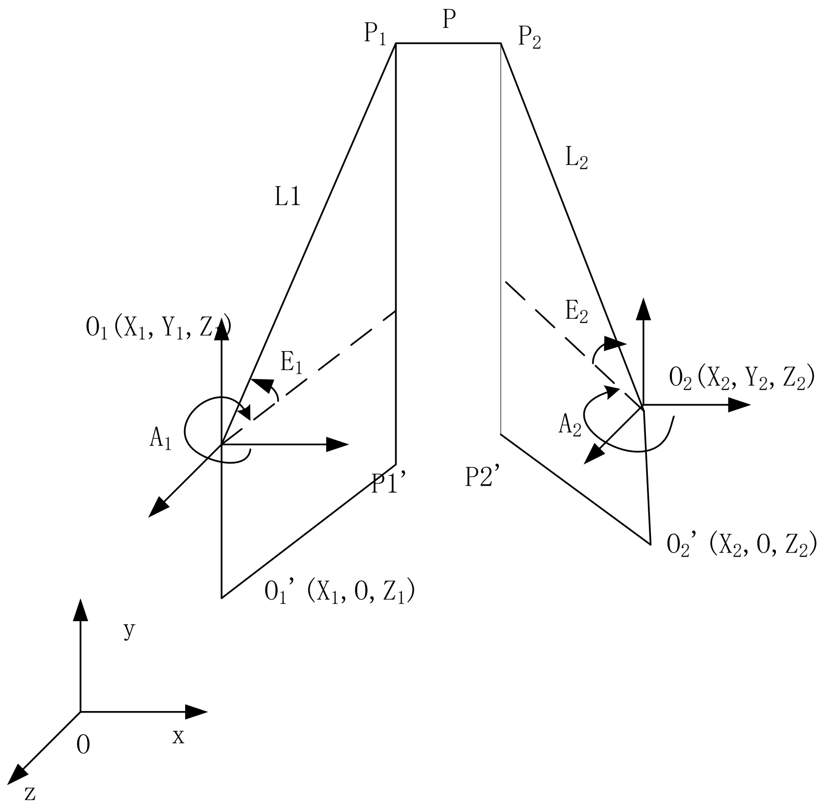

Figure 2. Measurement theory of dis-coplanar intersection

The references of target are as below:

$$p_1 = \cos A_1(X_1 - X_2) + tgE_1(Y_1 - Y_2) + \sin A_1(Z_1 - Z_2)$$

$$p_2 = \cos A_2(X_2 - X_1) + tgE_2(Y_2 - Y_1) + \sin A_2(Z_2 - Z_1)$$

$$K = [\cos(A_1 - A_2) + tgE_1 tgE_2]^2 - \sec^2 E_1 \sec^2 E_2$$

$$L_1 = \frac{p_2[\cos(A_1 - A_2) + tgE_1 tgE_2] + p_1 \sec^2 E_2}{K}$$

$$L_2 = \frac{p_1[\cos(A_1 - A_2) + tgE_1 tgE_2] + p_2 \sec^2 E_1}{K}$$

$$\theta = ar\cos[\cos(A_1 - A_2)\cos E_1 \cos E_2 + \sin E_1 \sin E_2]$$

$$d = \sqrt{(X_1 - X_2 + L_1\cos A_1 - L_2\cos A_2)^2 + (Y_1 - Y_2 + L_1 tgE_1 - L_2 tgE_2)^2 + (Z_1 - Z_2 + L_1\sin A_1 - L_2 \sin A_2)^2}$$

$$X = \rho(X_1 + L_1\cos A_1) + (1-\rho)(X_2 + L_2\cos A_2)$$

$$Y = \rho(Y_1 + L_1 tgE_1) + (1-\rho)(Y_2 + L_2 tgE_2)$$

$$Z = \rho(Z_1 + L_1 \sin A_1) + (1-\rho)(Z_2 + L_2 \sin A_2) \tag{26}$$

Thereinto $\rho \subseteq [0,1]$ is the weighting coefficient on the basis of different measure precision, *d* is length of P_1P_2, θ is the included angle between $O_1\, P_1$ and $O_2\, P_2$.

According to rule of error propagation, if A_i, E_i have root-mean-square error $\sigma_{A_i}, \sigma_{E_i}$, point coordinate x_i, y_i, z_i has root-mean-square error $\sigma_{x_i}, \sigma_{y_i}, \sigma_{z_i}$, the space root-mean-square error passed is :

$$\sigma_x = \sqrt{\begin{matrix}(\frac{\partial X}{\partial A_1})^2 \partial_{A_1}^2 + (\frac{\partial X}{\partial E_1})^2 \partial_{E_1}^2 + (\frac{\partial X}{\partial x_1})^2 \partial_{x_1}^2 + (\frac{\partial X}{\partial y_1})^2 \partial_{y_1}^2 + (\frac{\partial X}{\partial z_1})^2 \partial_{z_1}^2 \\ + (\frac{\partial X}{\partial A_2})^2 \partial_{A_2}^2 + (\frac{\partial X}{\partial E_2})^2 \partial_{E_2}^2 + (\frac{\partial X}{\partial x_2})^2 \partial_{x_2}^2 + (\frac{\partial X}{\partial y_2})^2 \partial_{y_2}^2 + (\frac{\partial X}{\partial z_2})^2 \partial_{z_2}^2\end{matrix}} \tag{27}$$

The source of error would be separated two parts[30]: one is the angle measurement error of photoelectric theodolite, another is geodetic coordinate system error. The former would bring error inevitably. While the latter would make $\sigma_{x_i}, \sigma_{y_i}, \sigma_{z_i}$ little enough, so that the space coordination *x* can be ignored. There will be abbreviated:

$$\sigma_x = \sqrt{(\frac{\partial X}{\partial A_1})^2 \partial_{A_1}^2 + (\frac{\partial X}{\partial A_2})^2 \partial_{A_2}^2 + (\frac{\partial X}{\partial E_1})^2 \partial_{E_1}^2 + (\frac{\partial X}{\partial E_2})^2 \partial_{E_2}^2} \tag{28}$$

Because the two angle measure systems of photoelectric theodolite are uncorrelated, the variance of azimuth and elevation given are equal, there are $\sigma_{A_i} = \sigma_{E_i} = \delta$. With the figure 2 there comes that the smaller the *d* is, the higher the intersection accuracy is. The intersection angle $0 \le \theta \le 180$, and $\theta = \theta(A_i, E_i)$. There be derived:

$$\sigma_x = \sqrt{(\frac{\partial X}{\partial A_1})^2 + (\frac{\partial X}{\partial A_2})^2 + (\frac{\partial X}{\partial E_1})^2 + (\frac{\partial X}{\partial E_2})^2} \tag{29}$$

Similarly from intersection formula, it may work out the space coordinate *X,Y,Z* being the function of $A_i, E_i, X_i, Y_i, Z_i, (i=1,2)$, as:

$$\begin{cases} x = X(A_i, E_i, X_i, Y_i, Z_i) \\ y = Y(A_i, E_i, X_i, Y_i, Z_i) \\ z = Z(A_i, E_i, X_i, Y_i, Z_i) \end{cases} \tag{30}$$

According to rule of error propagation[31], if A_i, E_i have root-mean-square error: $\sigma_{A_i}, \sigma_{E_i}$, the space coordinate X、Y、Z would have root-mean-square error as the following:

$$\begin{cases} \sigma_x = \sqrt{(\frac{\partial X}{\partial A_1})^2 \sigma_{A_1}^2 + (\frac{\partial X}{\partial A_2})^2 \sigma_{A_2}^2 + (\frac{\partial X}{\partial E_1})^2 \sigma_{E_2}^2 + (\frac{\partial X}{\partial E_2})^2 \sigma_{E_2}^2} \\ \sigma_y = \sqrt{(\frac{\partial Y}{\partial A_1})^2 \sigma_{A_1}^2 + (\frac{\partial Y}{\partial A_2})^2 \sigma_{A_2}^2 + (\frac{\partial Y}{\partial E_1})^2 \sigma_{E_2}^2 + (\frac{\partial Y}{\partial E_2})^2 \sigma_{E_2}^2} \\ \sigma_z = \sqrt{(\frac{\partial Z}{\partial A_1})^2 \sigma_{A_1}^2 + (\frac{\partial Z}{\partial A_2})^2 \sigma_{A_2}^2 + (\frac{\partial Z}{\partial E_1})^2 \sigma_{E_2}^2 + (\frac{\partial Z}{\partial E_2})^2 \sigma_{E_2}^2} \end{cases} \quad (31)$$

Because two angle measure systems of photoelectric theodolite are uncorrelated, the errors of $A_1, A_{2,} E_1, E_2$ are mutual independence. Here we can suppose that the errors of angle are equal.

$$\sigma_{A_1} = \sigma_{A_2} = \sigma_{E_1} = \sigma_{E_2} = \delta \quad (32)$$

There would be conducted the random error variance as:

$$\begin{cases} \sigma_x = \delta\sqrt{(\frac{\partial X}{\partial A_1})^2 + (\frac{\partial X}{\partial A_2})^2 + (\frac{\partial X}{\partial E_1})^2 + (\frac{\partial X}{\partial E_2})^2} \\ \sigma_y = \delta\sqrt{(\frac{\partial Y}{\partial A_1})^2 + (\frac{\partial Y}{\partial A_2})^2 + (\frac{\partial Y}{\partial E_1})^2 + (\frac{\partial Y}{\partial E_2})^2} \\ \sigma_z = \delta\sqrt{(\frac{\partial Z}{\partial A_1})^2 + (\frac{\partial Z}{\partial A_2})^2 + (\frac{\partial Z}{\partial E_1})^2 + (\frac{\partial Z}{\partial E_2})^2} \end{cases} \quad (33)$$

Random noise is:

$$C = \begin{bmatrix} \delta_x^2 & 0 & 0 \\ 0 & \delta_y^2 & 0 \\ 0 & 0 & \delta_z^2 \end{bmatrix} \quad (34)$$

Thereinto：

$$\begin{cases} \dfrac{\partial p_1}{\partial A_1} = -\sin A_1 (X_1 - X_2) + \cos A_1 (Z_1 - Z_2) \\ \dfrac{\partial p_1}{\partial E_1} = \sec^2 E_1 (Y_1 - Y_2) \\ \dfrac{\partial p_1}{\partial A_2} = -\sin A_2 (X_2 - X_1) + \cos A_2 (Z_2 - Z_1) \\ \dfrac{\partial p_1}{\partial E_2} = \sec^2 E_2 (Y_2 - Y_1) \end{cases} \tag{35}$$

$$\begin{cases} \dfrac{\partial K}{\partial A_1} = -2[\cos(A_1 - A_2) + tgE_1 tgE_2]\sin(A_1 - A_2) \\ \dfrac{\partial K}{\partial E_1} = 2\sec^2 E_1 [\cos(A_1 - A_2) tgE_2]\sin(A_1 - A_2) \\ \dfrac{\partial K}{\partial A_1} = 2[\cos(A_1 - A_2) + tgE_1 tgE_2]\sin(A_1 - A_2) \\ \dfrac{\partial K}{\partial E_2} = 2\sec^2 E_2 [\cos(A_1 - A_2) tgE_1 - tgE_2] \end{cases} \tag{36}$$

$$\begin{cases} \dfrac{\partial l_1}{\partial A_1} = [\dfrac{\partial p_1}{\partial A_1}\sec^2 E_2 - p_2 \sin(A_1 - A_2) - \dfrac{\partial K}{\partial A_1} L_1] / K \\ \dfrac{\partial l_1}{\partial E_1} = [\dfrac{\partial p_1}{\partial E_1}\sec^2 E_2 + p_2 \sec^2 E_1 tgE_2 - \dfrac{\partial K}{\partial E_1} L_1] / K \\ \dfrac{\partial l_1}{\partial A_2} = \left\{ \dfrac{\partial p_2}{\partial A_2}[\cos(A_1 - A_2) + tgE_1 tgE_2] + p_2 \sin(A_1 - A_2) - \dfrac{\partial K}{\partial E_2} L_1 \right\} / K \\ \dfrac{\partial l_1}{\partial E_2} = \left\{ \dfrac{\partial p_2}{\partial E_2}[\cos(A_1 - A_2) + tgE_1 tgE_2] + p_2 tgE_1 \sec^2 E_2 + 2 p_1 tgE_2 \sec^2 E_2 - \dfrac{\partial K}{\partial E_2} L_1 \right\} / K \end{cases} \tag{37}$$

$$\begin{cases} \frac{\partial x}{\partial A_1} = \frac{1}{2}(\frac{\partial l_1}{\partial A_1}\cos A_1 - l_1 \sin A_1 + \frac{\partial l_2}{\partial A_1}\cos A_2) \\ \frac{\partial x}{\partial E_1} = \frac{1}{2}(\frac{\partial l_1}{\partial E_1}\cos A_1 + \frac{\partial l_2}{\partial E_1}\cos A_2) \\ \frac{\partial x}{\partial A_2} = \frac{1}{2}(\frac{\partial l_1}{\partial A_2}\cos A_1 - l_2 \sin A_2 + \frac{\partial l_2}{\partial A_2}\cos A_2) \\ \frac{\partial x}{\partial E_2} = \frac{1}{2}(\frac{\partial l_1}{\partial E_2}\cos A_1 + \frac{\partial l_2}{\partial E_2}\cos A_2) \\ \sigma_x = \delta\sqrt{(\frac{\partial X}{\partial A_1})^2 + (\frac{\partial X}{\partial E_1})^2 + (\frac{\partial X}{\partial A_2})^2 + (\frac{\partial X}{\partial E_2})^2} \end{cases} \tag{38}$$

$$\begin{cases} \frac{\partial x}{\partial A_1} = \frac{1}{2}(\frac{\partial l_1}{\partial A_1}\cos A_1 - l_1 \sin A_1 + \frac{\partial l_2}{\partial A_1}\cos A_2) \\ \frac{\partial x}{\partial E_1} = \frac{1}{2}(\frac{\partial l_1}{\partial E_1})\cos A_1 + \frac{\partial l_2}{\partial E_1}\cos A_2) \\ \frac{\partial x}{\partial A_2} = \frac{1}{2}(\frac{\partial l_1}{\partial A_2}\cos A_1 - l_2 \sin A_2 + \frac{\partial l_2}{\partial A_2}\cos A_2) \\ \frac{\partial x}{\partial E_2} = \frac{1}{2}(\frac{\partial l_1}{\partial E_2})\cos A_1 + \frac{\partial l_2}{\partial E_2}\cos A_2) \\ \sigma_x = \delta\sqrt{(\frac{\partial x}{\partial A_1})^2 + (\frac{\partial x}{\partial E_1})^2 + (\frac{\partial x}{\partial A_2})^2 + (\frac{\partial x}{\partial E_2})^2} \end{cases} \tag{39}$$

Ⅳ.SIMULATION AND EXPERIMENT ANALYSIS

A. Comparison and analysis error estimation related to coordinate

Within two-station intersection measure systems, the target should better be in the area of two occupied stations straight ahead, and the elevation should better below in 60°. Suppose the coordinate of two occupied stations are (100, 3000, 100) and (2800, 2600，500,) in launch space rectangular coordinates system, the angle error is $\sigma_{A_1} = \sigma_{A_2} = \sigma_{E_1} = \sigma_{E_2} = \delta = 20''$, the elevation A_1 and A_2 are below 60°, base length is bigger than 3Km and smaller than 5Km, the error estimation according to coordinate and angle is simulated as below.

The experiment system is two station intersection measure systems, constituted by three parts: photoelectric theodolite A, photoelectric theodolite B, compute center C and moving target T, shown as figure 3. When T is flying, A and B are initiative detecting, and get sync testing parameters to transmit C, where it can work out accurate trail of dynamic target.

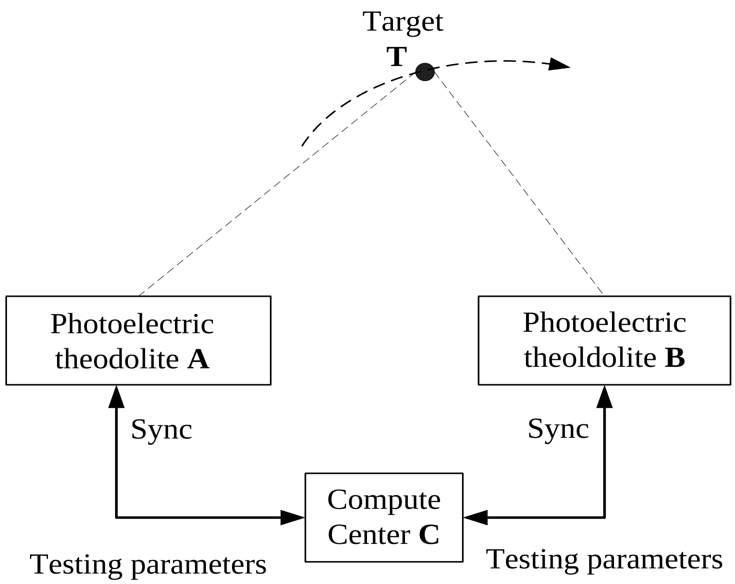

Figure 3. Experiment system of two occupied stations

The error estimation of σ_x according to coordinate x is below 0.23 shown in figure 4. The error estimation of σ_x according to azimuth is below 0.23 shown in figure 5. The error estimation of σ_z according to coordinate z has a decrease phrase and increase phrase shown in figure 6.

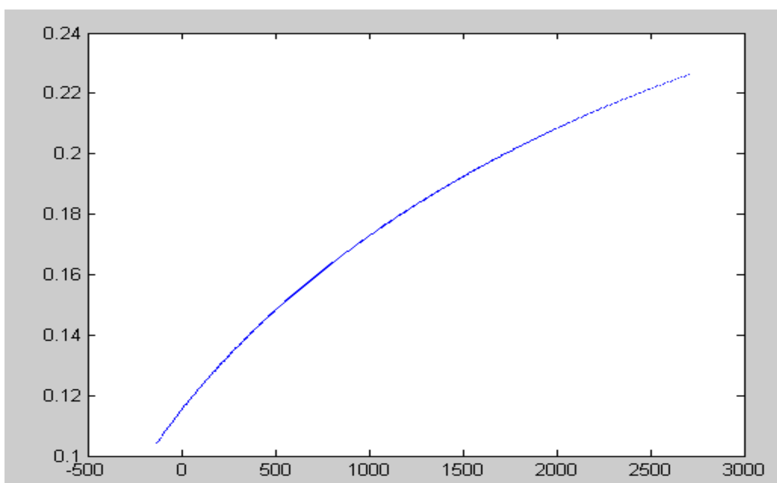

Figure 4. The error estimation of σ_x according to coordinate x

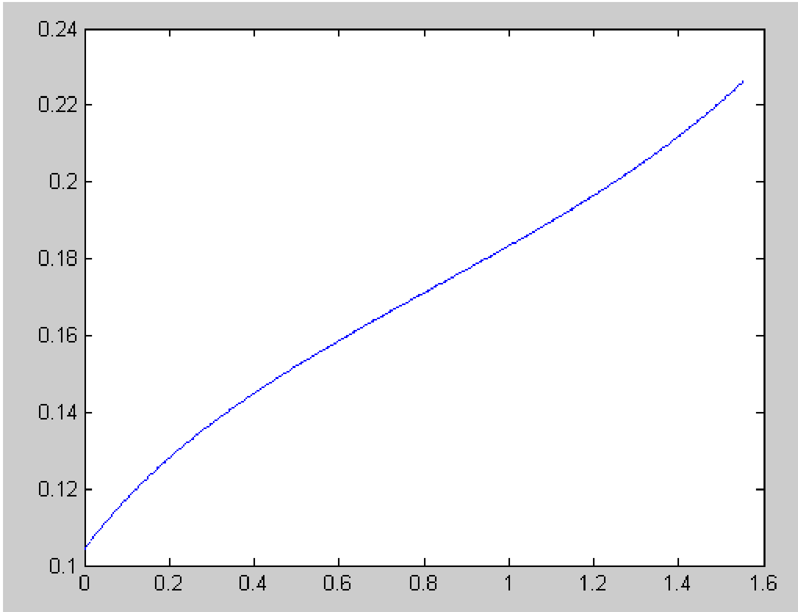

Figure 5. The error estimation of σ_x according to azimuth A_2

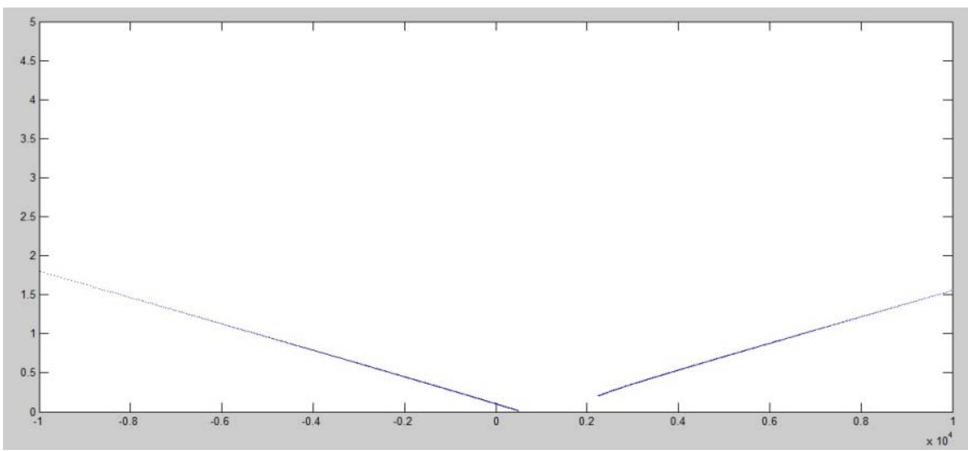

Figure 6. The error estimation of σ_z according to coordinate z

Through the comparison of result, it shows that base length effects coordinate measure accuracy distinctly. Along with the base length increasing, measure error in x coordinate amplifies gradually, while measure error in z coordinate decrease gradually and would increase after a smooth transition.

B. Comparison and analysis error estimation related to azimuth

According to the principle mentioned above, The error estimate of δ_x δ_y δ_z according to azimuth A_2 are shown in figure 7 , figure 8 and figure 9.

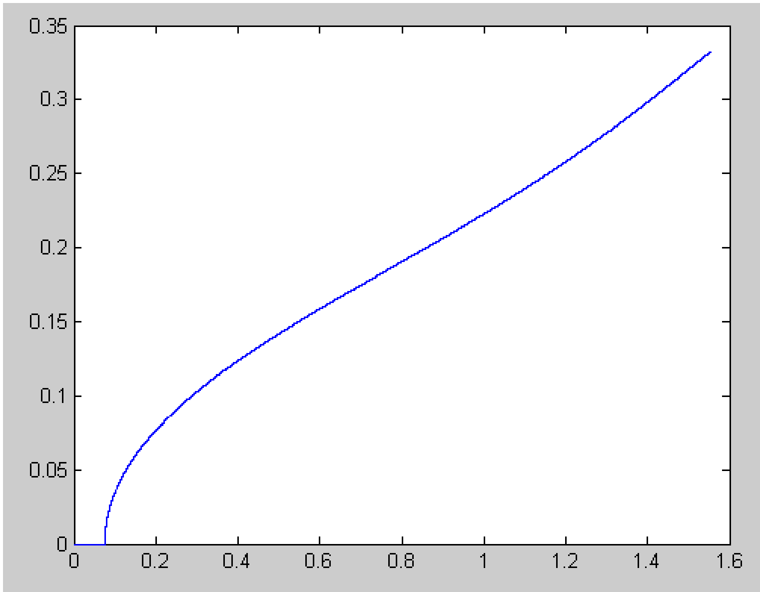

Figure 7. The error estimation of δ_x according to azimuth A_2

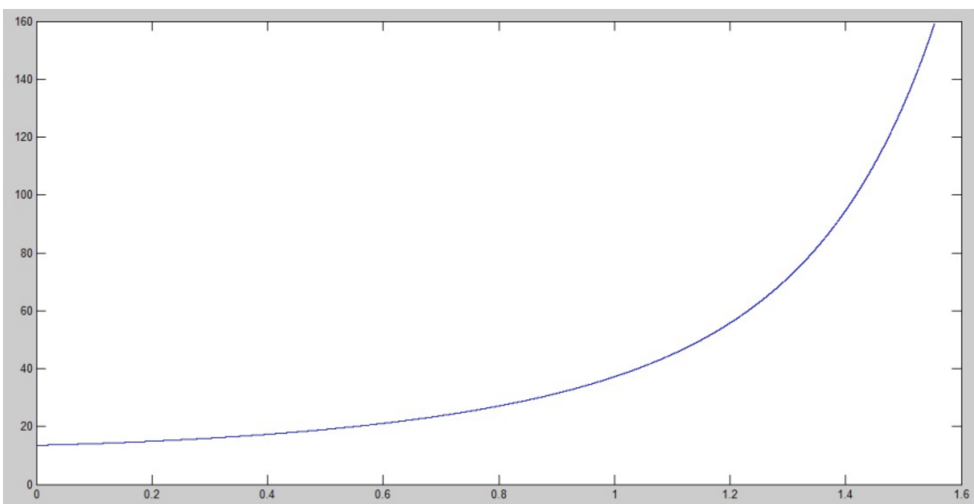

Figure 8. The error estimation of δ_y according to azimuth A_2

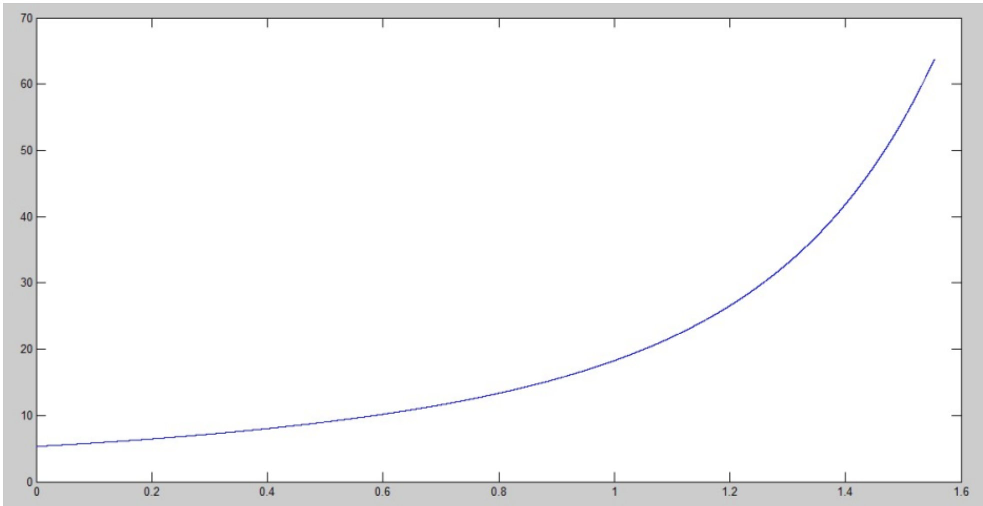

Figure 9. The error estimation of δ_z according to azimuth A_2

According to the principle mentioned above, The error estimate of δ_x δ_y δ_z according to azimuth A_1 are shown in figure 10 , figure 11 and figure 12.

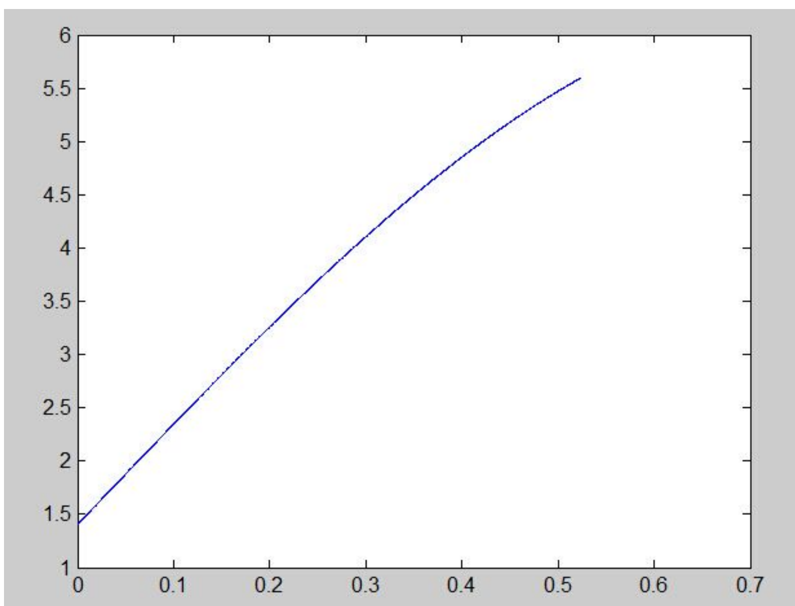

Figure 10. The error estimation of δ_x according to azimuth A_1

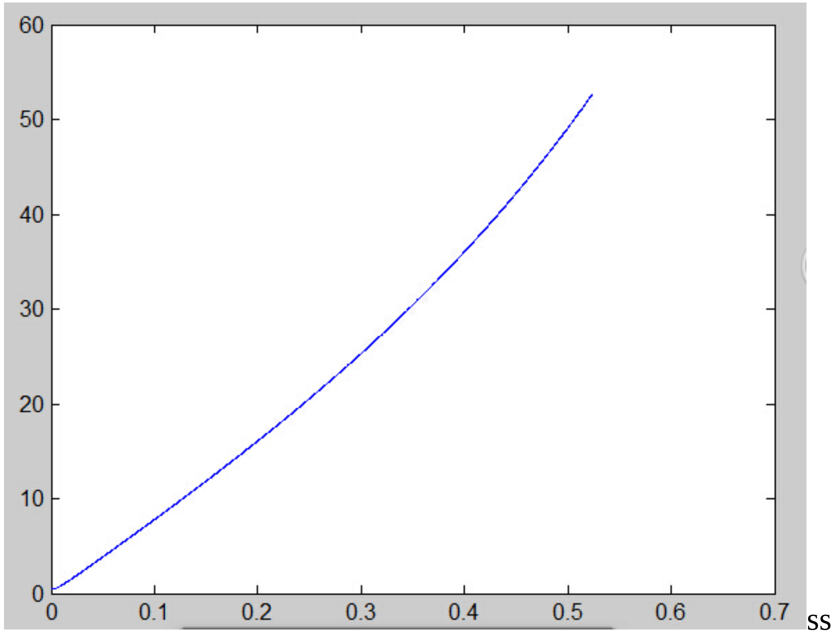

Figure 11. The error estimation of δ_y according to azimuth A_1

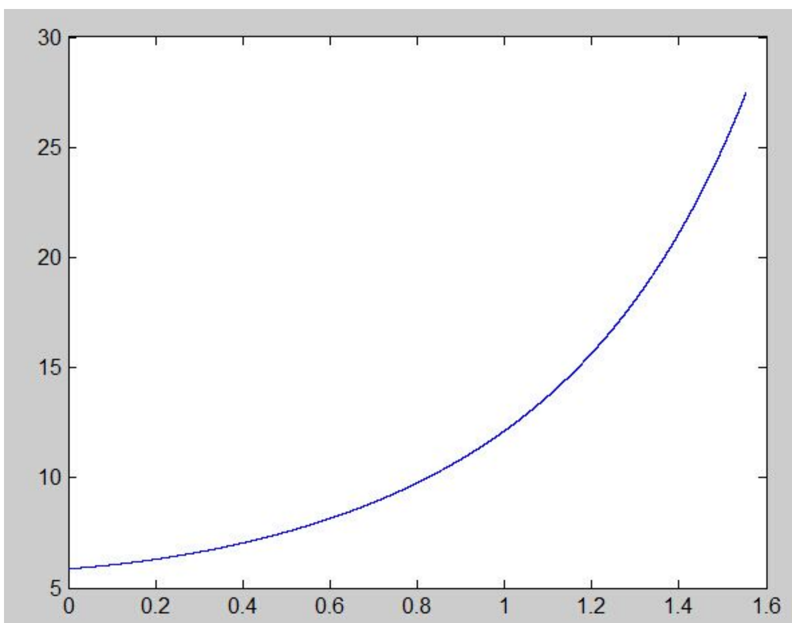

Figure 12. The error estimation of δ_z according to azimuth A_1

When the error of measure angle is definite, the measure accuracy is related to device accuracy and occupied station layout. So a rational layout would bring out good performance in measure device. When using multi-occupied stations, it should better to shorten the distance between occupied station and target. Especially if the base length shapes equilateral triangle, the location accuracy can reach centimeter level.

V. CONCLUSIONS

This paper researches testing error problem of photoelectric theodolite. It analyses the forming cause, divides them to three sorts: system error, random error and outlier error, then instructs resolution method to each sort. Particularly for random error, it builds an error model, analyses the properties of unbiased, equal variance and uncorrelated, conducts best error estimate, discusses choosing and effect of kernel function and smooth parameter. Afterwards it researches measure theory of coplanar intersection and dis-coplanar intersection of photoelectric theodolite, derives a series of measure formulas, builds and analyses random error model respectively. Finally the simulation result proves the error model and processing method are correct. By comparing simulation of model with experiment measurement, the result shows that if the error of measure angle is definite, the measure accuracy is related to device accuracy and occupied station layout. So a rational layout would bring out good performance in measure device. When using multi-occupied stations, it should better shorten the distance between occupied station and target. Especially if the base length shape equilateral triangle, the location accuracy can reach centimeter level.

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