



PERFORMANCE BOUND ANALYSIS ON TRANSMIT ANTENNAS SELECTION SYSTEMS CONSIDERING ERRONEOUS CHANNEL INFORMATION

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Abstract- We consider multiple-inputs-single-output (MISO) systems equipped with N transmit and single receive antennas, and concentrate on transmit antenna selection (TAS) for flat Rayleigh fading channels. We assume perfect channel state information (CSI) at receiver and imperfect CSIs through an feedback link at transmitter. The metric of TAS are those with the largest instantaneous signal-to-noise ratio (SNR), the best 2 out of N available transmit antennas always are selected. We use Alamouti encoding scheme and derive the probability density function (PDF) of Frobenius norm of column vector of the channel matrix. Using the known PDF we can derive the joint PDF of order statistics channel for the subset $\{i, j\}$. Assuming that the transmitted signals employ Mary phase-shift keying (MPSK) constellation, we consider the impact of imperfect antenna selection subsets on system

performance, and explicitly derive a closed-form BER expression of Chernoff upper bounds (CUB). For two special cases with 2 out of $N = 3$ and 4 possible transmit antennas, we analyse asymptotic performance of selected subsets. Numerical and simulating results show that we can achieve full spatial diversity with antenna selection in the presence of imperfect CSI and the largest ordinal number antenna selected as if all the transmit antennas were used; there may be some loss in the diversity order without the best transmit antenna.

Index terms: transmit antennas selection (TAS), Alamouti scheme, multiple-input-single-output (MISO), channel state information (CSI), bit error rate (BER), Rayleigh fading channel, ordinal number, diversity order, Chernoff upper- bound (CUB).

I. INTRODUCTION

Multiple input multiple output (MIMO) systems are one of the most significant technical breakthroughs in modern wireless digital communications. Compared with single inputs single output (SISO) systems, the capacity and reliability of a wireless communication system can be improved dramatically by employing multiple antennas at the transmitter and/or receiver without increasing bandwidth and transmit power [1]. MIMO technology has been drawn significant research interests recently due to its advantages. Most importantly, the several standard wireless networks, e.g. the third-generation cellular phones (3rd generation partnership project, 3GPP), IEEE802.11, IEEE802.16, have enjoyed the MIMO technology. The MIMO technology has been widely applied to Beyond 3G and 4G systems. We can foresee that the mobile communication systems in the future, including the 5G system, will be implemented by massive MIMO technology [2]. In addition, with the booming of the Internet of things (IOT), the close communication systems, such as wireless sensor networks (WSN) [3]-[4], radio frequency identification (RFID) [5], and cognitive radio (CR) [6] et al, will use MIMO technology.

Nevertheless, MIMO systems require additional antenna elements compared to the traditional SISO systems. These additional antenna elements are usually not of high cost. However, the radio frequency (RF) chains include amplifiers, up-down converters, as well as the analog to digital to analog conversions (A/D/A) are expensive. Therefore, the application of multiple antennas has been restricted by the hardware cost and power consumption of the RF chains. How

to reduce the hardware complexity and at the same time maintain most of the advantages of MIMO systems become an important research topic. A MIMO system with antenna selection (AS) has been shown to significantly outperform a system exploiting the same number of RF chains without AS. To deal with such challenges, a promising technique referring to antenna subset selection has been proposed in [7]-[10]. The key idea of AS is to use a limited number of RF chains that are adaptively switched to a subset of the available antennas, which can effectively reduce the number of RF chains required, yet preserving the selection diversity gains.

In a more practical implementation, multiple antenna systems can also be employed to achieve full diversity order through space-time coding techniques such as space-time block codes (STBC) [11]. Because STBC technique can offer simple maximum likelihood decoding using linear processing at the receiver, in the past decade, STBC systems have received much attention since they can greatly improve the system performance over flat fading channels with a reasonable level of complexity. However, STBC with more than two antennas reduces the data rate below one, as a particular case with two transmit antennas, the Alamouti space-time transmit diversity scheme [12] can achieve both fully diversity and rate one. Hence, selection of two out of N antennas and use of Alamouti transmit diversity has been of major interest in the research community [13]-[17]. They assume that corresponding systems have perfect knowledge of the channel gains, and examine the BER performance of Alamouti scheme with transmit and /or receive antenna selection for complex constellation signals.

However, most of the works on antenna selection assume perfect channel state information (CSI) at the receiver and /or the transmitter. In this paper, we focus on the research to the case of erroneous CSI system [18]-[29]. The first effort in determining the effect of imperfect antenna selection policy on system performance appeared in [18] for a multiple-input-single-output (MISO) system with two transmit antenna selected for STBC transmission at the transmitter. Assuming that channel is an independent and flat Rayleigh fading channel and the system employs binary phase-shift keying (BPSK) modulation, and the presence of the erroneous CSIs taken into consideration. For two special cases with 2 out of $N = 3$ and 4 possible transmit antennas, [19] derived the accurate analytical expressions of bit-error rate (BER) of Alamouti scheme with TAS, and [20] derived Chernoff bounds for same system and used these bounds to quantify the diversity gain of the system using antenna selection. For a special case with 2 out of $N = 3$ possible transmit antennas, [21] presented the performance analysis for an antenna selection

system considering both erroneous CSI and transmit power allocation. Same assumptions as in [19]-[21], [22] applies the results of asymptotic performance of wireless communications with generalized selection combining over random fading channels at high signal-to-noise ratio (SNR) in [23], the asymptotic performance of the Alamouti scheme with transmit antenna selection has been investigated taking into account imperfect antenna subset selection. It is shown that the asymptotic transmit diversity order is equal to the largest ordinal number of the antenna within the selected subset. However, execution of transmit antenna selection (TAS) algorithm assume usually that the perfect channel gains can be achieved at the receiver, and requirement of ideal feedback channel to the transmitter increases complexity and overheads. Therefore, sub-optimum TAS with less complexity and feedback channel with low data rate are of interest [24]-[28]. In Nakagami- m fading channels, [29] proposes a novel reduced feedback-rate transmit diversity scheme employing transmit antenna selection (TAS) and Alamouti scheme yielding robust error performance at erroneous feedback conditions.

This paper concentrates on antenna subset selection at the transmitter for independent and identically distributed (*i.i.d.*) quasi-static flat Rayleigh fading channels. For a MISO system equipped with N transmit antennas and single receive antenna (without loss of generality) with encoding STBC, the transmitter chooses arbitrary 2 out of N available transmit antennas, denoted by $\{N, 2;1\}$. The selected antennas are those with the largest instantaneous SNR. This is achieved by comparing the magnitude square of the fading coefficient at each transmit antenna and selecting the best 2 out of them. The adopted selection criterion is clearly optimal in the sense that it maximizes the output SNRs at the receiver. In practical scenarios, it is not feasible to assume that the best antenna subset can always be selected owing to channel estimation errors, feedback delay and feedback errors. Therefore, the impact of imperfect antenna selection subsets on system performance should be investigated.

On the basis of the [19], [20], and [22], we discuss the impact of imperfect transmit antenna subset selection on system performance. When we verify and assess BER performance of communication systems, Chernoff upper bounds is a kind of effective tool [30]. Therefore, in this paper, we explicitly derive Chernoff upper bounds on the bit error probability and study their diversity order. Numerical analysis shows that by using antenna selection, we can achieve the same diversity order as the full-complexity MIMO system. An interesting conclusion as in [22] is

reached that the system diversity order is proportional to the ordinal number of the selected antenna.

The organization of the paper is as follows. In section II, we present the system and selection channel model under consideration, and introduce order statistics for the $\{N, 2;1\}$ TAS system. For the special case of $\{4,2;1\}$, we discuss the joint PDF of imperfect TAS subset. In section III, we present analysis of BER Chernoff upper bound (CUB) and diversity advantage. In section IV, the numerical curves depict that antennas selection system can offer diversity advantage. Finally, we make some conclusions in section V.

II. SYSTEM CHANNEL MODEL AND ORDER STATISTICS

a. System channel model

Suppose that a MIMO system consists of N transmit and M receive antennas in figure 1. If the serial incoming data streams have been encoded with a STBC encoder, the output of the encoder is then fed into a serial-to-parallel converter that converts the input streams into N parallel streams. The resulting N streams are transmitted from the N transmit antennas simultaneously. We assume that the blocks that involve modulation and demodulation, etc., have been suppressed from figure 1 due to their irrelevancy in the analysis. At the receiver, after demodulation, matched filtering, and sampling, the signal $y_j(k)$ received by the j th antenna at time kT (T denotes a symbol period) is given by

$$y_j(k) = \sum_{i=1}^N h_{ij}(k)x_i(k) + n_j(k). \quad (1)$$

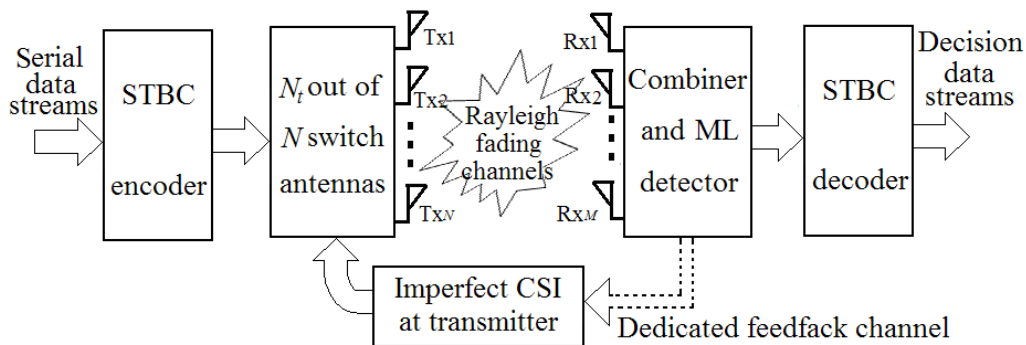


Figure 1. Block diagram of MIMO system with TAS/STBC

Thus, the MIMO channel is given by the $M \times N$ matrix \mathbf{H} with

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1N} \\ h_{21} & h_{22} & \cdots & h_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ h_{M1} & h_{M2} & \cdots & h_{MN} \end{bmatrix}. \quad (2)$$

Where the element h_{ij} of matrix \mathbf{H} denotes channel fading coefficient between the i th transmit antenna and the j th receive antenna is modeled as independent samples of complex Gaussian random variables with a zero mean and the variance of $\sigma^2=0.5$ per dimension, where $1 \leq i \leq N$ and $1 \leq j \leq M$.

The input-output relation for the MIMO channel may be expressed in matrix notation as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \quad (3)$$

where $\mathbf{y} = [y_1(k), y_2(k), \dots, y_M(k)]^T$ is the $M \times 1$ vector of received signals. $\mathbf{x} = [x_1(k), x_2(k), \dots, x_N(k)]^T$ is the $N \times 1$ vector of transmitted signals with STBC coding (whose components are from complex modulation constellation). And \mathbf{H} denotes $N \times M$ channel matrix. $\mathbf{n} = [n_1(k), n_2(k), \dots, n_M(k)]^T$ is the $M \times 1$ vector of complex Gaussian noise terms, the noise terms are independent samples of circularly symmetric zero-mean complex Gaussian random variables with variance $N_0/2$ per dimension, N_0 is single-sided noise power spectral density.

b. The PDF of Frobenius norm of column vector of the matrix \mathbf{H}

Based on MIMO channel matrix \mathbf{H} in (2), let

$$C_i = \sum_{j=1}^M |h_{ij}|^2, \quad 1 \leq i \leq N \quad (4)$$

which is the instantaneous channels power gain between transmit antenna i and all the receive antennas. We rearrange the random variables C_i in ascending order of magnitude and denote them by X_l , where $1 \leq l \leq N$ and $X_1 \leq X_2 \leq \dots \leq X_N$. The index l is referred to as the ordinal number of the antenna associated with X_l . We employ Alamouti's STBC with full data rate. Therefore, at any symbol period, a transmit antenna subset consisting of two antennas, associated with X_n and X_m , $1 \leq m < n \leq N$, is selected among N transmit antennas. Such a subset comprises one antenna with ordinal number n and one with m , namely subset $\{n, m\}$.

For a system employing Alamouti scheme, the output signal-to-noise ratio (SNR) of system decoding, denoted by γ_0 , for transmit antenna subset selected $\{n, m\}$, is given by [12,22]

$$\gamma_0 = \gamma_s [X_n + X_m], \quad (5)$$

in which $\gamma_s = E_s/N_0$, where E_s is the average total energy per symbol at transmitter and N_0 is the one-side power spectral density of the additive white Gaussian noise (AWGN) per receive antenna.

In a flat Rayleigh MIMO channel, C_i in (4) is i.i.d. χ^2 random variables with $2 \times M$ degrees of freedom. The probability density function (PDF) of C_i is given by [31]

$$f(x) = \frac{1}{(N-1)!} x^{M-1} e^{-x}, \quad x \geq 0, \quad (6)$$

and the cumulative distribution function (CDF) of C_i is expressed as [31]

$$F(x) = 1 - e^{-x} \sum_{i=0}^{M-1} \frac{x^i}{i!}, \quad x \geq 0 \quad (7)$$

For simplicity, let $M=1$, then (6) and (7) can be, respectively, rewritten as

$$f(x) = e^{-x}, \quad x \geq 0 \quad (8)$$

and

$$F(x) = 1 - e^{-x}, \quad x \geq 0 \quad (9)$$

c. Order statistics of subset $\{i, j\}$

We Consider a $\{N, 2; 1\}$ system equipping with N transmit antennas and single receive antenna, and take the presence of the erroneous channel state information into consideration. In practical applications of antennas selection at the transmitter side, it is true that the system will not always choose two antennas for the best and second-best channel. Therefore, in addition to selecting the most optimal subset $\{N-1, N\}$, the TAS system also chooses other non-ideal subsets $\{i, j\}$, where $1 \leq i < j \leq N$. There are $N \times (N-1)/2$ kinds of antenna selection subsets in $\{N, 2; 1\}$ system. Using results above equations, we can derive the joint PDF of order statistics channel for the subset $\{i, j\}$.

According to a equation of order statistics as below [32],

$$f(x_{j_1}, x_{j_2}) = \frac{N!}{(j_1-1)!(j_2-j_1-1)!(N-j_2)!} \times [F(x_{j_1})]^{j_1-1} \times [F(x_{j_2}) - F(x_{j_1})]^{j_2-j_1-1} \times [1-F(x_{j_2})]^{N-j_2} \times f(x_{j_1})f(x_{j_2}), \quad (10)$$

where $1 \leq j_1 < j_2 \leq N$, $x_{j_1} \geq 0$ and $x_{j_2} \geq 0$.

For the interesting system $\{N;1\}$ with transmit antennas and one receive antenna, we substitute the equations (8) and (9) into the equation (10), we can obtain the joint PDF of x_{j_1} and x_{j_2} as follow

$$f(x_{j_1}, x_{j_2}) = \frac{N!}{(j_1 - 1)!(j_2 - j_1 - 1)!(N - j_2)!} \times \left[1 - e^{-x_{j_1}}\right]^{j_1 - 1} \times \left[e^{-x_{j_1}} - e^{-x_{j_2}}\right]^{j_2 - j_1 - 1} \times \left[e^{-x_{j_2}}\right]^{N - j_2} \times e^{-(x_{j_1} + x_{j_2})}. \quad (11)$$

For the transmit antenna selection case of $N=4$ and $M=1$, i.e. the system of choosing two transmit antennas in four available antennas which may be simply called the 2 out of 4, denoted by $\{4,2;1\}$, thus the equation (11) can be expressed in the equation (12)

$$f(x_{j_1}, x_{j_2}) = \frac{24}{(j_1 - 1)!(j_2 - j_1 - 1)!(4 - j_2)!} \times \left[1 - e^{-x_{j_1}}\right]^{j_1 - 1} \times \left[e^{-x_{j_1}} - e^{-x_{j_2}}\right]^{j_2 - j_1 - 1} \times \left[e^{-x_{j_2}}\right]^{4 - j_2} \times e^{-(x_{j_1} + x_{j_2})}, \quad (12)$$

where $1 \leq j_1 < j_2 \leq N$. Because the error may possibly be appeared in the CSIs, there are $C_4^2=6$ possible TAS subsets that can be used for transmission, i.e. $\{1,2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}$, and $\{3,4\}$, where the term $C_m^n = m! / n! / (m - n)!$. i.e. for the case of $j_1=2$ and $j_2=4$ over $\{4,2;1\}$ system, namely x_1 and x_2 are selected, the joint PDF of x_1 and x_2 can be derived by

$$f_{24}(x_2, x_4) = \frac{24}{(2 - 1)!(4 - 2 - 1)!(4 - 4)!} \times \left[1 - e^{-x_2}\right]^{2 - 1} \times \left[e^{-x_2} - e^{-x_4}\right]^{4 - 2 - 1} \times \left[e^{-x_4}\right]^{4 - 4} \times e^{-(x_2 + x_4)} \\ = 24e^{-(2x_2 + x_4)} - 24e^{-(x_2 + 2x_4)} - 24e^{-(3x_2 + x_4)} + 24e^{-(2x_2 + 2x_4)}. \quad (13a)$$

Similarly, using equation (12), we can, respectively, obtain the joint PDF of other five pairs except for the case above as follow

$$f_{12}(x_1, x_2) = 12e^{-x_1} e^{-3x_2}, \quad (13b)$$

$$f_{13}(x_1, x_2) = 24e^{-2(x_1 + x_3)} - 24e^{-x_1 - 3x_3}, \quad (13c)$$

$$f_{14}(x_1, x_4) = 12e^{-(3x_1 + x_4)} - 24e^{-(2x_1 + 2x_4)} + 12e^{-(x_1 + 3x_4)}, \quad (13d)$$

$$f_{23}(x_2, x_3) = 24e^{-(x_2 + 2x_3)} - 24e^{-(2x_2 + 2x_3)}, \quad (13e)$$

and

$$f_{34}(x_3, x_4) = 12(1 - e^{-x_3})^2 e^{-(x_3 + x_4)}. \quad (13f)$$

It can be known from the theory of antenna selection that if and only if the transmit antennas x_3 and x_4 are selected, it is just what we wanted, while other five pairs are caused by the imperfect CSI. In order to realize the influence of them on the error probability performance of the system, we have to solve the analytical expressions in theory of average BER of the corresponding system and discuss their diversity order.

III. BIT ERROR PERFORMANCE BOUND

a. BER in AWGN channel

We assume that a system transmission Mary phase-shift keying (MPSK) modulation has perfect CSI at the receiver and uses maximum-likelihood (ML) decoder. In the additive white Gaussian noise (AWGN) channel, the bit error rate (BER) for MPSK is [31]

$$P_e(\gamma_b) = \frac{2}{k} Q\left(\sqrt{2\gamma_s} \sin \frac{\pi}{M}\right) = \frac{2}{k} Q\left(\sqrt{2k\gamma_b} \sin \frac{\pi}{M}\right), \quad (14)$$

where $Q(\cdot)$ is Gaussian Q function, and $k=\log_2 M$, $M=2^i$, $i=2,3,\dots$. For the convenience of expression, let $\gamma_M=k\gamma_b \sin^2(\pi/M)$, thus (14) can be written as

$$P_e(\gamma_M) = \frac{2}{k} Q\left(\sqrt{2\gamma_M}\right). \quad (15)$$

As a special case of $M=2$, binary PSK (BPSK) digital modulation, the BER expression is[31]

$$P_e(\gamma_b) = Q\left(\sqrt{2\gamma_b}\right). \quad (16)$$

In MIMO Rayleigh fading channel, as discussion above, we choose arbitrarily an antenna selection subset $\{j_1, j_2\}$, the conditional BER of system employing Alamouti scheme conditioned on X_{j_1} and X_{j_2} can be expressed as

$$P_{e_{j_1 j_2}}(\gamma_M | X_{j_1}, X_{j_2}) = \frac{2}{k} Q\left(\sqrt{2\gamma_M(x_{j_1} + x_{j_2})}\right). \quad (17)$$

In particular, applying equation (16), equation (17) for BPSK modulation can be simplified as

$$P_{e_{j_1 j_2}}(\gamma_b | X_{j_1}, X_{j_2}) = Q\left(\sqrt{2\gamma_b(x_{j_1} + x_{j_2})}\right). \quad (18)$$

b. CUB bit error performance and diversity order

In this subsection, we mainly discuss bit error performance and diversity order on $\{3, 2;1\}$ and $\{4, 2;1\}$ systems, and assume that system use BPSK modulation. For example, the $\{4, 2;1\}$ system

appears error CSIs in the transmitter, antenna selector takes $j_1=1$ and $j_2=2$, two transmit antennas corresponding to X_1 and X_2 are selected. Then $M=2, k=1$, we rewrite (18) as (19), thus we have

$$P_{e12}^4(\gamma_b|X_1, X_2) = Q(\sqrt{2\gamma_b(x_1 + x_2)}). \quad (19)$$

Combining the joint PDF of X_1 and X_2 given by (13b) with (19), we can average the right side in (19). Thus the integral formula of average BER for transmit subset $\{1,2\}$ is given by

$$\bar{P}_{e12}^4(\gamma_b) = \int_0^\infty \int_0^\infty Q(\sqrt{2\gamma_b(x_1 + x_2)}) f(x_1, x_2) dx_1 dx_2. \quad (20)$$

Chernoff upper-bound (CUB) of Q function $Q(x) \leq \frac{1}{2} e^{-x^2/2}$ is applied to above equation, we lead to

$$\bar{P}_{e12}^4(\gamma_b) \leq \frac{1}{2} \int_0^\infty \int_{x_1}^\infty e^{-2\gamma_b(x_1+x_2)/2} \times 12e^{-x_1} e^{-3x_2} dx_1 dx_2.$$

Integrating the right side of the above inequality with respect to x_1 and x_2 , we obtain

$$\begin{aligned} & 6 \int_0^\infty \left[e^{-(1+\gamma_b)x_1} \right] dx_1 \int_{x_1}^\infty e^{-(\gamma_b+3)x_2} dx_2 \\ &= \frac{6}{3+\gamma_b} \int_0^\infty e^{-(2\gamma_b+4)x_1} dx_1 \\ &= \frac{3}{(\gamma_b+2)(\gamma_b+3)}. \end{aligned} \quad (21)$$

If $\gamma_b \rightarrow \infty$, the right side of the above equality average CUB BER in (21) can be approximated as

$$\frac{3}{(\gamma_b+2)(\gamma_b+3)} \approx \frac{3}{\gamma_b^2}.$$

The above CUB approximation show that subset $\{1,2\}$ for $\{4,2;1\}$ system can achieve 2 diversity order. Similarly, we can derive BER CUB expressions and their diversity orders of other five TAS subsets except for the above case, which are summarized in Table 1.

Table 1: Performance bound for $\{4, 2;1\}$ TAS/STBC

Subset $\{j_1, j_2\}$	BER CUB($\gamma_b \rightarrow \infty$)	Diversity order
X_1X_3	$6/[(\gamma_b+2)^2(\gamma_b+3)] \approx 6/\gamma_b^3$	3
X_1X_4	$6/[(\gamma_b+1)(\gamma_b+2)^2(\gamma_b+3)] \approx 6/\gamma_b^4$	4
X_2X_3	$6/[(\gamma_b+2)^2(2\gamma_b+3)] \approx 3/\gamma_b^3$	3
X_2X_4	$3/[(\gamma_b+1)(\gamma_b+2)^2(\gamma_b+3/2)] \approx 3/\gamma_b^4$	4
X_3X_4	$3/[(\gamma_b+1)^2(\gamma_b+2)(2\gamma_b+3)] \approx 1.5/\gamma_b^4$	4

c. CUB performance for $\{N,2;1\}$ system with MPSK constellation

We have just presented a special case with $N=4$, and the transmitting signals use BPSK digital modulation. In this subsection, the special case will be generalized to Mary-PSK constellation and $\{N, 2;1\}$ TAS system for any subset $\{i, j\}$, where $1 \leq i < j \leq N$. And a BER CUB expression on TAS system with erroneous CSIs will be derived as follows.

Combining (10) with (17), we obtain the integral expression of average BER as follows

$$\bar{P}_{ej}^N(\gamma_M) = \frac{2}{k} \int_0^\infty \int_{x_i}^\infty Q(\sqrt{2\gamma_M(x_i + x_j)}) f(x_i, x_j) dx_i dx_j, \quad (22)$$

where γ_M defined as in the equation (14). Obviously, it is quite difficult to integrate (22). We utilize the Chernoff upper bound, consequently, the expression in (22) can be upper-bounded as

$$\begin{aligned} \bar{P}_{ej}^N(\gamma_M) \leq & \frac{N!}{k(i-1)!(j-i-1)!(N-j)!} \int_0^\infty \int_{x_i}^\infty e^{-\gamma_M(x_i+x_j)} (1-e^{-x_i})^{i-1} \\ & \times (e^{-x_i} - e^{-x_j})^{j-i-1} e^{-(N+1-j)x_j} e^{-x_i} dx_i dx_j. \end{aligned} \quad (23)$$

In order to integrate the right side of the above inequality with respect to two random variables (RVs) x_i and x_j , we apply binomial expansion

$$(a+b)^n = \sum_{i=0}^n C_n^i a^i \times b^{(n-i)},$$

where the term $C_n^i = n!/[i!(n-i)!]$ denotes combination. Thus we get

$$(1-e^{-x_i})^{i-1} = \sum_{u=0}^{i-1} \{C_{i-1}^u \times (-1)^{i-1-u} \times e^{-(i-1-u)x_i}\},$$

and

$$(e^{-x_i} - e^{-x_j})^{j-i-1} = \sum_{v=0}^{j-i-1} \{C_{j-i-1}^v \times (-1)^{j-i-1-v} \times e^{-vx_i} \times e^{-(j-i-1-v)x_j}\}.$$

Substituting the two binomial expansions into (23), we yield

$$\begin{aligned} \bar{P}_{ej}^N(\gamma_M) \leq & \frac{N!}{k(i-1)!(j-i-1)!(N-j)!} \int_0^\infty \int_{x_i}^\infty e^{-\gamma_M(x_i+x_j)} \sum_{u=0}^{i-1} \{C_{i-1}^u \times (-1)^{i-1-u} e^{-(i-1-u)x_i}\} \\ & \times (e^{-x_i} - e^{-x_j})^{j-i-1} e^{-(N+1-j)x_j} e^{-x_i} dx_i dx_j. \end{aligned}$$

To organize on the right side of above inequality, and exchange order of between two accumulative operations and double integral, finally, we obtain

$$\begin{aligned} \bar{P}_{ej}^N(\gamma_M) \leq & \frac{N!}{k(i-1)!(j-i-1)!(N-j)!} \sum_{u=0}^{j-1} \sum_{v=0}^{j-i-1} \int_0^\infty \int_{x_i}^\infty (-1)^{j-u-v} C_{i-1}^u \times C_{j-i-1}^v \\ & \times (e^{-(\gamma_M+i+v-u)x_i} e^{-(\gamma_M+N-v-i)x_j}) dx_i dx_j. \end{aligned} \quad (24)$$

To integrate the right side of inequality (24) with respect to x_j , we can obtain integral form with respect to x_i as follows

$$\bar{P}_{ej}^N(\gamma_M) \leq \frac{N!}{k(i-1)!(j-i-1)!(N-j)!} \times \sum_{u=0}^{j-1} \sum_{v=0}^{j-i-1} \frac{(-1)^{(j-u-v)} C_{i-1}^u C_{j-i-1}^v}{(\gamma_M + N - v - i)} \int_0^\infty e^{-(2\gamma_M + N - u)x_i} dx_i.$$

To integrate the right side of above inequality with respect to x_i , finally we can obtain the theoretical BER expression of Chernoff upper bound as follows

$$\bar{P}_{ej}^N(\gamma_M) \leq \frac{N!}{k(i-1)!(j-i-1)!(N-j)!} \times \sum_{u=0}^{j-1} \sum_{v=0}^{j-i-1} \frac{(-1)^{(j-u-v)} C_{i-1}^u C_{j-i-1}^v}{(\gamma_M + N - v - i)(2\gamma_M + N - u)}. \quad (25)$$

We notice the following relationship

$$1 \leq i < j \leq N, \quad 1 \leq j - i \leq N - 1,$$

and

$$0 \leq u \leq j - 1, \quad 0 \leq v \leq j - i - 1.$$

By analyzing the double accumulative operation expression on the right side of the (25), which is composed of the sum of $(j-1) \times (j-i-1)$ terms fraction. We observe from (25) that the probability of error varies as $1/\gamma_M$ or $1/\gamma_b$ raised to the j th power after organizing the $(j-1) \times (j-i-1)$ terms fraction. When $\gamma_M (=k\gamma_b)$ is sufficiently large (e.g. greater than 10dB), denoted as $\gamma_M \rightarrow \infty$ or $\gamma_b \rightarrow \infty$, average BER in (25) can be approximated as

$$\bar{P}_{ej}^N(\gamma_M) \propto 1/\gamma_M^j,$$

or

$$\bar{P}_{ej}^N(\gamma_b) \propto 1/\gamma_b^j.$$

The error rate decreases inversely with the j th power of the SNR. Thus, TAS subset (i, j) can get j diversity order, so diversity order is equal to ordinal number j .

IV. RESULTS AND DISCUSSION

In Rayleigh fading channel, we assume that channel fading coefficients obey independent samples of complex Gaussian random variables with a zero mean and the variance of $\sigma^2=0.5$ per dimension, and system employ BPSK, QPSK, 8PSK, and 16PSK digital modulation, respectively. The corresponding systems involve double transmit antennas and single receive antenna, $\{2;1\}$,

without AS, and both $\{3,2;1\}$ and $\{4,2;1\}$ with TAS. We use the theoretical BER expression of Chernoff upper bound (CUB) in the equation (25), and present the CUB BER performance comparison of TAS system with Alamouti encoding scheme in figure 2, figure 3, figure 4, figure 5, figure 6, and figure 7. In order to convenient comparison, in figure 2 and figure 3, we assume that the transmitter has perfect knowledge of CSI and it selects the two optimal antennas to Alamouti's space time block coding. The analytical results show the performance curves of 2 out of 3, 2 out of 4 and no selection $\{2;1\}$. In order to compare with their CUB BER performance of two transmitted antennas, performance curve of the $\{2;1\}$ system without AS also is plotted in figure 2. In the figure 2, the curves with solid line denote theoretical CUB BER performance, and the dotted lines denote Monte-Carlo (MC) computer simulation, the same below in the figure 4 and figure 6. The figure 2 shows that when the largest ordinal number of antenna selected CUB BER performance with TAS outperforms by far without TAS. In addition, we also can see in the figure 4 and figure 6 that when the ordinal number of one of the selected antennas is greater than the number of radio-frequency (RF) chains at the transmitter 2, such as $(x1x3)$ of $\{3,2;1\}$ and $(x1x3)$, $(x2x3)$ and $(x1x4)$ of $\{4,2;1\}$, the graphs show that the BER performance with TAS performs better than without TAS. For the other kinds of PSK modulation, in figure 3, the performance curves of the dot dash line, dotted line, and solid line denote QPSK, 8PSK, and 16PSK, respectively (the same below in figure 5 and figure 7). The same conclusions as AWGN channel, BER performance of high order modulation is poorer than low on both $\{3,2;1\}$ and $\{4,2;1\}$ TAS systems.

If the CSI emerges error in the transmitter, the non-optimal selection of transmitted antenna subset should be occurred in the transmitter. In figure 4, figure 5, figure 6, and figure 7, we consider that the transmitter has imperfect knowledge of CSI and it selects any two of transmitting antennas to encode. For $\{3,2;1\}$ and $\{4,2;1\}$ TAS/Alamouti scheme systems, figure 4, figure 5, figure 6 and figure 7 give respectively the BER CUB performance curves of the several selected TAS subsets for imperfect CSI and MPSK Modulation. Either $\{3,2;1\}$ or $\{4,2;1\}$, these figures show that the influences of the antenna selection subset on the performance of the TAS system are very distinctness. When the two worst antennas are selected as antenna selection subset, i.e. $(x1, x2)$ antenna pair, the performance of system is less than that of the non-antenna selection system of the same numbers of transmit antenna. When the largest ordinal number is not given in the subset, such as $(x1,x2)$ of $\{3,2;1\}$ and $(x1,x2)$, $(x1,x3)$, $(x2,x3)$ of $\{4,2;1\}$, the

performance of the system is worse. As long as the largest ordinal number is given in the subset, such as (x_1, x_3) , (x_2, x_3) of $\{3, 2; 1\}$ and (x_1, x_4) , (x_2, x_4) , (x_3, x_4) of $\{4, 2; 1\}$, the performance of the system is better. Consequently, for a practical system, when the error is existed in CSI, we ought to choose the subset consisting of largest ordinal number as transmit that, thereby the worst case is escaped, i.e. the case of ordinal number 1 and 2 are not selected in $\{3, 2; 1\}, \{4, 2; 1\}$, and even general $\{N, 2; 1\}$.

Finally, in order to validate and support our CUB theoretical analyses, for the $\{2; 1\}$ without TAS and both $\{3, 2; 1\}$ and $\{4, 2; 1\}$ with TAS, and the BPSK modulation signals are transmitted, we perform extensive computer simulation experiments with Monte-Carlo (MC) method. Same conclusions can be obtained from MC method as CUB theoretical results in the figure 2, figure 4, and figure 6, when the CSI is known, the larger the ordinal number of selected antenna in transmitter are, the better the performance of the system.

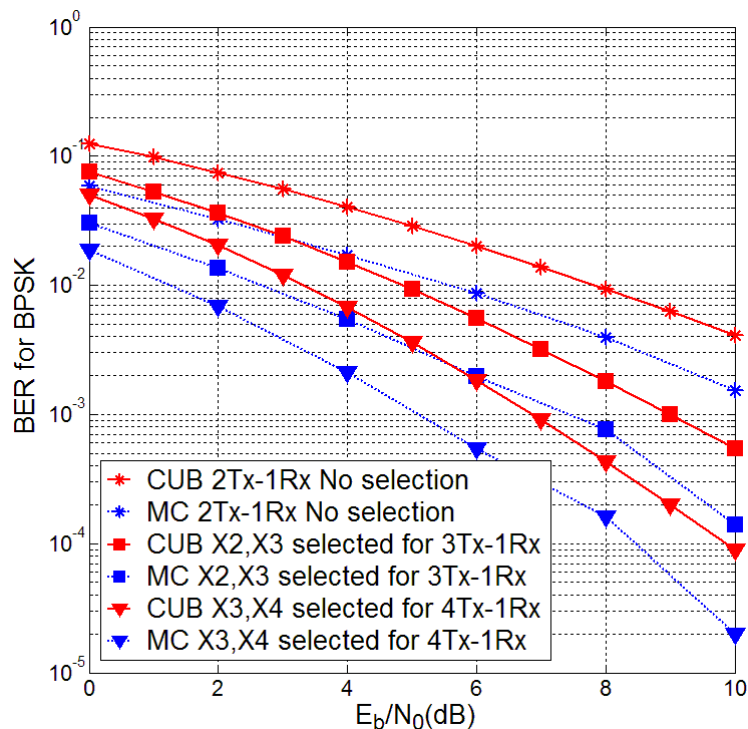


Figure 2. BER CUB comparison of perfect CSI antenna selection

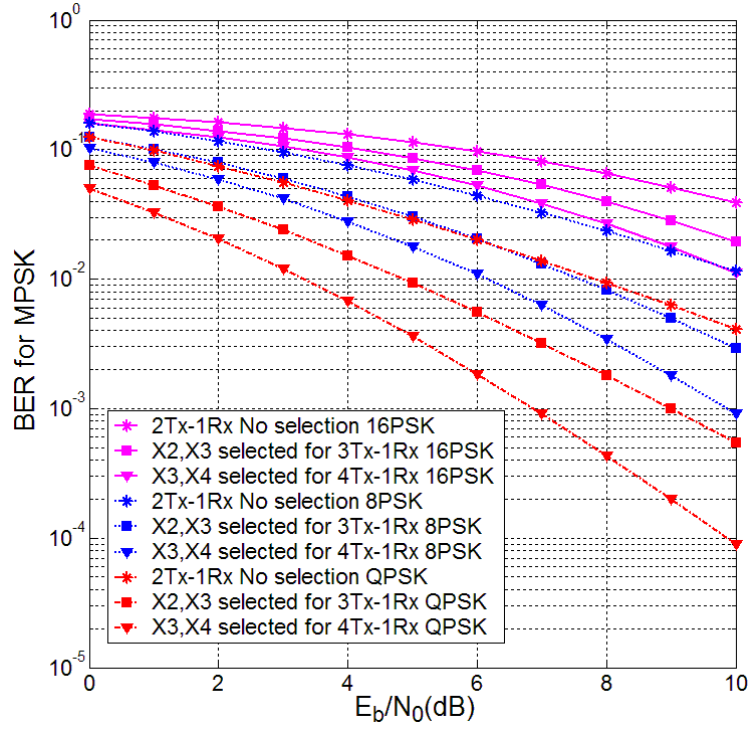


Figure 3. BER CUB comparison of perfect CSI AS for MPSK modulation

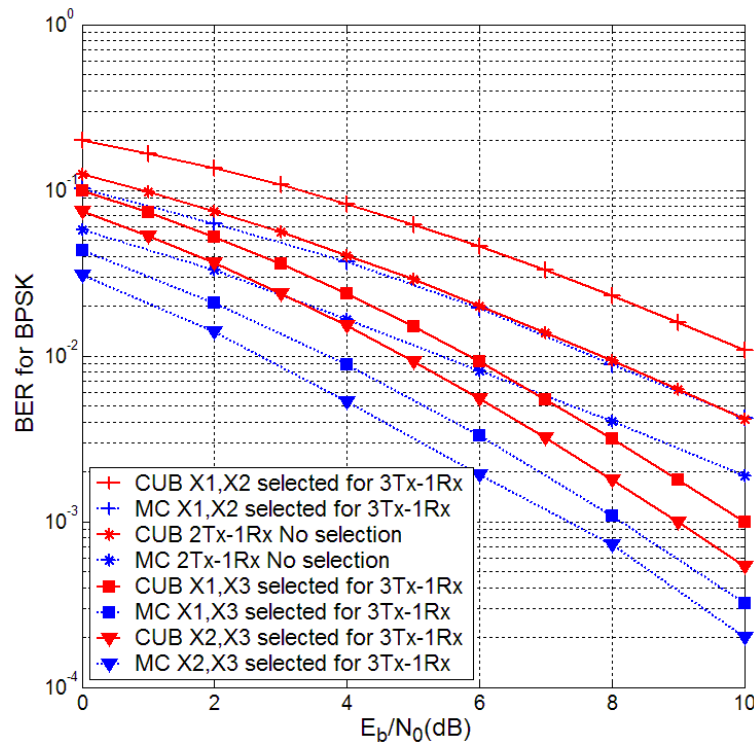


Figure 4. BER CUB comparison of {3, 2; 1} TAS/STBC system for imperfect CSI

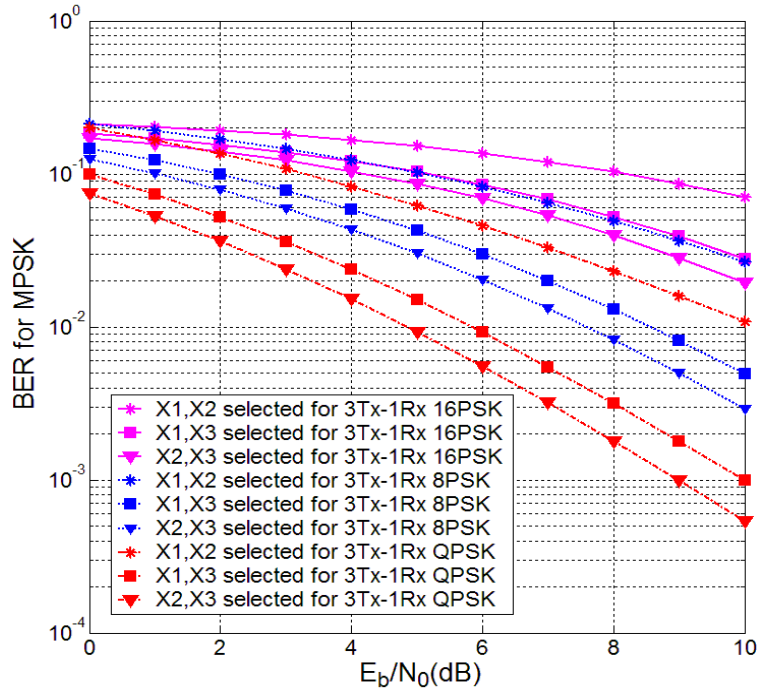


Figure 5. BER CUB comparison of {3, 2; 1} TAS/STBC system for imperfect CSI and MPSK

Modulation

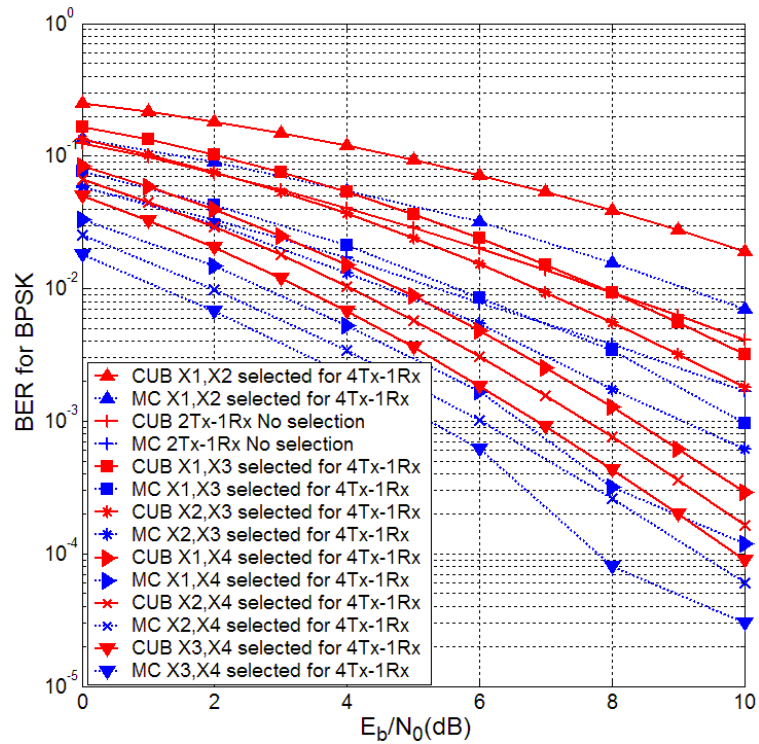


Figure 6. BER CUB comparison of {4, 2; 1} TAS/STBC system for imperfect CSI

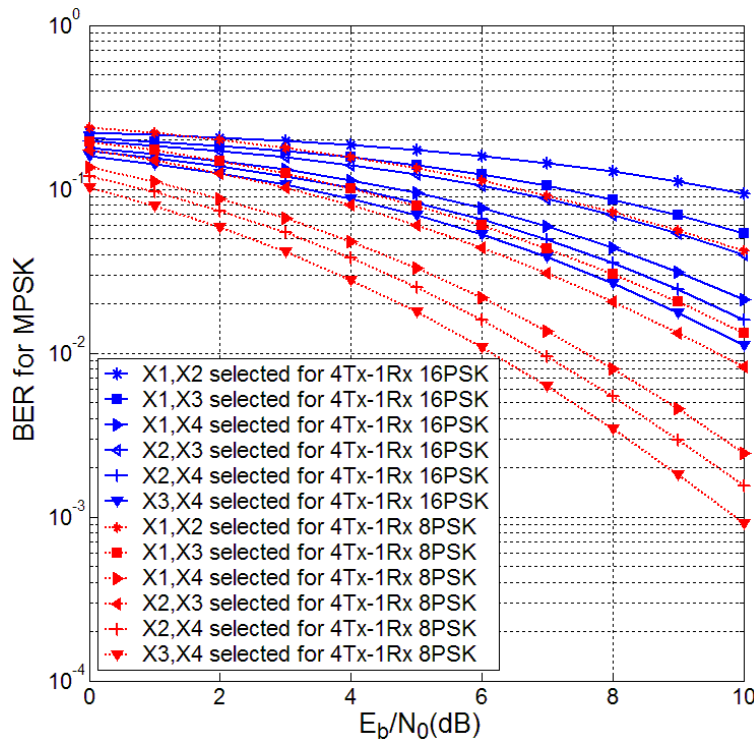


Figure 7. BER CUB comparison of $\{4, 2; 1\}$ TAS/STBC system for imperfect CSI and MPSK Modulation

V. CONCLUSIONS

Asymptotic performance, closed-form analytical expression, and Monte-Carlo simulation are commonly methods that communication systems are used to calculate error performance. However, the asymptotic method is suitable for high signal to noise ratio, Monte-Carlo simulation spends long computing time, and closed-form BER expression is too complex and physical concept is not clear. The paper introduces Chernoff upper bounds (CUB) method for $\{N, 2; 1\}$ system with arbitrary MPSK modulation, we explicitly derive a closed-form BER expression of Chernoff upper bounds applying the joint PDF of order statistics channel for the subset $\{i, j\}$. With respect to both asymptotic method and closed-form expression, the proposed method simplifies greatly the derivation process, and the result is more concise. Same as asymptotic performance, closed-form BER expression, and Monte-Carlo simulation, we make the same conclusions: the numerical results of CUB performance show that $\{N, 2; 1\}$ system can achieve full spatial diversity with antenna selection in the presence of imperfect CSI and the largest ordinal number antenna selected as if all the transmit antennas were used, these results are

promising in the sense that the diversity order is retained with antenna selection. Otherwise, numerical results indicate that there may be some of the losses in the diversity order without the best transmit antenna. Therefore, we should try to ensure the largest ordinal number antenna selected in the practical antenna selection.

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