A NEW THREE-DIMENSION SPATIAL LOCATION ALGORITHM OF WIRELESS SENSOR NETWORK

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Abstract- Aiming at three-dimensional space localization problem in wireless sensor network, a method of three-dimensional centroid algorithm with spherical perception radius (3DCSPR) is proposed. The algorithm uses Gaussian probability density function to estimate the sensor node radius, and uses tetrahedron method and three-dimensional center to locate unknown node with estimated. Firstly, RSSI value between beacon nodes and destination node is selected by RSSI distribution density function, that confidence interval of nodes perception radius is estimated. Secondly, satisfied tetrahedron is got by the intersections sphere coverage of beacon nodes in three-dimensional. Finally, destination node localization is calculated with weighted centroid in three-dimensional. Simulation results show that the proposed method can significantly reduce positioning errors in three-dimensional network environment in reality and can be widely applied to many fields.

Index terms: wireless sensor network; three-dimensional location; perception radius; centroid algorithm.
I. INTRODUCTION

Nowadays, wireless sensor network (WSN) becomes a hot research technology among short-distance communication network. It is applied in many fields such as military, industry, agriculture, environmental monitoring and others, which embodies its wide application prospects. In wireless sensor network, it plays a key role for occurred position, when circumstance information is sampled by monitoring nodes in real time. Sensor node localization problem is one of main technologies in wireless sensor network technology.

Localization in wireless sensor network is a premise problem before solving routing and topological problem. Usually in real engineering applications, sensor nodes are placed around the monitoring area in a random fashion, which are distributed in three-dimensional space, thus two-dimensional solution method cannot meet the requirements for actual. So it’s need to study the three-dimensional localization to meet the WSN application scenarios [1-3].

There are some research results in acquiring the network node localization. Result of two-dimensional localization algorithm is relatively mature. Based on distance between nodes during localization process, localization algorithm can be divided into range-based algorithm and range-free algorithm, the range-based algorithm has been widely researched and applied. Range-based algorithms includes: Received Signal Strength Indicator (RSSI), Time of Arrival (TOA), The Difference of Arrival(TDOA), Angle of Arrival(AOA) [4]. Because node has RF module itself, localization algorithm does not need to add extra hardware. The distance between nodes can be derived by the RSSI values based on RSSI range-based algorithm, which is a general way to measure distance in wireless sensor network. Alexander Wessels [5] proposed a spatial dynamic RSSI-filter to apply in dynamic indoor localization by using multi-measurement with RSSI. There would require no further additional hardware or process during transportation process. For RSSI value is affected by signal frequency, CEYLAN O [6] proposed a new range-based algorithm based on RSSI. The main idea is to transmitter different frequency electromagnetic signals at same transceiver in same distance, different RSSI values are got, and distance accuracy will be increased by analyzing the different RSSI values.

By comparing with two-dimensional localization algorithm, three-dimensional localization algorithm research is relatively less. AMK Tahan [7] proposed a cubes judging rules of three-dimensional coverage strategy, that sensors can be laid anywhere in cube to transform three-dimensional coverage model into two-dimensional plane, then transform two-dimensional
coverage model into one-dimensional. The cube covered region method just adapts to rules region application. J.Dend [8] proposed a network topology information judgment method based on direction traffic and track data gathering in the target network, then the node localization is derived by the topology information. But with the network size increasing, the package forwarding path will become very long, and store topology information space will be very large.

The paper proposes a new three-dimensional localization algorithm with measure distance based on RSSI. Probability density function is used to select the RSSI value, and the node’s perception radius can be calculated. Then node’s position is got in three-dimensional by using tetrahedron method and three-dimensional center in math. The 3DCSPR algorithm is the three-dimensional within spherical network, which is meet to the real application requirements [9].

II. ALGORITHM DESIGN

A. The probability density function of signal strength

In three-dimensional space, the localization problem can be abstracted as a three-dimensional sphere coverage issue. With the position of nodes in network, it can be ensured that the least number of nodes can be transmitted and worked, which make several nodes covered overlapping. As shown in Figure 1, the coverage of beacon nodes are sphere which composed by the beacon nodes localization as sphere’s center and the perception radius as sphere’s radius respectively. Based the sensor node’s perception area in three-dimensional, we can calculate the size of monitor scope, the number of the beacon nodes and its location, in order to getting the good utilization of nodes realization and ensuring the characteristic of full coverage with small overlap rate.

![Figure 1. Perception area in three-dimension](image)

In the application of three-dimensional environment, the map between RSSI value and the square of distance is effected randomly by the actors: terrain, neighbor nodes, noise interference and
antenna gain. But not all RSSI measurement values are help for resolving covered region, so we need to discuss the valid values first.

Based on the theory of electromagnetic wave propagation, the distance relation $d$ between receiving antenna and transmitting antenna can be derived, that uses electromagnetic wave propagation model in free space, transmit power $P_T$ and receive power $P_r$ [10]. As shown in equation (1):

$$P_r = P_T G_r G_t \left( \frac{\lambda}{4\pi d} \right)^2$$  \hspace{1cm} (1)

During the signal propagation in space, the RSSI value would change with the distance between receiving node and transmitting node. Then there is a linear relationship between the distance $d$ and the signal strength RSSI. For the signal strength path loss parameters are uncertain under different circumstances. If using the multi-sensor fusion technology and signal processing technology for environmental parameters to judge, it will result high consumption. It’s better to build the mathematical model for the network, the relationship between RSSI value and the distance $d$ can be derived based on an empirical model of multiple measurements. As shown in equation (2):

$$RSSI = -(10^n \log_{10}^d + A)$$  \hspace{1cm} (2)

Where $A$ is a RSSI value in which the destination node is 1 meter away from the beacon node, $n$ is the loss factor of path transmission [11-12]. In the different circumstance parameters, the absolute RSSI value, multi-value $A$ and loss factor $n$ are retrieved by multiple measurements, and get the value of $A$ and $n$ can minimized error by curve fitting.

In the logarithms-constant model, as shown in equation (2), the RSSI obeys the distribution of normal distribution $(0, \sigma^2)$, the Gaussian density distribution function is:

$$f(RSSI) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(RSSI-\mu)^2}{2\sigma^2}}$$  \hspace{1cm} (3)

The equation (3) is for the single node, which $\mu$ and $\sigma$ are the average value and standard deviation respectively. When there are $n$ nodes, the both sides are taking the partial derivation, and then taking $\mu$ and $\sigma$ is derivation by $\ln f(x)$, and makes the partial derivation equation equaling zero. $\mu$ and $\sigma$ can be solved as formula (4).
\[
\mu = \frac{1}{n} \sum_{i=1}^{n} RSSI_{i,0}
\]
\[
\sigma = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n-1} (RSSI_{i,0} - \mu)^2}
\] (4)

Standard normal Gaussian distribution interval is \((\mu - \sigma \leq RSSI_{i,0} \leq \mu + \sigma)\), and the probability is:
\[
p(\mu - \sigma \leq RSSI_{i,0} \leq \mu + \sigma) = F(\mu + \sigma) - F(\mu - \sigma)
= \varphi(1) - \varphi(-1) = 2\varphi(1) - 1 = 0.6826
\] (5)

Therefore, we selected the point of 0.6826 as the critical point of RSSI value. The RSSI value is probably in accordance with true measurement, when the Gaussian distribution function value is greater than or equal to 0.6826. otherwise the RSSI value is small in probability. As shown in equation (4), the mean of \(\mu\) and the variance of \(\sigma^2\) can be calculated, RSSI value can be chose in \(0.6826 \leq f(RSSI) \leq 1\).

![Gaussian density distribution function](image)

Figure 2. Gaussian density distribution function

As figure 2, the mean value \(\mu = 0\) and the variance value \(\sigma = 3\). It shows that the RSSI values obey the Gaussian distribution. So when estimating the RSSI value, we should discard the nodes with signal strength probability are less than 0.6826, that it can reduce signal strength errors resulted by noise.
B. Selection of signal strength

In order to make the correct inference, the paper assumes that:

1. \( d_{ij} \): the distance between beacon node \( i \) and destination node \( j \), which the value is \( i = 1,2,\ldots,n, \quad j = 1,2,\ldots,n \)

2. \( RSSI_{ij} \): the prefer RSSI value between beacon node \( i \) and destination node \( j \)

A mapping relationship between distance \( d_{ij} \) and the \( RSSI_{ij} \) measurement can be expressed as \( d_{ij} \leftrightarrow RSSI_{ij} \) called DR relationship. The effectively RSSI can be got between the beacon nodes and the destination node. The steps are: firstly, gets the RSSI value through multiple times measurement between the beacon nodes and the destination node; secondly, gets the effectively RSSI with the given threshold, which comes from the signal strength distribution density function. The data is shown in matrix as formula (6).

\[
\begin{pmatrix}
R_{00} & R_{01} & \ldots & R_{0n} \\
R_{10} & R_{11} & \ldots & R_{1n} \\
\vdots & \vdots & \ddots & \vdots \\
R_{m0} & R_{m1} & \ldots & R_{mn}
\end{pmatrix}
\]  

(6)

Mapping relationship matrix is a \( m \times n \) matrix which constructed from the number of beacon nodes and the times of measurement, where \( m \) is the number of beacon nodes, \( n \) is the measurement times. In DR matrix, the row represents the RSSI of same node in different time, the line represents the RSSI of different nodes at the same time. Then there is a mapping relationship matrix between distance and signal strength is formed in single-to-several. By taking the RSSI weighted average value, the reliable RSSI value of the beacon node \( i \) and the destination node \( j \) can be derived.

\[
RSSI_i = \frac{1}{m} \sum_{i=0}^{m} R_{ij}
\]  

(7)

Assumed that there are \( N \) beacon nodes to participate in getting localization, and it has derived the RSSI weighted average for each beacon node, as shown in the equation (7). Then the RSSI collection is \( RSSI_{ij} \leftrightarrow \{RSSI_1, RSSI_2, \ldots, RSSI_N\} \).
C. Estimation of perceived radius

The paper uses RSSI average weighted and three-dimensional center of mass to estimate the nodes perception radius interval of three-dimensional. The collection of RSSI is constructed by weighted average with N RSSI numbers. Hence, according to formula (8), the $d$ estimate value is:

$$\hat{d} = 10^{\frac{RSSI + A}{10^N}}$$  \hspace{1cm} (8)

To the equation (8), we make estimation values $\hat{d}_1, \hat{d}_2, ..., \hat{d}_N$ corresponding to $RSSI_1, RSSI_2, ..., RSSI_N$. Therefore, the distance errors can be derived as shown in formula (9).

$$\epsilon_d = \sqrt{(d_1 - \hat{d}_1)^2 + (d_2 - \hat{d}_2)^2 + ... + (d_N - \hat{d}_N)^2}$$  \hspace{1cm} (9)

So the beacon node interval perception distance is $R = \frac{1}{N} \sum_{i=0}^{N} \hat{d}_i \pm \epsilon_d$.

D. Tetrahedron Constitution

We can freely choose three beacon nodes with multiple iterations, and form a sphere, of which the sphere center is the localization and the sphere radius is the perception distance respectively. The main steps to get the tetrahedron constitution can be refined as follow. Firstly, it computes the intersection circle which belongs to any two of the three beacon nodes; Secondly, it computes the intersection with intersection circle and the other sphere surface. Thirdly, it forms multiple intersection points by multiple iterations, and links four intersection points randomly, there would be formed several tetrahedrons. At last, it deduces the tetrahedron’s volume, the relationship between tetrahedrons and the destination node. If both the tetrahedron volume is minimum and the destination node is inside, the tetrahedron is chose to calculate centroid.

![Figure 3. Constitution of tetrahedron](image.png)
Assuming the coordinate of destination node is \(O(x, y, z)\). The coordinates of chosen beacon nodes are \(A_1(x_1, y_1, z_1), A_2(x_2, y_2, z_2), A_3(x_3, y_3, z_3)\), and the perception distance are \(R_{a1}, R_{a2}, R_{a3}\) respectively. The algorithm processes are:

1) Circle area
The perception radius of sphere \(A_1\) and sphere \(A_2\) are \(R_{a1}^2, R_{a2}^2\) respectively. As shown in figure 3, the equation is:

\[
\begin{align*}
(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2 &= R_{a1}^2 \\
(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2 &= R_{a2}^2
\end{align*}
\]

(10)

The intersection circle plane between two spheres is \(S_2, S_3, S_1, S_4\). The circle plane equation can be obtained by sphere equation \(A_1\) minus sphere equation \(A_2\) in geometry.

\[
(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2 - (x - x_2)^2 - (y - y_2)^2 - (z - z_2)^2 = R_{a1}^2 - R_{a2}^2
\]

(11)

2) Intersection point
The problem is converted to the intersection issue of circle and sphere. According to the geometry, projection is made sphere \(A_3\) in the intersection circle plane, and circle plane equation is obtained. Then the circle equation is got from the circle plane equation, by combining circle equation \(S_2, S_3, S_1, S_4\) with the projection circle equation, the common points coordinate can be calculated. Similarly, the other intersection points can be got in the same way. Finally, the destination node localization can be derived based on centroid algorithm of the construct tetrahedron, that the construct tetrahedron is built by selecting intersections.

**E. Centroid node selection**

The key idea of the centroid algorithm is a tetrahedron centroid with the destination localization estimate interval, from the beacon nodes in communication range. We definite a certain threshold \(k\), if the destination node can receives different nodes signals exceeded \(k\), it can deduce that the communication is good between destination node and beacon nodes, then the destination node coordinate is the polygons centroid, which is constructed by the intersections of communicating beacon nodes.

\[
(x_0, y_0, z_0) = \left( \frac{\sum_{i=1}^{k} x_i}{k}, \frac{\sum_{i=1}^{k} y_i}{k}, \frac{\sum_{i=1}^{k} z_i}{k} \right)
\]

(12)
Traditionally the destination node localization is calculated by centroid algorithm, while the distance $d$ value is got by using RSSI value, which would be impact from external environment, the selection of centroid would be influenced also. The paper introduces a weight parameters to react internal relations. Some key judgment is summarized as follows.

1) whether the intersections could construct a tetrahedron

Supposing the intersections localization are $A_1(x_1, y_1, z_1), A_2(x_2, y_2, z_2), A_3(x_3, y_3, z_3), A_4(x_4, y_4, z_4)$, matrix to judge whether the intersections are coplanar is formula (13):

$$V = \begin{vmatrix}
  x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\
  x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \\
  x_4 - x_1 & y_4 - y_1 & z_4 - z_1 \\
\end{vmatrix}$$

If the matrix $V = 0$, it shows that the four intersections are coplanar. If matrix $V \neq 0$, it means that the four intersections are not coplanar, then they can form a tetrahedron.

2) whether the destination node could in the tetrahedron

Supposing intersection coordinates are $A_1(x_1, y_1, z_1), A_2(x_2, y_2, z_2), A_3(x_3, y_3, z_3), A_4(x_4, y_4, z_4)$, so the tetrahedron is:

$$V'_{1234} = \frac{1}{3!} \begin{vmatrix}
  x_1 & y_1 & z_1 & 1 \\
  x_2 & y_2 & z_2 & 1 \\
  x_3 & y_3 & z_3 & 1 \\
  x_4 & y_4 & z_4 & 1 \\
\end{vmatrix}$$

The RSSI weighted average value can be derived by the signal strength assembling between $A_1, A_2, A_3$ and destination node $o(x_o, y_o, z_o)$. Thus the distance $d_{10}, d_{20}, d_{30}$ can be estimated for each other between tetrahedron apexes and destination node.

$$\begin{align*}
  (x_1 - x_o)^2 + (y_1 - y_o)^2 + (z_1 - z_o)^2 &= d_{10} \\
  (x_2 - x_o)^2 + (y_2 - y_o)^2 + (z_2 - z_o)^2 &= d_{20} \\
  (x_3 - x_o)^2 + (y_3 - y_o)^2 + (z_3 - z_o)^2 &= d_{30}
\end{align*}$$

From equation (15), there are $O_1(x_o, y_o, z_o), O_2(x_b, y_b, z_b)$, and the distance between $A_4$ and $O_1, O_2$ can be derived by the equation (16) respectively.
\[
\begin{align*}
\frac{d(O_1, A_4) = \sqrt{(x_4 - x_a)^2 + (y_4 - y_a)^2 + (z_4 - z_a)^2}}{d(O_2, A_4) = \sqrt{(x_4 - x_b)^2 + (y_4 - y_b)^2 + (z_4 - z_b)^2}}
\end{align*}
\] (16)

By comparing the value between \(d_{40} - d(O_1, A_4)\) and \(d_{40} - d(O_2, A_4)\), the minimum is selected as \(O_{123}\). Similarly, coordinates \(O_{123}, O_{124}, O_{134}\) and \(O_{234}\) can be calculated. Then the nearest in \(O_{123}, O_{124}, O_{134}\) and \(O_{234}\) can be regarded as the approximate destination node \(O(x_i, y_i, z_i)\). Finally the relationship between the approximate destination node and the tetrahedron is judged by the theory of tetrahedral nature.

\[
V_{1234} = \frac{1}{3!} \begin{vmatrix} x_i & y_i & z_i & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_4 & y_4 & z_4 & 1 \end{vmatrix}, \quad V_{134} = \frac{1}{3!} \begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_4 & y_4 & z_4 & 1 \\ x_i & y_i & z_i & 1 \end{vmatrix}, \quad V_{124} = \frac{1}{3!} \begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_4 & y_4 & z_4 & 1 \\ x_i & y_i & z_i & 1 \end{vmatrix}, \quad V_{134} = \frac{1}{3!} \begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_i & y_i & z_i & 1 \end{vmatrix}.
\] (17)

If the results of \(V_{1234}, V_{1234}, V_{134}, V_{124}, V_{134}\) are in the same direction, it indicates destination node is in the tetrahedron, which is constructed by the beacon nodes \(A_1, A_2, A_3, A_4\).

**F. Coverage area in three-dimensional destination node**

For RSSI value would change with transmission distance. When RSSI value decreasing, it contributes would reduce. So the weighting factor \(r_i\) is inversely to distance, as \(r_i = \frac{1}{d_i}\). The destination node coordinate calculated formula is (18). Where \((x_i, y_i, z_i)\) is tetrahedron vertexes coordinate.

\[
(x_0, y_0, z_0) = \left(\frac{\sum_{i=1}^{4} r_i x_i}{4}, \frac{\sum_{i=1}^{4} r_i y_i}{4}, \frac{\sum_{i=1}^{4} r_i z_i}{4}\right)
\] (18)
III. EXPERIMENT

In this section we analyze the performance of our proposed algorithm. Our localization algorithm is evaluated by comparing the errors, which estimate the destination node localization with different percentage of beacon nodes.

A. Experimental equipment

1) A portable computer
2) The wireless network nodes of TEXAS INSTRUMENTS (34 nodes)
   a. 8-bit low power, 8051 CPU Core and temperature & humidity sensor are integrated in nodes.
   b. cc2530, 2.4GHz and Zigbee RF transceiver are integrated.
   c. the core flash is up to 256Kb, and the SRAM is 8Kb.
   d. operating VCC range: 2V to 3.6V.
   e. operating temperature range: -40 to 125.
3) USB to serial port converter wire (1 line)

![Figure 4. The wireless sensor node](image)

B. Experiment circumstance

The experiment environment is aisle, the farthest beacon node is 5m from destination node, and others are dispersed around the destination node interval 0.5m increment. The beacon node coordinates are (1, 3, 1.2), (2, 0, 24.45), (3, 3.5, 16.95), (12, 4, 8.7), (5, 7, 7.2), (2, 7.5, 6.2), (5, 7, 8.2), (3.5, 5, 7.2), (4, 6, 7.2), and the environment noise is neglected. As shown in figure 4, the red star represents the destination node localization in three-dimensional, the blue circle represents the beacon nodes localization in three-dimensional experiment. We test the RSSI values of destination
node from ten beacon nodes and get map relationship between RSSI value and distance \( d \). The sampling times are 5 of 2s for each node. DR map relationship and the distance estimation error is as shown in tables. The experiment is estimating 3DCSPR algorithm in different beacon nodes, which includes distance estimation errors and localization estimation errors.

![Figure 5. Localization of beacon nodes in simulation](image)

**C. Experiment**

For wireless sensor network has remarkable features as small size, low cost, and large scale scenario. Usually in network deployment, sensor node uses limit energy batteries for power supply, busy node dies easily with lack of energy, it needs to provide a certain number of beacon nodes. Beacon node is a feasible way to gauge performance of destination node localization algorithm. During localization algorithm design, it is necessary to get a high accuracy degree with fewer beacon nodes. Therefore, we set transmit packets as 30 \%, 40\% and 50\% respectively in the network, we make an distance estimation experiment and localization errors experiment with different number of beacon nodes in network.

The distance between beacon node and destination node is calculated by RSSI weighted average method. \( d \) estimate value is \( \hat{d} = 10^{\frac{RSSI + A}{10^n}} \), RSSI is signal weighted average value in the \( d \) distance, \( A \) and \( n \) are circumstance parameters. From the experiment data, the environmental parameters can be calculated by \( A = 25, n = 2.25 \).

Destination node coordinate is calculated by centroid in three-dimensional and tetrahedron method, and its distance estimation errors is derived by the destination node’s real localization, Localization
distance errors formula is defined as \( \delta = \sqrt{(x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2} \), truth destination node localization is \((x,y,z)\), estimated destination node coordinate is \((x_i,y_i,z_i)\), \(d\) represents the distance between beacon nodes and destination node.

1) beacon nodes are 30%

a. distance estimation errors

<table>
<thead>
<tr>
<th>Distance d</th>
<th>Measurement value of RSSI</th>
<th>Estimation errors of distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>-19, -18.8, -18.5, -18, -18.3</td>
<td>-0.02</td>
</tr>
<tr>
<td>1</td>
<td>-21.7, -22, -21.8, -22, -21.7</td>
<td>-0.2</td>
</tr>
<tr>
<td>1.5</td>
<td>-29, -30.5, -30.2, -30, -29.5</td>
<td>0.08</td>
</tr>
<tr>
<td>2</td>
<td>-35, -34, -32, -33.2, -33</td>
<td>0.3</td>
</tr>
<tr>
<td>2.5</td>
<td>-35, -34.7, -34.5, -34.9, -35</td>
<td>0.19</td>
</tr>
<tr>
<td>3</td>
<td>-37.2, -37.3, -37.1, -37, -37</td>
<td>0.46</td>
</tr>
<tr>
<td>3.5</td>
<td>-37.5, -37.8, -37.2, -38, -37</td>
<td>-0.04</td>
</tr>
<tr>
<td>4</td>
<td>-40, -39, -39.5, -39.3, -39.8</td>
<td>0.3</td>
</tr>
<tr>
<td>4.5</td>
<td>-40.2, -39, -39.9, -40, -39.9</td>
<td>0.07</td>
</tr>
<tr>
<td>5</td>
<td>-42, -42.2, -41.6, -42, -41.8</td>
<td>0.6</td>
</tr>
</tbody>
</table>

In table 1, the left column are actual values between destination node and beacon nodes, the middle column are RSSI measurement values, and right column are distance errors.

When the beacon nodes are 30%, we can see the estimation errors are slightly difference in -0.2m ~ + 0.6m, And from above formula (4), the average of RSSI values are

\[
\mu \leftrightarrow \{-18.52, -21.84, -29.84, -33.44, -34.82, -37.12, -37.5, -39.52, -39.8, -41.92\},
\]

From the formula (3), it shows that the measured RSSI values meet the threshold probability. Thus RSSI weighted average is

\[
\text{RSSI} \leftrightarrow \{-18.52, -21.84, -29.84, -33.44, -34.82, -37.12, -37.5, -39.52, -39.8, -41.92\} .
\]

The estimation errors of distance simulation as shown in figure 7.
In figure 6, the red star point represents destination node localization estimation errors in three-dimensional, the blue one represents true localization estimation errors. We can get that the distance estimation error is increasing with the distance, which as the relationship between RSSI and distance in formula (7).

b. localization errors

Table 2. The result of distance $d$ and localization errors within 5m (beacon nodes are 30%)

<table>
<thead>
<tr>
<th>Distance $d$</th>
<th>Estimation errors of localization</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.178</td>
</tr>
<tr>
<td>1</td>
<td>0.301</td>
</tr>
<tr>
<td>1.5</td>
<td>0.256</td>
</tr>
<tr>
<td>2</td>
<td>0.315</td>
</tr>
<tr>
<td>2.5</td>
<td>0.315</td>
</tr>
<tr>
<td>3</td>
<td>0.372</td>
</tr>
<tr>
<td>3.5</td>
<td>0.369</td>
</tr>
<tr>
<td>4</td>
<td>0.432</td>
</tr>
<tr>
<td>4.5</td>
<td>0.401</td>
</tr>
<tr>
<td>5</td>
<td>0.413</td>
</tr>
</tbody>
</table>

The left column is actual value between destination node and beacon node, the right column is localization errors. In table 2, the localization errors are theoretical errors. Making the localization algorithm for the node which distance is 3m in randomly, and the simulation as shown in figure 8.
As we can see from the figures, the estimation error interval of localization estimation errors and truth localization errors are reduce not obviously.

2) beacon nodes are 40%

a. distance estimation errors

Table 3. Test result of $d$ and RSSI within 5m (beacon nodes are 40%)

<table>
<thead>
<tr>
<th>Distance $d$</th>
<th>The measurement of RSSI</th>
<th>The estimation errors of distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>-18, -18.5, -18, -19, -19.2</td>
<td>0.02</td>
</tr>
<tr>
<td>1</td>
<td>-20, -22, -21.5, -20.5, -28</td>
<td>-0.2</td>
</tr>
<tr>
<td>1.5</td>
<td>-35, -25, -24, -32, -32.5</td>
<td>0.08</td>
</tr>
<tr>
<td>2</td>
<td>-35, -30, -34.5, -33, -32.5</td>
<td>0.01</td>
</tr>
<tr>
<td>2.5</td>
<td>-34, -35, -34.5, -34.8, -35</td>
<td>0.01</td>
</tr>
<tr>
<td>3</td>
<td>-35, -36.8, -36.6, -37, -37.2</td>
<td>0.2</td>
</tr>
<tr>
<td>3.5</td>
<td>-37.6, -37.9, -33.3, -38.6, -38.3</td>
<td>-0.05</td>
</tr>
<tr>
<td>4</td>
<td>-42, -41, -41.5, -35, -35</td>
<td>0.07</td>
</tr>
<tr>
<td>4.5</td>
<td>-42.4, -41.3, -41.5, -36, -38</td>
<td>0.07</td>
</tr>
<tr>
<td>5</td>
<td>-42, -39.5, -40.8, -41.3, -43</td>
<td>0.2</td>
</tr>
</tbody>
</table>

In table 3, the left column are actual values between destination node and beacon nodes, the middle column are RSSI measurement values, and right column are distance errors. When the beacon nodes are 40%, we can see the estimation errors are slightly difference in -0.2m ~ + 0.2m, And from above formula (4), the average of RSSI values are
\( \mu \leftrightarrow \{-18.54, -22.4, -29.7, -33, -34.66, -36.52, -37.14, -38.9, -39.84, -41.32\} \), From the formula (3) it shows that the measured RSSI values meet the threshold probability. Thus RSSI weighted average is 

\[ \text{RSSI} \leftrightarrow \{-18.54, -22.4, -29.7, -33, -34.66, -36.52, -37.14, -38.9, -39.84, -41.32\} \). The estimation errors of distance simulation as shown in figure 9.

![Distance and localization errors](image)

**Figure 8. Distance and localization errors (beacon nodes are 40%)**

In figure 8, the red star represents the destination node localization estimation errors in three-dimensional, the blue represents the true localization estimation errors. From the figure 9, we can get that the distance estimation error is changed not obviously with the distance.

**b. Localization errors**

**Table 4. Result of \( d \) and localization errors within 5m (beacon nodes are 40%)**

<table>
<thead>
<tr>
<th>Distance ( d )</th>
<th>Estimation errors of localization</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.176</td>
</tr>
<tr>
<td>1</td>
<td>0.219</td>
</tr>
<tr>
<td>1.5</td>
<td>0.259</td>
</tr>
<tr>
<td>2</td>
<td>0.295</td>
</tr>
<tr>
<td>2.5</td>
<td>0.327</td>
</tr>
<tr>
<td>3</td>
<td>0.364</td>
</tr>
<tr>
<td>3.5</td>
<td>0.36</td>
</tr>
<tr>
<td>4</td>
<td>0.406</td>
</tr>
<tr>
<td>4.5</td>
<td>0.401</td>
</tr>
<tr>
<td>5</td>
<td>0.396</td>
</tr>
</tbody>
</table>
The left column is actual values between destination node and beacon nodes, the right column is localization errors. We make the localization algorithm for the node which distance is 3m in randomly, and the simulation as shown in figure 9.

![Figure 9. Sphere coverage of destination node (beacon nodes are 40%)](image)

As we can see from the figure 9, the estimation error is reduced between localization estimation and truth localization with the higher repetition. When beacon nodes are 40%, the localization estimation error is reduced as few as possible. Therefore, beacon nodes are 40% in the networks, which would be a significant number to get the destination node localization.

3) beacon nodes are 50%

a. distance estimation errors

<table>
<thead>
<tr>
<th>Distance $d$</th>
<th>Measurement value of RSSI</th>
<th>Estimation errors of distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>-18, -18.2, -17, -18.9, -20</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>-21, -21.8, -22, -21.7, -22</td>
<td>0.3</td>
</tr>
<tr>
<td>1.5</td>
<td>-30, -28, -27, -27.9, -29</td>
<td>0.1</td>
</tr>
<tr>
<td>2</td>
<td>-32, -32.8, -33.2, -35, -33</td>
<td>0.25</td>
</tr>
<tr>
<td>2.5</td>
<td>-34, -34.8, -34.4, -35.2, -36</td>
<td>0.29</td>
</tr>
<tr>
<td>3</td>
<td>-37, -36.8, -37.2, -37, -37.1</td>
<td>0.1</td>
</tr>
<tr>
<td>3.5</td>
<td>-38, -37.2, -36, -37.4, -37.8</td>
<td>0.05</td>
</tr>
<tr>
<td>4</td>
<td>-38.7, -39.2, -39.1, -40, -38.1</td>
<td>-0.1</td>
</tr>
<tr>
<td>4.5</td>
<td>-40.4, -39.1, -40.2, -40, -39.9</td>
<td>0.07</td>
</tr>
<tr>
<td>5</td>
<td>-42, -41.8, -42.1, -41.8, -40</td>
<td>0.3</td>
</tr>
</tbody>
</table>
In table 5, the left column are actual values between destination node and beacon nodes, the middle column are RSSI measurement values, and right column are distance errors. When the beacon nodes are 50%, we can see the estimation errors are slightly difference in -0.1m ~ + 0.3m, And from above formula (4), the average of RSSI values are

\[ \mu \leftrightarrow \{-18.42, -21.7, -28.38, -33.2, -34.88, -37.02, -37.28, -39.02, -39.92, -41.54\} \]

From the formula (3), it shows that the measured RSSI values meet the threshold probability. Thus RSSI weighted average is

\[ \mu \leftrightarrow \{-18.42, -21.7, -28.38, -33.2, -34.88, -37.02, -37.28, -39.02, -39.92, -41.54\} \]

The estimation errors of distance simulation as shown in figure 10.

![Distance and localization errors](image)

**Figure 10. Distance and localization errors (beacon nodes are 50%)**

In figure 11, the red star represents the destination node localization estimation errors in three-dimensional, the blue represents the true localization estimation errors. Compared with the figure 7 and figure 9, we can get that the distance estimation error is changed gently with the distance.

b. the localization errors

<table>
<thead>
<tr>
<th>Distance (d)</th>
<th>The estimation errors of localization</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.179</td>
</tr>
<tr>
<td>1</td>
<td>0.219</td>
</tr>
<tr>
<td>1.5</td>
<td>0.259</td>
</tr>
<tr>
<td>2</td>
<td>0.315</td>
</tr>
<tr>
<td>2.5</td>
<td>0.357</td>
</tr>
<tr>
<td>3</td>
<td>0.424</td>
</tr>
<tr>
<td>3.5</td>
<td>0.362</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>------</td>
</tr>
<tr>
<td>4</td>
<td>0.412</td>
</tr>
<tr>
<td>4.5</td>
<td>0.401</td>
</tr>
<tr>
<td>5</td>
<td>0.406</td>
</tr>
</tbody>
</table>

The left column is actual values between destination node and beacon nodes, the right column is localization errors. We make the localization algorithm for the node which distance is 3m in randomly, and the simulation result is as shown in figure 11.

Figure 11. Sphere coverage of destination node (beacon nodes are 50%)

In figure 11, estimation localization and the truth localization error is small. While comparing with figure 10, the estimation error interval of localization would not reduce obviously, then the beacon nodes are 50%, which is not the best choice to estimate destination node localization.

**D. Complexity analysis**

As we can see from the figures by comparing, in three-dimensional space when packets transmitted more, the communication overhead is higher and the error is smaller. While when transfer data packets reach a certain number, the estimation error interval of distance would not reduce obviously. Moreover beacon nodes have much functions and high power consumption, with the number of beacon nodes increasing, total costs of network would be higher. Therefore, it needs to select a certain transmit packages and nodes number to get the destination node localization based on the environment parameters. In the experiment, beacon nodes are 40% of total nodes number is help to reduce unnecessary communication overhead and localization estimation.
By making Comprehensive analysis of the above situation, a summary can be got of beacon nodes selection method:
(1) Gently changed of distance estimation error can reduce accuracy of error estimation. When distance estimation error value is changing very small between beacon node and destination node (errors change gently), estimation of localization errors would reduce based on 3DCSPR, as shown in table 3 and figure 10.
(2) It’s uncertain that more beacon nodes number is help for getting accuracy localization. In three-dimensional space, the more the beacon nodes are, the more the data computational need, but estimation error interval would not reduce obviously. Therefore, it’s better to select a certain beacon node number to compute destination node localization based on the environment parameters.

IV. CONCLUSIONS

In this paper, 3DCSPR algorithm is proposed based on perception radius model and centroid algorithm principle, The simulation results show that the algorithm can get node localization in three-dimension effectively, which based on the weighted radius perception and centroid algorithm. Finally the experiment is tested to verify validity. In practical application when adopted 3DCSPR algorithm to handle node localization problem, it needs more data computational and workload, so how to improve work efficiency and reduce operation cost is still the focus of future research work. This work was supported by Research Foundation of Xi’an Science Technology Bureau of China (Grant No. CG201578), and supported by key science and technology program of Shaanxi province of China (Grant No. 2015GY041).

V. REFERENCES


